ON NON-LINEAR EFFECTS IN EDGE-CRACKED TENSION SPECIMENS

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A geometrically non-linear effect in fracture testing of unsymmetrically cracked slender elements is presented. This problem is studied by using the finite element method to show that actual stress intensity factor is smaller than the value which can be derived from external loads when displacements and rotations are supposed small. A simple method for estimating this non-linear effect is proposed.

INTRODUCTION

The application of fracture mechanics to thin cylinders such as high-strength prestressing steel wires has mainly been focused on the consideration of transverse surface cracks either on mode I (Astiz (1), Valiente (2)) or on mixed mode (Astiz et al. (3)). Testing of such thin wires (5 to 7 mm) makes it very difficult to use normalized specimens although some attempts have been achieved (Valiente et al. (4), Astiz et al. (5)). Nevertheless, the most reasonable approach consists in testing the pre-cracked wire in tension since this will be the actual situation during its service life.

Testing in tension such slender elements with edge cracks may result in non negligible lateral displacements which may modify substantially the stress distribution and the computed stress intensity factors. This geometrically non-linear effect may not only appear in thin wires but also wherever a slender cracked structural element is being tested.

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The object os this paper consists in producing some results to quantify this effect by means of full finite element computations as well as to introduce a simple method to evaluate lateral displacements for slender edge cracked specimens.

FEM ANALYSIS

The structural element which has been adopted for evaluation of the outlined problem is a plane rectangular strip with an edge crack, half-width deep (figure 1) and in plane strain conditions. The finite element mesh is made of quadratic elements (8 node quadrilateral and 6 node triangular); quarter-point elements (Barsoum (6), Henshell and Shaw (7)) were used around the crack tip. The NONSAP program (Bathe et.al (8)) with a total Lagrangian formulation was used to study the non-linear behaviour of the specimen.

Two different boundary conditions were considered: the doubly pin-jointed beam (uniform tension stress) and the doubly built-in beam (uniform end displacement) (figure 1). This choice covers most of the cases which may be encountered in a testing machine. The range of slenderness ratios (h/w) which has been considered is defined by the interval [3,20]. Two analyses (one linear and one non linear) were carried out for each specimen to compute the ratio of the stress intensity factors K_I^{NL}/K_I^L as a funtion of the applied stress. Results are summarized in figure 2 for the built-in beam and in figure 3 for the pin-jointed beam.

By looking at both figures one can conclude that the non-linear effect tends to reduce the stress intensity factor (up to 30% in the cases which have been analyzed). This fact is positive for the structural element since by computing the stress intensity factor after neglecting the non-linear effect we would overestimate this figure and we would be on the safe side. Nevertheless, if we use standard formulas in a laboratory test we would also overestimate the critical stress intensity factor and we would then be on the unsafe side.

Comparison of results for built-in and pin-jointed beams indicate that, as expected, the non-linear effect is less important in the most constrained case, which is the built-in condition.

EVALUATION OF NON-LINEAR EFFECTS

The reason for the existence of lateral displacements is the non-symmetrical behaviour of the cracked structural element when an axial force is applied to it. The simplest way to analyze this effect consists in modelling the specimen (actually half of it because of symmetry) as a bar element with bending stiffness EI and length h which shows a rotation discontinuity at the plane of the crack (figure 4), θ_c (this is the part of the discontinuity corresponding to half of the specimen).

Although the effect of the crack on the deformation of the full cracked element is more complex, most of the nonlinear behaviour may be explained by supposing that the angular discontinuity is a linear function of axial force, P, and of bending moment at the crack plane, M_c.

$$\theta_c = \alpha P - \beta M_c \tag{1}$$

where α and β are compliance values which can be determined through experimental work or by means of standard linear elastic finite element analyses.

By imposing the moment equilibrium equation for non negligible lateral displacements

$$M_c = M + P u \tag{2}$$

and the boundary condition M=0 for the pin-jointed beam or θ=0 for the built-in beam, it is possible to derive, after some lengthy algebra, the lateral displacement, u, at the crack tip as:

$$u = \frac{3EIh\alpha P}{3EI + (3EI\beta + h)hP} \tag{3}$$

for the pin-jointed element and

$$u = \frac{3EIh\alpha P}{6EI(1 + \beta EI/h) + (2EI\beta + h/2)hP} \tag{4}$$

for the built-in element.

This simplified method of evaluation of lateral displacement u gives very reasonable results as it may be checked on figure 5 where it has been plotted as a function of the applied tension stress for the pin-jointed plane strip with h/w=10 and a/w=0.5; this displacement is compared to the linear and the non-linear values resulting from finite element analyses. The same result is shown on figure 6 for the built-in plane-strip. It may be observed that the non-linear effect may be very important even for low values of the tension load. The differences between the estimated lateral displacement and the best approximation (resulting from non-linear finite element analysis) is not significant specially for stress intensity factor computation since the induced bending moment (Pu) is only responsible for a fraction of this factor (K_I values are computed by adding tension, P, and bending, M_c, contributions).

CONCLUSIONS

A geometrically non-linear effect in fracture testing of unsymmetrically cracked slender elements has been presented. This effect reduces stress intensity factor with respect to standard linear solutions and may drive to unsafe estimations of fracture toughness. A simple method to estimate this non-linear effect may be derived; this method is applicable to any slender fracture specimen (not necessarily plane).

