ON FAST CRACK MOTION IN ELASTIC-PLASTIC MATERIALS
II - EQUATIONS OF MOTION

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In the present article the results of the paper (1) has been utilized to discuss the various forms of potential equation of motion. The new form of equation motion for Dugdale-Panasyuk crack has been proposed and compared to the equation based on the CTOD concept. The equation of motion based on the concept of the CTOD is equivalent to the equation of motion based on the concept of the energy rate conservation for the steady-state motion only. They have different physical meaning as well as produce different results in acceleration/deceleration domain.

INTRODUCTION

One of the most important problems in the crack propagation analysis is a proper formulation of the crack-tip equation of motion in order to select an actual motion from the class of all dynamically admissible motions. The useful form of the crack motion equation would be the one that in limit, when crack tip speed $v \to 0$, leads to a criterion of crack motion initiation. For the linear-elastic materials the best candidates to be applied to construct equation of crack motion are either the stress intensity factor (SIF) or energy release rate (ERR). By analogy to the criterion predicting the onset of the crack motion one can postulate the following equation of the crack-tip propagation:

$$k_i(\sigma_a, a, v, \text{geometry}) = k_{ic}(v, \text{temp. environment}) \quad (1)$$

where k_i is dynamic stress intensity factor that can be calculated from the boundary-value problem and k_{ic} represents the resistance of material to the crack motion and is considered as a material parameter. (sub. i = I, II or III) For linear and nonlinear elastic

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materials an equivalent to Eq.(1) equation of motion can be proposed utilizing the concept of the crack-extension force G

$$G_i^d(\sigma_a, a, v) = G_{ic}^d(v, \text{temp. environment}) \tag{2}$$

Unfortunately, the above simple equation can not be utilized to the analysis of the moving cracks in the non-elastic materials. The asymptotic analyses presented by Achenbach and Dunayevsky (2), Gao and Nemat-Nasser (3) for elastic-perfectly-plastic materials, or by Achenbach et al (4) for plastic-strain- hardening materials, or by Lo (5) for elastic-visco-plastic materials do not provide us with an appropriate amplitude factor that might be adopted to propose an equation of motion, equivalent to Eq.(1). Thus, for elastic-plastic materials another crack motion equation must be proposed. For example the concept of the critical strains at a certain point ahead of the crack tip, as proposed by Freund and Douglas (6) for steady-state motion of a Mode III crack may be utilized. The above approach is conceptually similar to the equation of crack motion based on a crack tip opening displacement concept that is particularly useful in the Dugdale (7) - Panasyuk (8) crack propagation analysis. Both equations can be written in the following form:

$$e_{23}^d(\sigma_a, a, v, \text{spec. geom.})\Big|_{x_1 = x_{cr}} = e_{cr}^d(v, \text{ temp., environment}), \tag{3}$$

$$\delta_{Ti}^d(\sigma_a, a, v, \text{spec. geom.}) = \delta_{Tic}^d(v, \text{temp., environment}), \tag{4}$$

where e_{23}^d is the strain component (in Mode III) ahead of the moving crack tip, δ_{Ti}^d denotes the crack tip opening displacement, the subscript i = I, II,or III depending on Mode of loading, the superscript d indicates that particular quantity follows from the dynamic analysis and e_{cr}^d, δ_{Tic}^d are the material parameters.

EQUATIONS OF MOTION FOR DUGDALE-PANASYUK CRACK

In the first paragraph three equations of motion based on three different parameters have been presented. For the D-P crack the SIF is equal to zero. Therefore, we will concentrate on two remaining quantities

a) One of the most often equations of motion used is the one based on a concept of the energy release rate G_i^d that for a moving crack can be defined as follows (9),(10):

$$G_i^d = \lim_{L\to 0} \frac{1}{v} F_{III} = \lim_{L\to 0} \frac{1}{v} \int_L \left[\sigma_{ij} n_j \dot{u}_i + \frac{1}{2}\left(\sigma_{ij} u_{i,j} + \rho \dot{u}_i \dot{u}_i\right) v n_1\right] dl \tag{5}$$

