ON FAST CRACK MOTION IN ELASTIC-PLASTIC MATERIALS. I - MECHANICAL FIELDS AROUND MOVING CRACK TIP.

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The results of the calculation of the fundamental parameters characterizing the arbitrary fast Dugdale-Panasyuk crack motion are presented. The simplified model of the crack kinetics has been adopted: the smooth crack trajectories were approximated by a piecewise-linear functions. Acceleration, deceleration or steady state crack motion are represented by a rate of change of the plastic zone length. The results obtained will be utilized in the Part II to propose and discuss crack motion equations.

INTRODUCTION

The Dugdale-Panasyuk model of the crack plays a very specific role in Fracture Mechanics. Thanks to its basic postulates (1) the complex problem of the crack in elastic-plastic materials is replaced by a much easier one of a crack in linear-elastic material with an additional boundary condition on a crack faces. It was shown in the several experimental studies e.g. (1),(2) that despite of a strong simplifying assumptions the D-P model provides a very good results, at least for a stable cracks. Weak point of the model is its rather limited applicability, only for the plane stress situation. Nevertheless the simplicity of the model encourages to utilize it in crack dynamics analysis hoping for certain results that are impossible or very difficult to obtain using more realistic models. This article is a first of a series of papers on a fast crack motion with varying velocity according to the Dugdale-Panasyuk model. It is three-fold purpose of the two papers presented during EGF8 conference.

a) to recast the Achenbach and Neimitz (3) analysis of the fast, Mode III, Dugdale-Panasyuk (D-P) crack motion in the framework of an energy balance including varying crack-tip speed,

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b) to reconsider, the validity of an Eq.

$$G_i^d\left(\sigma_a, a, v\right) = G_{ic}^d\left(v, \text{temp.}, \text{environment}\right)$$

for an arbitrary D-P fast crack motion and to propose a new equation of motion,

c) to compare the above mentioned equation with equation of motion based on the CTOD concept.

BASIC ASSUMPTIONS

The fast motion of the D-P cracks has already been discussed by several authors. Goodier and Field (4), Kanninen (5), Achenbach and Neimitz (3) and Neimitz (6),(7), utilizing various techniques, adopted this model to analyse crack motion with a constant, high velocity. Glennie and Willis (8) were the first who included effect of a varying crack tip speed on certain features of the fast crack motion phenomenon. They utilized Friedlander's method (Friedlander (9)) to obtain the stress and displacement fields within the crack tip region, moving with an arbitrary velocity, and adopted it to the D-P model. Kostrov (10) and Achenbach (11) solved the same problem adopting Evvard's method (Evvard (12)). This method was later adopted by Achenbach and Neimitz (3) to the analysis of the D-P crack motion. Both methods lead to similar results, but Glennie and Willis' method is more general since it provides displacement field in an arbitrary point around the crack tip, not only in the plane of the crack. Besides, in the Glennie and Willis method the effect of body forces can also be considered. However, the advantage of the Kostrov and Achenbach method lies, in the author's opinion, in a clear, convincing, physical picture of this complex phenomenon and in an easy computational procedure. There is, however, one serious disadvantage in both methods (if one accepts the D-P model) that is particularly in computing displacement within the D-P zone or crack tip opening displacement. Namely, one should know both leading and trailing edge trajectories prior to computing displacements. But, in turn, a particular motion can be determined only with the help of the crack opening displacement. Thus the analysis may only be an approximate one and such was proposed by Glennie and Willis (8). They assumed that the critical crack tip opening displacement was constant, independent of the crack tip speed, the length of the plastic zone was constant as for steady state motion, the equation of energy flow out of the body consisted only of one term, neglecting the influence of the variation in the length of the D-P zone. The real crack tip speed was approximated by a piecewise-linear v(t) function. With all the above assumptions they solved given equations on a computer. It turned out that the energy released during crack tip acceleration increased over its steady-state value and decreased during crack tip deceleration. In the present paper the analysis will be carried out in another direction. It is assumed that both crack tip (trailing edge) and plastic zone tip (leading edge) move with constant velocities and β_L= const $\neq$ β_T= const,

where $\beta_L = v_L/c_T$, $\beta_T = v_T/c_T$.

