

ELASTOPLASTIC INTERPRETATION OF RESULTS OF THE  
FRACTURE MIXED MODES UNDER CYCLIC BIAXIAL LOAD

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New gradient characteristic of elastoplastic state having a physical sense of structurally-responsive parameter of material is introduced. Its influence on elastoplastic stress and strain fields is taken into account when formulating the model of prediction of crack growth rate for mixed mode fracture. A comparison of the model suggested with experimental data concerning to crack growth rate in aluminium alloys under biaxial load of arbitrary direction is presented.

INTRODUCTION

Modern models of prediction of crack propagation are based on combination of both the Manson-Coffin type equation of low cycle fatigue and the Paris type equation. Above equations, as a rule, comprise parameter connected with a material micro-structure. However, the proper evaluation of its influence on elastoplastic stress and strain fields in region around the crack tip has not been carried out. With a help of this structurally-responsive parameter of material one can give an elastoplastic interpretation of results of crack growth rate for mixed mode fracture on the base of small scale yielding model. In given paper one introduces the new gradient parameter of material state, evaluates its influence on elastoplastic fields and formulates model of prediction of crack propagation.

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GRADIENT CHARACTERISTICS OF MATERIAL

It is known that HRR-solution (by Hutchinson (1) and Rice and Rosengren (2)) describes well the boundary-value problem with the corresponding boundary conditions. One of problems is to determine above mentioned boundary conditions for mixed problems of nonlinear crack mechanics. For the plane stress HRR-solution has the following peculiarity. In the known point of  $\Theta$ -distribution the situation of physical field infinite gradients takes place due to the discontinuity of higher order derivative. This situation results in necessity of artificial and ungrounded transfer over the region adjacent to this point. So far as the real materials have the finite gradient properties, we have introduced criterial characteristics as follows

$$G_r = \max \left[ \frac{d}{dr} \tilde{\sigma}_{ij} \right]_{r=r_c} \quad (1)$$

in which  $r_c$  is the structurally-responsive parameter of material. Thus, to set the limiting gradients in  $\Theta$ -distribution of dimensionless stress tensor components we propose to use the criterial value determined in orthogonal direction. Elastoplastic singularity mode stipulating for the value of  $G_r$  has a sense of limiting gradient due to the fact that the stress concentration in region around the crack tip has a limiting character for each concrete problem. Therefore, condition which has to be satisfied when solving symmetric and antisymmetric problems of nonlinear crack mechanics is as follows:

$$G_\theta = \frac{d}{d\theta} \tilde{\sigma}_{ij} \leq G_r \quad (2)$$

Condition (2) permits to use criterion introduced in dimensionless form, moreover its value is determined only by the material properties  $n$  and  $r_c$  ( $n$  is the strain hardening exponent). In fig. 1 there are presented  $\Theta$ -distributions of elastoplastic stress intensity obtained taking into account (2) against the criterial value (1). From these data it follows that for  $r \leq 0.0024$  the stress  $\Theta$ -distribution coincides with HRR-solution. In all other cases distinction in results is directly proportional to  $r_c$  growth. Thus, it is shown the influence of structurally-responsive parameter on elastoplastic state of material in region around the crack tip. Corresponding values of  $r_c$  are ta-

ken into account when interpreting the results of crack growth rate for mixed mode fracture of aluminium alloys under uni- and biaxial tension.

MODEL OF PREDICTION OF CRACK GROWTH  
RATE FOR MIXED MODE FRACTURE

Let us assume that process of crack propagation is connected with material strain characteristics. Those are both the static or cyclic margin of plasticity  $\epsilon_f$  and the plastic zone size  $r_p$  (or the parameter of material micro-structure  $r_c$ ). In series of works there is the connection between the low cycle fatigue and crack propagation rate characteristics describing by the Manson-coffin and Paris equations respectively. Similar results being generalized one can write as follows

$$\frac{da}{dN} = K^* \epsilon_{if}^{\beta} K_{i,max}^m \quad (3)$$

in which  $\epsilon_{if}$  is the intensity of plasticity margin;  $k^*, m$  are the Paris constants and  $\beta$  is the Manson-Coffin exponent. So far as  $\epsilon_{if}$  depends on the stressed state mode, then the crack propagation rate for biaxial load of arbitrary direction also depends on the available plasticity in the crack propagation direction. Thus, the crack growth rate under an arbitrary biaxial load (analogously to by Ahmad (3) and Brown and Miller (4)) may be expressed by the corresponding parameter of some fixed stressed state

$$\left[ \frac{da}{dN} \right]_{\eta} = K_0^* \left[ \frac{\epsilon_{if\eta}}{\epsilon_{if0}} \right]^{\frac{1}{\beta_0}} K_{i,max}^{m_0} \quad (4)$$