where v is a crack tip speed, L is an arbitrary contour that begins on one traction-free face of the moving crack, surrounds the tip and ends on the opposite traction-free face, σ_{ij} and u_i are the stress tensor and displacement vector components respectively, n_i

is a component of the unit vector normal to the contour L and dot denotes derivative with respect to time. The integral over contour L in Eq.(5) represents the rate of energy flow F through L and can be calculated from the power balance equation according to procedure proposed by Freund (9) and it was first proposed by Atkinson and Eshelby (11). Thus, the integral in (5) must be path independent. In general it is true for the steady-state crack motion only as was shown e.g. by Nilsson (10). However, it turns out that for relatively wide class of materials, for which the displacement of the crack faces satisfies the relation $u_i \sim r^{\alpha}$ where $0 < \alpha < 1$ and r is a distance from the crack tip, an arbitrary motion of the crack can be considered as an asymptotically steady-state (Nilsson (10), Freund (9)). Thus for the linear and nonlinear elastic materials the equation of motion in a form (2) can be adopted to discuss certain crack propagation problems. However, in any case, path independence of the definition of G^d_i should be checked before Eq.(2) is used for a particular problem. Before we proceed to discuss the D-P crack motion equation the function $\mathcal{G}^d_{III}$ will be introduced in the form

$$\mathcal{G}^d_{III} = \frac{1}{\beta_L c_T} F_{III} = \frac{1}{v} F_{III} \qquad (6)$$

and ploted in Fig.1. The function $\mathcal{G}^d_{III}$ is in some sense similar to the function G^d_{III} but in general it is not the energy release rate. Henceforth, it will be called the "driving force" (DF) on the D-P crack. According to the definition (5) the energy flow into the plastic zone, divided by the crack tip speed may be interpreted as an energy release rate for the steady state motion only. For arbitrary motion with varying velocity of the D-P zone it can not be proved that the motion is asymptotically steady-state since the contour L is precisely defined. The quantity G^d_{IIIC} in Eq.(2) is usually called the resistance of the material to the crack motion. It may be noticed that for β_T smaller than certain critical value the quantity $\mathcal{G}^d_{III}$ is greater than G_{IIIC}. There is no doubt that for many materials the material parameter G^d_{IIIC} is a function of a crack tip speed. The physical intuition and the observation of the experimental results suggest that the G^d_{IIIC} is either increasing function of β_T (initially increases slowly and from a certain value of β_T rapidly) or initially decreases with β_T (because for higher values of β_T there is little time for plastic deformation) and then increases with β_T (because of crack branching processes). Thus the equation in the form (2) is certainly not correct for the first stage of propagation and probably is not also true for later stages except for a steady- state motion. Taking into account above arguments one may assume the equation of motion of the D-P crack in the form:

$$\mathcal{G}^{d}_{III} - \mathcal{G}^{d}_{IIIC} = Ma_{t} \qquad (7)$$

interpreting $\mathcal{G}^{d}_{III}$ as a "driving force" on D-P crack, $\mathcal{G}^{d}_{IIIC}$ as a resistance of the material to the crack motion, M as an "equivalent" mass of the plastic zone and a_T as a crack tip acceleration (in the proposed model of the crack tip kinetics a_T should be interpreted as a "mean" acceleration of a trailing edge). The equivalent mass M can be assumed to be proportional to the plastic zone mass. When the crack motion is steady-state Eq.(7) reduces to (2). In general Eq.(7) contains three unknowns β_T, β_L and a_T. This differential equation could be solved for a given loading history and material parameter $\mathcal{G}^{d}_{IIIC}(\beta_T, \beta_L)$ if is supplemented by an additional relation between β_T and β_L. For proposed model of crack kinetics (Fig.2) one may introduce two additional equations derived from purely geometrical analysis along the crack trajectories. From Eq.(7) and relation