With this assumption, the length of the plastic zone either increases ($\beta_L > \beta_T$) or decreases ($\beta_T > \beta_L$) in time. The length of the plastic zone r_p follows from the stress analysis and Barenblatt's postulate that stresses should be finite ahead of the crack tip (Achenbach and Neimitz (3)):

$$r_p = \frac{\pi}{8} \frac{K^2_{III}(1)}{\tau_f^2} \left(1 - \beta_T\right) \quad . \tag{1}$$

From the above relation one can conclude that when crack tip speed increases the length of the plastic zone decreases to zero value at $\beta_T = 1$. For $\beta_T = 0$ one obtains the static value of r_p. Thus the assumption that $\beta_T = \text{const} \neq \beta_L = \text{const}$ simulates the process of acceleration or deceleration of the crack while the real shapes of the crack trajectories are approximated by a piecewise-linear functions. We hope that such an approach will provide us with certain additional information not available from the elastic or elastic-plastic steady state analysis and will be a good starting point to more advanced computer simulation procedures. It should be noted, however, that while discussing the whole history of the crack tips propagation the approximation procedure of the crack trajectories by a piecewise-linear functions can not be arbitrary. This problem will be discussed in more details later on (13). It is here also assumed that the shearing yield stress within plastic zone is constant, independent of the crack tip speed. We simply neglect (as a first approximation) the viscous effect. The viscous effect within the plastic zone was discussed by E.B.GLENNIE (14). It is here assumed that the strain rate, or more precisely the crack tip speed, will influence the parameters characterizing fracture toughness of the material only. It should also be recalled here that presented analysis is strictly valid for semi-infinite crack within an infinite body only.

RESULTS

The boundary value problem leading to the results presented in this article was formulated in the paper (3). The formulas defining the length of the plastic zone r_p (Eq.(1) in this paper) and the crack tip opening displacement (CTOD) were the main results of the paper (3) that is the starting point to the present analysis. The CTOD can be expressed by the relation

$$\delta^d_T = \frac{K^2_{III}}{2\mu\tau_f} \left(\frac{1 - \beta_T}{1 + \beta_L}\right)^{1/2} \left[2 - \left(\frac{1 + \beta_T}{1 + \beta_L}\right)^{1/2}\right] \tag{2}$$

In (3) the motion of the D-P crack was also analysed but limiting discussion to the steady-state situation only. In the present paper the analysis is extended for an arbitrary motion with a varying velocity. One of the most often equations of motion used is the one based on a concept of the energy release rate G_i^d that for a moving crack can be defined as follows (15) :

$$G_i^d = \lim_{L\to 0} \frac{1}{v} F_{III} = \lim_{L\to 0} \frac{1}{v} \int_L \left[\sigma_{ij} n_j \dot{u}_i + \frac{1}{2}\left(\sigma_{ij} u_{i,j} + \rho \dot{u}_i \dot{u}_i\right) v n_1\right] dl \tag{3}$$

The integral in Eq.(3) denotes the energy flow through an arbitrary contour L, surrounding the crack tip and moving along with it with a speed v. For Mode III it will be denoted by F_{III}. Analysing the D-P model, it is convenient to select contour L in this way that it begins at the lower crack face at the trailing edge, terminates at the upper face of the crack at the trailing edge and it is drawn along the lower and upper faces, surrounding the leading edge. For such a contour the integral in (3) reduces to the form:

$$F_{III} = v \int_0^{\delta_T} \tau_f d\delta + \int_0^{r_p} \tau_f \frac{\partial \delta}{\partial t} d\zeta \quad . \tag{4}$$

The first integral in (4) can be evaluated yielding to the following relation

$$v \int_0^{\delta_T^d} \tau_f d\delta = v \tau_f \delta_T^d = v \frac{K_{III}^2}{2\mu} \left[\frac{1-\beta_T}{1+\beta_L}\right]^{1/2} \left[2 - \left[\frac{1+\beta_T}{1+\beta_L}\right]^{1/2}\right]. \tag{5}$$

To evaluate the second integral in (4) we must first calculate the function $\delta(\zeta, t)$ for a moving crack. It may be done utilizing Egs (4.8),(4.9) of (3)and certain geometrical relations following from the assumed piecewise-linear trajectories of the leading and trailing edges of the crack.

$$\delta(\psi, s) = \left\{ 2A \; 2r_p^{1/2}\psi^{1/2} - \psi^{1/2}\left(r_p + b\,\psi\right)^{1/2} + \right.$$

$$\left. \frac{1}{c^{1/2}} \left(r_p - \psi\right) \ln \left[\frac{\left(r_p + b\,\psi\right)^{1/2} - \left(c\,\psi\right)^{1/2}}{\left(r_p - \psi\right)^{1/2}}\right]\right\} , \tag{6}$$

where

$$A = \frac{2}{\pi} \frac{\tau_f}{\mu} \left(1+\beta_L\right)^{-1/2} \left(1-\beta_T\right)^{-1/2} , \quad b = \frac{\beta_T - \beta_L}{1+\beta_L} , \quad c = \frac{1+\beta_T}{1+\beta_L} ,$$

$$\psi = -\zeta , \quad r_p = r_{po} + s\left(\beta_L - \beta_T\right) , \quad s = c_T t$$

and r_{po} is a length of the D-P zone, measured at the instant when either β_T or β_L or both changed their values. Inserting (5), and

