# 3D NUMERICAL COMPUTATION OF THE STRESS INTENSITY FACTOR IN A CRACKED BOLT WITH A NUT

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This paper presents the calculation of stress intensity factor (K) solutions for surface cracks in bolts directly loaded by the nut. A virtual crack extension technique is used to compute the energy release rate and so the stress intensity factor. Two modifications are made to improve the accuracy of the results: the displacement not only of the main node, but also of the quarter-point nodes located in the normal plane and the adjacent nodes in the crack line, avoiding both the change of the singularity and the crack curving.

The results show that direct loading on the thread by the nut increases the stress intensity factor. This trend decreases with the crack length. However, the tendency is opposite for the largest circumferential cracks, due to a bending effect.

## INTRODUCTION

Threaded fasteners, usually in the form of bolted joints, are widely used in the assembly of many large structural components. However, some difficulties have arisen in this point, due to the fact that bolt geometry is very complex and loading conditions are not well defined. A great effort has been made, consequently, during recent years in this area, as part of a research programme supported by the European Space Agency (Toribio and Sánchez-Gálvez (1)).

Published stress intensity factor solutions referring to cracked bolts are quite scarce. References (2) and (3) deal with certain configurations of the cracked threaded bar, and present stress intensity results, expressed as correction factors. Papers (4) and (5) present interesting reviews of K-solutions in scientific literature applicable to cracks in bolts. The surprising conclusion is that, although a great number of K-solutions have been developed for cracks in unnotched round bars, few solutions have dealt with cracks in the vicinity of threads.

The present paper presents a new - very realistic - stress intensity factor numerical solution for the cracked bolt loaded directly by the nut. The main objective of the computation is to determine the influence of the nut on the stress intensity factor. Analytical expressions - obtained by means of a second order interpolation with the numerical results - are provided, which are very useful for fatigue life prediction.

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## GEOMETRY AND LOADING CONDITIONS

The geometry analyzed consists of an ISO M8 x 1.0 bolt with an elliptical crack at the surface subjected to different loading conditions. The bolt was modeled in separate threads, without taking into account the screw, to preserve the axial symmetry of the problem. A surface crack was considered in the open thread ground perpendicular to the bolt axis. The shape of the crack is semi-elliptical (Fig.1) with its center located on the surface of the inner -minimum diameter- cylinder and semi-axes $a$ and $b$, corresponding to the crack depth and the transversal length of the crack. Three aspect ratios were considered: shallow (a/b = 0.2), intermediate (a/b = 0.6) and circular (a/b = 1). Five crack depths were used in the computations (a/d = 0.1, 0.2, 0.3, 0.4 and 0.5).

The following loading conditions were considered (see Fig.2):

(a) **Tension loading**, which consists of a uniform stress distribution applied far enough from the crack.
(b) **Nut loading**: In this case the load, due to the nut, was directly applied on the thread. The design of the load took into account the references containing numerical results (6) and experimental measurements (7) of the stress distribution along the thread faces of a bolt. According to these references, the loading system consists of constant pressure applied directly on the thread faces just below the crack, with a value p on the first thread (next to the crack) and the half of this value (p/2) on the second thread (below the first).

The nut loading was applied on the worst thread from the Fracture Mechanics point of view. The screw variation of the load (variable pressure) cannot be taken into account, to preserve the axial symmetry of the problem. The thread variation can be taken into account, but it represents a second order influence, according to Reference (2). Therefore, a constant pressure on each thread was chosen.

The main objective of the present work is the achievement of K-solutions (mode I only) for the cracked bolt with the above loading conditions. The stress intensity factor is a function of the crack depth, the position on the crack border and the crack aspect ratio. The results are expressed in terms of dimensionless correction factor Y:

$$Y = \frac{K}{\sigma\sqrt{\pi a}} \tag{1}$$

where $\sigma$ represents, in both loading cases, the net axial stress on the bolt.

## NUMERICAL COMPUTATION OF THE STRESS INTENSITY FACTOR

A computer program was written to automatically generate finite element meshes in general 3D geometries. Fig.3 shows a view of typical finite element mesh for the cracked bolt, and Fig.4 offers a detail of mesh around the crack. A standard mesh with 39 macroelements, 584 elements, 3057 nodes and 9171 degrees of freedom was designed for loading case (a). The mesh for loading case (b) contains 46 macroelements, 772 elements, 3961 nodes and 11883 degrees of freedom.