Here index "0" concerns to the base experiment, the uniaxial symmetric or equallybiaxial tension being considered as the latter. Relation  $(\epsilon_{if\eta} / \epsilon_{if0})$  we determine as by Braude (5) that results in by Shlyannikov and Braude (6)

$$\left( \frac{da}{dN} \right)_{\eta} = K_o^* \left\{ \frac{\sqrt{1-\lambda_o + \lambda_o^2} [A(\lambda_o) + \gamma B(\lambda_o)]}{\sqrt{1-\lambda + \lambda^2} [A(\lambda) + \gamma B(\lambda)]} \right\}^{\frac{n}{\beta_o}} K_{I, \max}^{m_o} (5)$$

in which  $\gamma$  is a constant of material determining by the results of cyclic tests;  $\lambda$  is relation of main elastoplastic stresses in the crack propagation direction determining by  $\theta^*$ ; fig.2 depends on the angle of crack initial orientation  $\lambda-\alpha$  for the different levels of nominal stresses  $\bar{\sigma}_y^o$  and the hardening exponent  $n$ . The formula (5) suggested considers the material properties ( $\gamma, n, \beta$ ); the level of acting stresses  $\bar{\sigma}_y^o$  and the local stress-strain state characteristics ( $\lambda, -\theta^*, \epsilon_f, r_c$ ). Calculation of crack propagation rate for mixed mode fracture under biaxial tension by the equation (5) includes the successive definition of crack propagation direction  $-\theta^*$  with an according value of  $r_p$  or  $r_c$ ; the value  $\lambda$  as a function of  $r_p$  and  $-\theta^*$  and also the crack trajectory for given  $\gamma, \alpha, n, \bar{\sigma}_y^o$ . A comparison of computational and experimental data concerning to crack propagation rate under biaxial tension ( $\gamma=0.5$ ) for mixed mode fracture ( $\alpha=90^\circ, 45^\circ, 25^\circ$ ) of aluminium alloys is illustrated in fig. 3. Above alloys properties are presented in Table I. From result presented it follows that the

TABLE I - Mechanical characteristics of materials.

	$\bar{\sigma}_{Ts}$ (MPa)	$\bar{\sigma}_{ys}$ (MPa)	Paris' constants $K_o^*$ (MPa m)	$m_o$	Ramberg-Osgud constant $n$
Al.alloy 1	445	310	11.405	3.23	9.68
Al.alloy 2	439	284	8.675	2.67	4.18

suggested method is correct for elastoplastic interpretation of results for mixed mode of biaxial cyclic load. Above presented step procedure for computation of angled crack growth rate by formula (5) was realized on the personal computer. The initial data for computation are the numerical values of  $\bar{\sigma}_y^o, n, \gamma, \alpha, \mu,$

$\bar{\epsilon}_i$  and region of  $\alpha$  change. Computational results are presented as a dependence between the crack growth rate  $da/dN$  and the stress intensity factor  $k$ . In similar cases for the structure mixed modes one generally uses some equivalent value  $K$ , which is the function of  $K_1$  and  $K_2$ . We have accepted that in the main computational equation (5) the exponent  $m$ , corresponding either to uniaxial symmetrical tension or to equibiaxial tension, is constant for all the versions of mixed load modes. This fact means that the diagrams of crack growth rate are equidistantly displaced about base curve ( $K_1^*, m_0$ ) of the given material. Therefore one can not use in equation (5) the equivalent value of  $K$ , as the presence of  $K_2$  results in rotation of the crack growth rate computational curve, which deviates from the framework of the model suggested definition. Proceeding from above reasons we consider that at  $m_0 = 0$  = const the usage of the only  $K_1$  for the given material is correct. Over the calculation of the current values of stress intensity factor  $K_{I,max}$  we have taken into account the  $K$ -taring functions of specific specimen, which are in their turn obtained numerically with a help of FEM.

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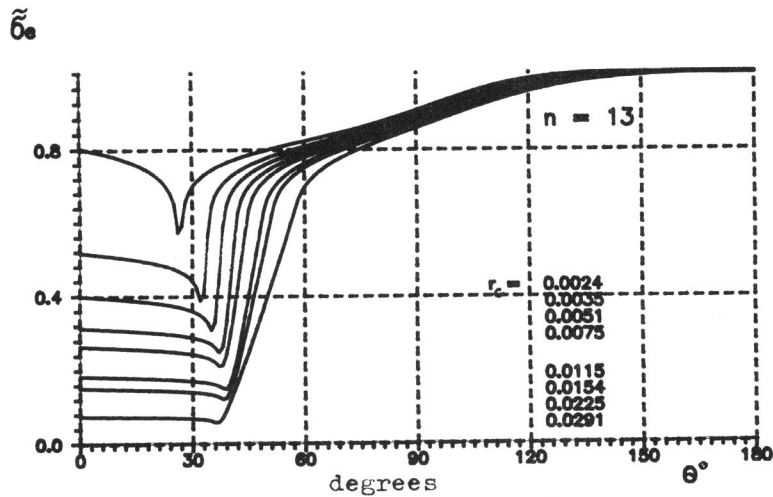


Figure 1 Dimensionless  $\theta$ -distributions of elastic stress intensity around the crack tip for different  $r_c$ .