$$r^{i}_{p} = r^{i-1}_{p} + c_T t \left(\beta^{i}_{L} - \beta^{i}_{T}\right)$$

the following, approximate formulae follow:

$$\beta^{i}_{L} \cong \beta^{i}_{T} + K_{III} \frac{dK_{III}}{dt} \frac{\pi}{4} \frac{1}{c_T \tau_f^2} \left(1 - \beta^{i}_{T}\right)$$

$$a_{T\,mean}^{\;i} = \frac{\beta^{i}_{T} - \beta^{i-1}_{T}}{2r^{i}_{p*}} \left(1 + \beta^{i}_{T}\right)$$

The computer simulation techniques may now be proposed to predict the crack motion history for given external loading and material properties. It will be a subject of the next paper.

b) Another equation of motion for the D-P crack can be postulated in more straight forward way. The concept of the CTOD is usually extended to dynamic case and is written in the form of Eq.(4). When both sides of Eq.(4) are multiplied by τ_f we obtain equation of motion in the form (2), that as was shown earlier is strictly true for steady-state motion only. The first term in Eq.(4), evaluated in Eq.(5) in the paper (1) may be interpreted as the irrecoverable work per unit area which has been done on the material element at the crack tip in bringing it to the point of separation. It does not take into account the energy dissipated within the plastic zone, that changes its length in time. This problem has been discussed in more details in the review article by Freund (12). Nevertheless, Eq.(4) has very often been utilized in a crack motion analysis. It can easily be applied in the presented model of crack kinetics. Eqs.(2) in (1) (the graphical representation of Eq.(2)

in (1) is shown in the Fig. 3) and (3) are sufficient to perform computer simulation procedure provided the crack tip trajectories are approximated by a piecewise-linear function. The assumed approximation of the real crack tips trajectories have been shown in Fig.(2). It is assumed that the crack starts propagation with a speed $\beta_T = \beta_T^o$ at $\beta_L = 0$. The information that the trailing edge has already started propagation reaches the leading edge at time S_1. At this moment the leading edge starts motion with the speed $\beta_L = \beta_T^o$ and sends information about that to the trailing edge. This information reaches the trailing edge at time $S = S_2$, and the latter corrects its speed to a new speed following from the equation of motion. Again the signal is sent to the leading edge which at time $S = S_3$ adjusts its speed to $\beta_L = \beta_T^1$ and so on until the steady-state is reached. It can easily be shown that if δ_{TC}^d = const and $\frac{dK_{III}}{dt} = 0$ (Fig.(2)) the steady state is reached after two steps. The situation is more complex for more general case, but computational procedure is the same.

REFERENCES

1. Neimitz, A., "On fast crack motion in elastic-plastic materials Part I ", this volume.
2. Achenbach, J.D.and Dunayevsky V. J.Mech.Phys.Solids, 29, (1981) pp. 283-303.
3. Gao, Y.C. and Nemat-Nasser S.,Mechanics of Materials, 2, (1983) pp. 47-60.
4. Achenbach, J.D., Kanninen,M.F. and Popelar,C.H., J.Mech.Phys. Solids, 29, (1981), 211-225.
5. Lo,K.K., J.Mech.Phys.Solids,31, (1983) pp.287-305.
6. Freund,L.B. and Douglas,A.S.,J.Mech.Phys.Solids,30,(1982), pp.59-74.
7. Dugdale,D.S.,J,Mech.Phys.Solids, 8, (1960),pp.100-104.
8. Panasyuk,V.V.,Dopovidi Akad.Nauk Ukr.RSR,9,(1960),pp.1185-1188.
9. Freund,L.B., J.Elasticity,2, (1972), pp.341-349.
10. Nilsson,F.,"Dynamic fracture theory", to be published by CISM Udine, Italy.
11. Atkinson,C. and Eshelby,J,D.,Int.J.Fracture Mech.,4, (1976), pp.3-8.
12. Freund,L.B.,"Dynamic crack propagation in the Mechanics of Fracture", ADM,19, Edited by F.Erdogan, ASME (1976)pp.105-134.