The numerical computations were carried out by using the Finite Element Method with an elastic code and isoparametric quadratic elements: 20-node brick elements and 15-node prismatic elements. In order to model the $r^{-1/2}$ singularity at the crack tip, singular quarter-point elements were used (8). In these elements the singularity is modelled by translating the mid-side nodes of a conventional element to the quarter-point position.

In order to obtain the stress intensity factor along the crack line, an energetic technique was used: the Stiffness Derivative Method, based on the computation of the energy release rate upon a virtual crack extension (9,10). It consists in the computation of the energy release rate when a virtual crack extension in a point of the crack front is carried out by shifting the node of that position in a perpendicular direction to the crack front. The expression to obtain the energy release rate is a second order quadratic form in displacements, that is:

$$G = \frac{1}{2}\ \{u\}^T\ \frac{K_{a+\Delta a} - K_a}{\Delta a}\ \{u\} \tag{2}$$

where K is the stiffness matrix before (a), and after (a+Δa) the virtual crack extension, and {u} the nodal displacement vector in the reference situation.

The Stiffness Derivative Method, which represents an energetic approach, has two important advantages: accuracy and computer economy. The accuracy is a consequence of the energetic character of the method; it is not necessary to know exactly the stress or strain field next to the singularity produced by the crack tip, but only to compute the stiffness matrices and the displacement vector in the body. The computer economy is due to the fact that in the virtual crack extension only the local stiffness matrices corresponding to the deformed elements in the vicinity of the crack tip are computed in the second situation (a+Δa).

The choice of the virtual node shift at the crack front was done according to the values proposed by Astiz (10,11) for a semielliptical crack in a cylindrical body. The proposed value is $\Delta a = 0.0001$ a.

Two modifications were introduced into the numerical method in order to improve the accuracy of the results:

- With regard to crack advance in the normal plane (Fig.5), a displacement was made not only of the main node *p* located in the crack front (which was shifted from *p* to *p'* ), but also of the whole core of quarter-point nodes *q* belonging to the normal plane (which were moved from *q* to *q'* ). Such a procedure maintains the $r^{-1/2}$ singularity at the crack tip, by maintaining the relationships 1/4 between distances along the finite element sides in the normal plane to the crack. Therefore, if the main node shift is $u_p = \Delta a$ the displacement of quarter-point nodes will be $u_q = (3/4)\ \Delta a$ in the same direction contained into the normal plane to the crack.

- In the case of local crack advances in the plane of the crack (Fig.6a), not only the main node *p* (shifted from *p* to *p'* ) was displaced, but also the two adjacent nodes *a* in the crack front (shifted from *a* to *a'* ). Whith such a system the cracked area increases in all its zones. This procedure avoids curving of the crack line and, therefore, a decrease in the cracked area. (Fig.6b). With a main node shift equal to Δa, the displacement of the two adjacent nodes will be $u_a = \Delta a / 2$.

## ANALYSIS OF RESULTS

The results for tension loading appear in Table 1, and they agree fairly well with values of the stress intensity factor in threaded bars published previously by Reibaldi (2) and Nord and Chung (3). However, the number of combinations between crack geometries and loading conditions analyzed in this research work is much higher in the matter of tension loading and bending moment.

The following comments may be made on the results for tension:

- S.I.F. values at the crack center are higher for shallow cracks.

- S.I.F. values at the crack surface are higher for circular cracks.

- For shallow cracks, the S.I.F. is higher at the crack center, and for circular cracks is higher at the crack surface.

TABLE 1- Dimensionless stress intensity factor (tension loading)

| a/b | a/d | $s/s_0 = 0$ | 1/4 | 2/4 | 3/4 | 1 |
|---|---|---|---|---|---|---|
| 0.2 | 0.1 | 0.99 | 0.89 | 0.84 | 0.76 | 0.65 |
| | 0.2 | 1.06 | 0.89 | 0.85 | 0.76 | 0.66 |
| | 0.3 | 1.25 | 0.89 | 0.85 | 0.78 | 0.70 |
| | 0.4 | 1.77 | 0.89 | 0.86 | 0.81 | 0.74 |
| | 0.5 | 2.08 | 0.89 | 0.87 | 0.83 | 0.78 |
| 1 | 0.1 | 0.75 | 1.01 | 1.03 | 1.10 | 1.22 |
| | 0.2 | 0.80 | 1.00 | 1.01 | 1.04 | 1.16 |
| | 0.3 | 0.88 | 1.00 | 1.02 | 1.08 | 1.22 |
| | 0.4 | 1.05 | 1.01 | 1.03 | 1.13 | 1.31 |
| | 0.5 | 1.27 | 1.01 | 1.06 | 1.19 | 1.43 |