SYMBOLS USED

EI = bending stiffness (Nm2)
Mc = bending moment at crack plane (Nm)
α = axial load compliance coefficient (rad/N)
β = bending moment compliance coefficient (rad/Nm)
θ_c = crack plane rotation (rad)

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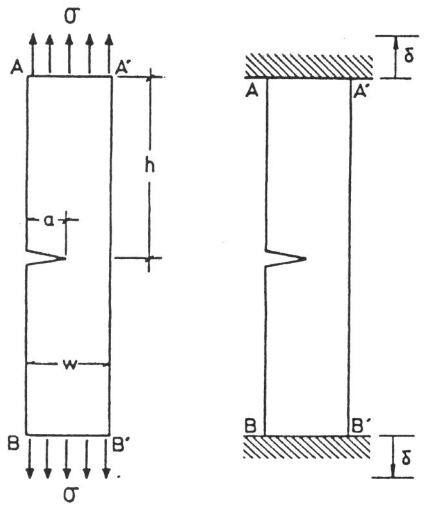

Figure 1 Geometrical description of the specimen.

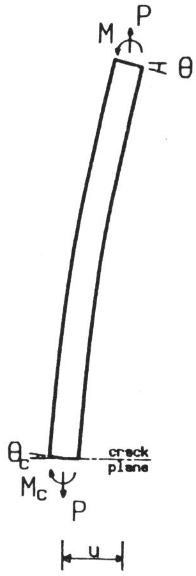

Figure 4 End forces and displacements for a cracked element

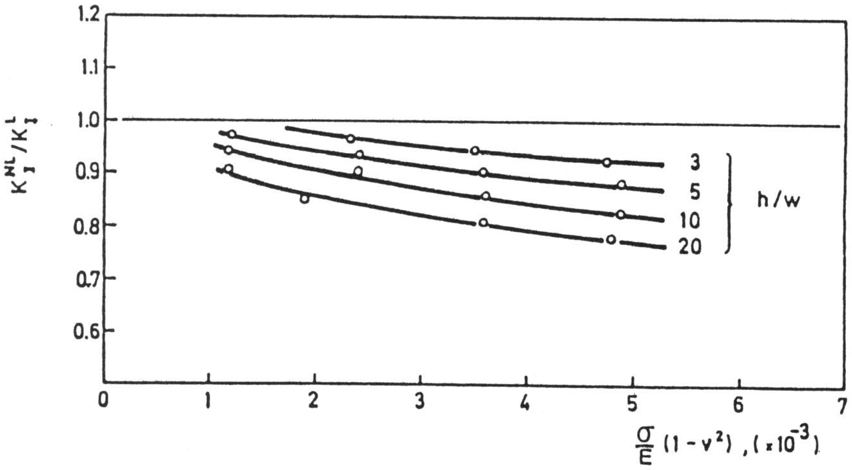

Figure 2 Stress intensity factor ratio for the doubly built-in tension specimen

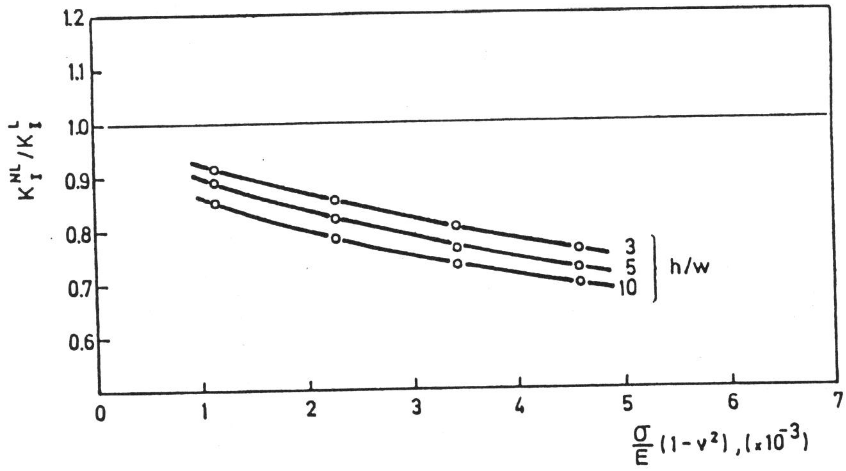

Figure 3 Stress intensity factor ratio for the doubly pin-jointed tension specimen

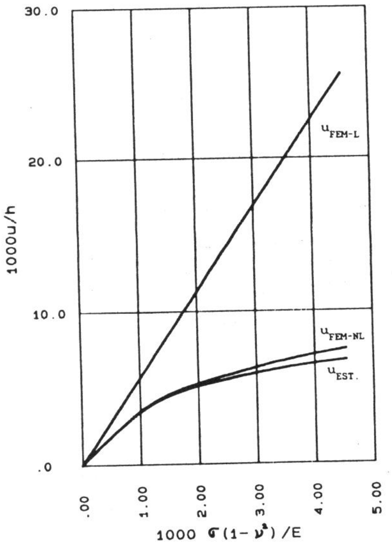

Figure 5 Lateral displacement for a pin-jointed strip with h/w=10 and a/w=0.5.

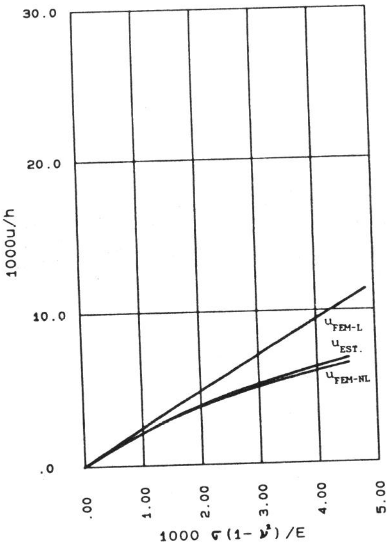

Figure 6 Lateral displacement for a built-in strip with h/w=10 and a/w=0.5.