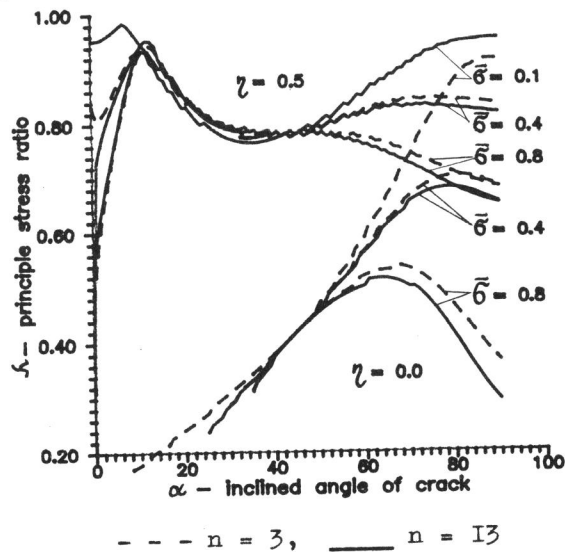


Figure 2 Relationship between  $\lambda$  and  $\alpha$  for different biaxial stress ratio  $\zeta$  and applied stress  $\bar{\sigma}$

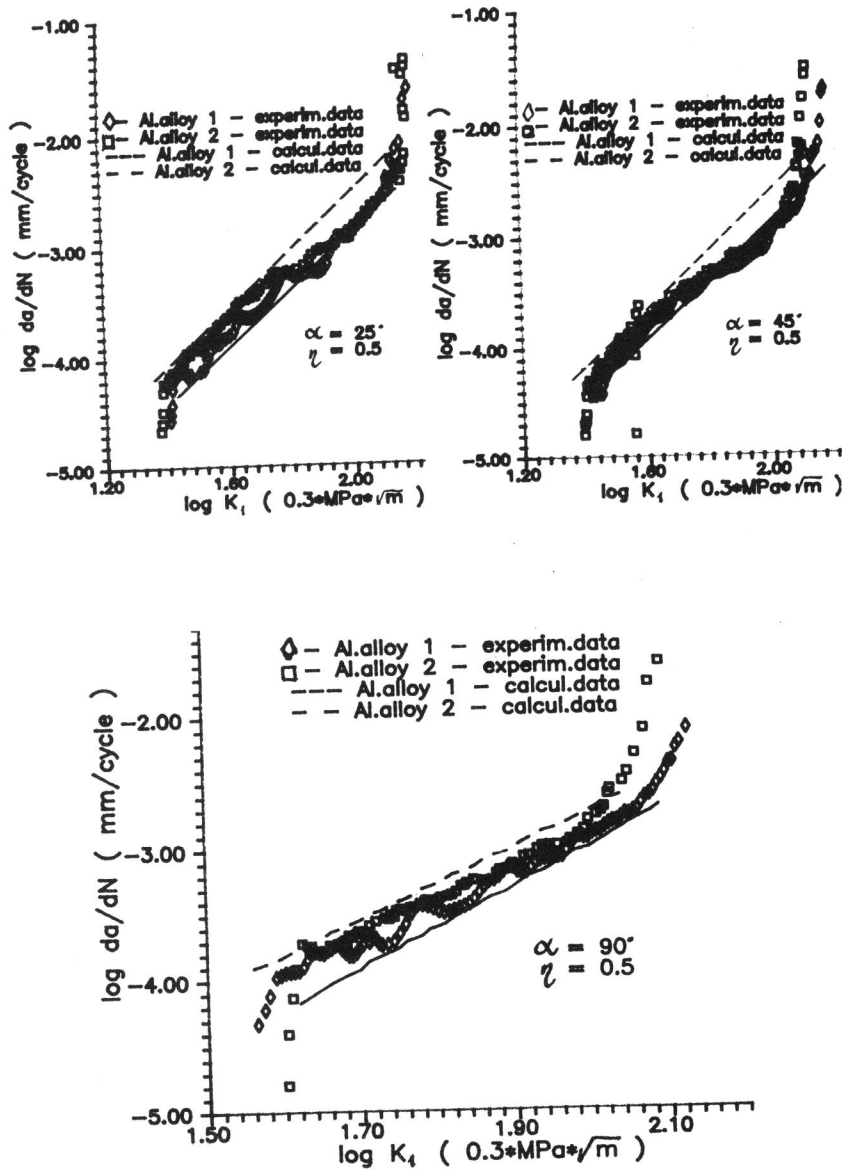


Figure 3 Comparison between experimental and computational data of crack growth rate under biaxial load