Starting from these results, a second-order interpolation was performed to provide useful analytical expressions to be used in fatigue life prediction. The dimensionless stress intensity factor Y is :

a / b = 0.2

$S/S_0 = 0)$ $Y = 0.968 - 0.581\ (a/d) + 5.786\ (a/d)^2$

$S/S_0 = 1/2)$ $Y = 0.838 + 0.027\ (a/d) + 0.071\ (a/d)^2$

$S/S_0 = 1)$ $Y = 0.634 + 0.082\ (a/d) + 0.429\ (a/d)^2$

a / b = 1

$S/S_0 = 0)$ $Y = 0.778 - 0.569\ (a/d) + 3.107\ (a/d)^2$

$S/S_0 = 1/2)$ $Y = 1.056 - 0.349\ (a/d) + 0.714\ (a/d)^2$

$S/S_0 = 1)$ $Y = 1.292 - 1.101\ (a/d) + 2.786\ (a/d)^2$

Figs. 7 and 8 show the solutions referring to nut loading, compared with those for tension loading. It should be noted that values for intermediate crack in tension were obtained by numerical interpolation, since only shallow and circular cracks were considered in the finite element computations with tension loading. Stress intensity factor versus crack depth was plotted at the crack center (Fig.7) and crack surface (Fig.8). In these figures Y represents the dimensionless S.I.F. for nut loading (full line) and Y* the same for tension loading (dashed line).

Referring to the crack center, it can be seen (Fig.7) that:

- Nut loading clearly increases the S.I.F. for the shortest cracks.
- This effect decreases with the crack depth
- The trend is opposite for the deepest circular and intermediate cracks. It can be explained as a bending effect, due to the forward loading, which produces two unloaded areas at the front of the crack tip beside the bolt surface.

With regard to the crack surface, Fig.8 shows that:

- Nut loading greatly increases (even more than in crack center) the S.I.F. for the shortest cracks.
- This effect increases with the crack aspect ratio a/b and decreases with the crack length.
- The trend is opposite for the deepest circular crack, due to the afore-mentioned bending effect.

Starting from results from nut loading, and through a second order interpolation, the next analytical expressions useful for fatigue crack growth computations can be obtained:

| | | |
|---|---|---|
| a/b = 0.2 | $s/s_o = 0$) | $Y = 1.914 + 7.424\ (a/d) - 14.36\ (a/d)^2$ |
| | $s/s_o = 1/3$) | $Y = 2.930 + 0.291\ (a/d) - 6.714\ (a/d)^2$ |
| | $s/s_o = 2/3$) | $Y = 3.058 - 5.036\ (a/d) + 5.643\ (a/d)^2$ |
| | $s/s_o = 1$) | $Y = 4.736 - 7.070\ (a/d) + 0.500\ (a/d)^2$ |
| a/b = 0.6 | $s/s_o = 0$) | $Y = 4.248 - 16.74\ (a/d) + 22.64\ (a/d)^2$ |
| | $s/s_o = 1/3$) | $Y = 5.662 - 27.93\ (a/d) + 39.50\ (a/d)^2$ |
| | $s/s_o = 2/3$) | $Y = 4.792 - 20.44\ (a/d) + 31.64\ (a/d)^2$ |
| | $s/s_o = 1$) | $Y = 10.32 - 47.42\ (a/d) + 60.64\ (a/d)^2$ |
| a/b = 1 | $s/s_o = 0$) | $Y = 1.918 - 1.941\ (a/d) + 0.786\ (a/d)^2$ |
| | $s/s_o = 1/3$) | $Y = 0.958 + 2.531\ (a/d) - 2.286\ (a/d)^2$ |
| | $s/s_o = 2/3$) | $Y = 5.686 - 24.49\ (a/d) + 29.50\ (a/d)^2$ |
| | $s/s_o = 1$) | $Y = 15.42 - 81.07\ (a/d) + 104.9\ (a/d)^2$ |

The stress intensity factor solution for a cracked bolt subjected to nut loading represents an important effort in Fracture Mechanics research. Moreover, damage tolerance computations are devoted to fatigue life estimation, and such a prediction involves many hard problems in a complex 3D geometry such as a cracked bolt.