(6) into (4) we finally obtain relatively simple relations for the energy flow F_{III}

$$F_{III}=\beta\, c_T \frac{K^2_{III}}{2\,\mu}\left(\frac{1-\beta}{1+\beta}\right)^{1/2} \quad \text{when} \quad \beta_T=\beta_L=\beta \tag{7}$$

$$F_{III}=\beta_L c_T \frac{K^2_{III}}{2\mu}\left[\frac{1-\beta_T}{1+\beta_L}\right]^{1/2}\left\{2-\left[\frac{1+\beta_T}{1+\beta_L}\right]^{1/2}+\frac{2(\beta_L-\beta_T)}{3\;\beta_T}+\right.$$

$$\left.\left[\frac{\beta_T-\beta_L}{\beta_T}\right]^{1/2}\left[\frac{1+\beta_L}{\beta_T}\right]^{1/2}\ln\left[\frac{(1+\beta_T)^{1/2}+(\beta_T-\beta_L)^{1/2}}{(1+\beta_L)^{1/2}}\right]\right\}, \tag{8}$$

for $\beta_T>\beta_L$

$$F_{III}=\beta_L c_T \frac{K^2_{III}}{2\mu}\left[\frac{1-\beta_T}{1+\beta_L}\right]^{1/2}\left\{2-\left[\frac{1+\beta_T}{1+\beta_L}\right]^{1/2}+\frac{2(\beta_L-\beta_T)}{3\;\beta_T}+\right.$$

$$\left.\left[\frac{\beta_L-\beta_T}{\beta_T}\right]^{1/2}\left[\frac{1+\beta_L}{\beta_T}\right]^{1/2}\operatorname{arc\,tg}\left[\frac{\beta_L-\beta_T}{1+\beta_T}\right]^{1/2}\right\}, \tag{9}$$

for $\beta_L>\beta_T$.

The F_{III} function will be used in the next article (13) as a one of the most fundamental quantities in formulating equations of motion. The crack tip opening angle CTOA is another parameter that can be utilized in a crack motion analysis. It is here defined as a mean value of the whole population of angles taken between the tangents to the D-P zone profile. For the analyzed model one can obtain

$$(\mathrm{CTOA})_{\mathrm{mean}}=\frac{4\,\tau_f}{\pi\,\mu}\frac{1}{(1-\beta_T)^{1/2}(1+\beta_L)^{1/2}}\left[2-\left[\frac{1+\beta_T}{1+\beta_L}\right]^{1/2}\right]$$

SYMBOLS USED

v_L, v_T - leading and trailing crack tip speed

c_T - speed of elastic shearing wave

σ_{ij}, u_i stress tensor and displacement vector components

τ_f - effective shearing yield stress

. dot defines derivative with respect to time

μ - shearing modulus

REFERENCES

1. Dugdale, D. S. J. Mech. Phys. Solids 8 (1960) pp. 100-104
2. Hahn, G. T. and Rosenfield, R. Acta Metall. 13, (1965) pp. 293-306
3. Achenbach, J. D. and Neimitz, A. , Engng Frac. Mech. 14 (1981) pp. 385-395
4. Goodier, J. N. and Field, F. A. "Plastic energy dissipation in crack propagation" in "Fracture of Solids" Edited by D. C. Drucker and J. J. Gilman, eds. (1963) pp. 103-118. Weley, New York
5. Kannien, M. F. J. Mech. Phys. Solids, 16 (1968) pp. 215-228
6. Neimitz, A. Arch. of Mech. 33 (1981) pp. 815-827
7. Neimitz, A. Engng Transac. 29 (1981) pp. 629-640
8. Glennie, E. B and Willis, J. R. , J. Mech. Phys. Solids, 19 (1871) pp. 11-30
9. Friedlander, F. G. Quart. J. Mech. Appl. Math 4 (1951)p. 344
10. Kostrov, B. V. , Prikl. Mat. Mekh. (English Translation), 30 (1966) pp. 1241-1248
11. Achenbach, J. D. ZAMP, 21 (1970) pp. 887-900
12. Evvard, J. C. "Distribution of wave drag and lift in the vicinity of wing tips at supersonic speeds", National Advisory Committee for Aeronautics, Technical Note No 1382 (1947)
13. Neimitz, A. Part II of the present article-this Volume
14. Glennie, E. B. J. Mech. Phys. Solids, 19 (1971) pp. 255-272
15. Freund, L. B. J. Elasticity, 2 (1972) pp. 341-349