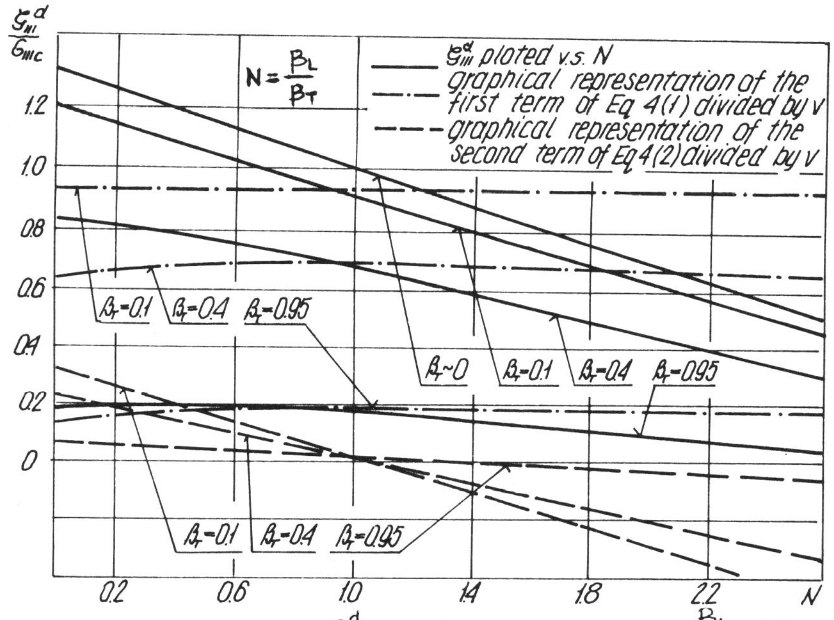

Fig.1.The plot of the G^d_{III} as a function of N= $\frac{\beta_L}{\beta_T}$ (1)

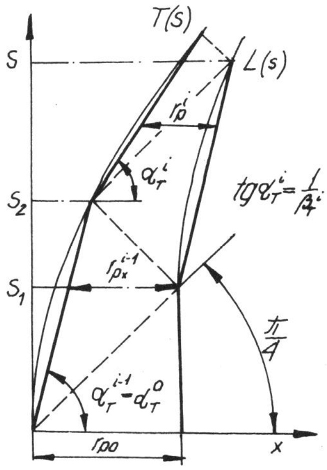

Fig.2.Hypothetical crack trajectories

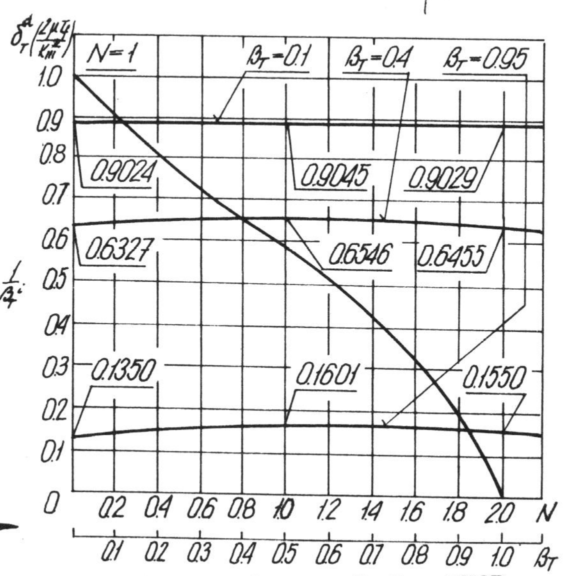

Fig.3.Dependence of the CTOD on β_T and N. (1)