## CONCLUSIONS AND FUTURE RESEARCH

1. The stress intensity factor solution for a cracked bolt subjected to tension loading and nut loading was obtained for various crack shapes and crack depths, in different crack points.
2. Two improvements were introduced into the computation in order to increase the accuracy of the results using the Stiffness Derivative Method: the displacement of the quarter -point nodes (normal plane) and the shifting of the adjacent nodes (crack line).
3. A polynomical second order interpolation was carried out with the numerical results, to provide analytical expressions to be used in fatigue life prediction.
4. The K-values for tension loading are higher at the crack center of shallow cracks and the crack surface of circular cracks.
5. Nut loading increases the stress intensity factor for the shortest cracks, mainly at the crack surface, due to the direct load on the thread by the nut.
6. This effect decreases with the crack length, which is good for the service life.
7. The trend is the opposite for the deepest circular crack, because of the bending effect.
8. The worst effect of the nut is related to the next three simultaneous conditions:
   - short cracks
   - points next to the free surface
   - circular cracks

## ACKNOWLEDGEMENTS

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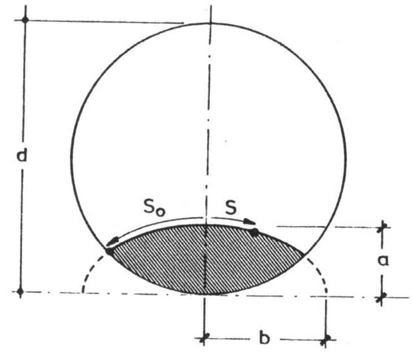


Figure 1. Crack shape

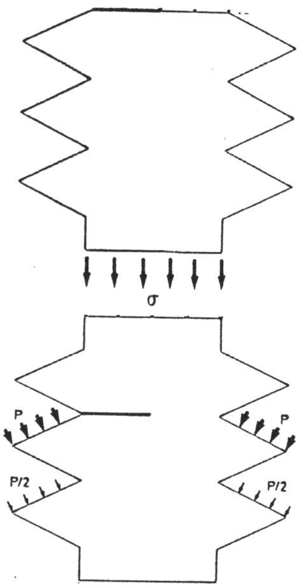


Figure 2. Loading conditions: (a) tension (b) nut loading.

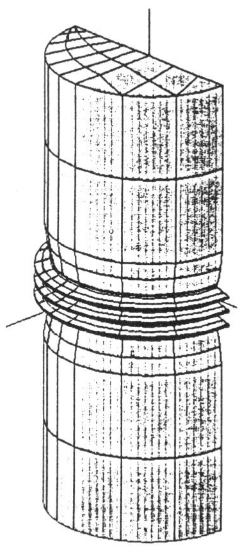

Figure 3. Finite element mesh for the cracked bolt

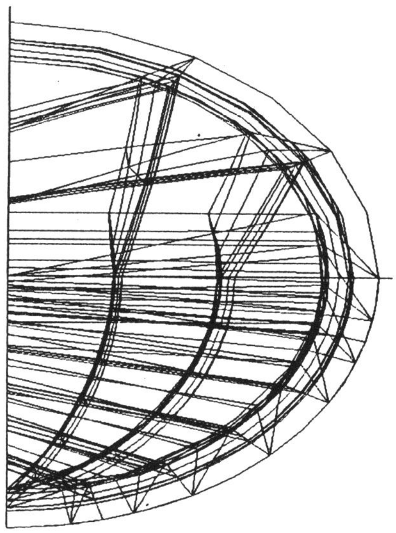

Figure 4. Detail of mesh around the crack

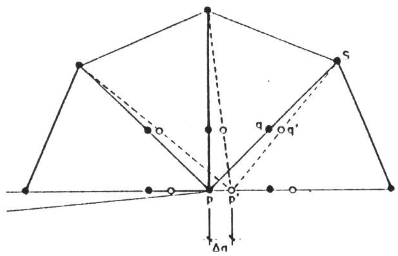


Figure 5. Modeling of crack advance in the normal plane

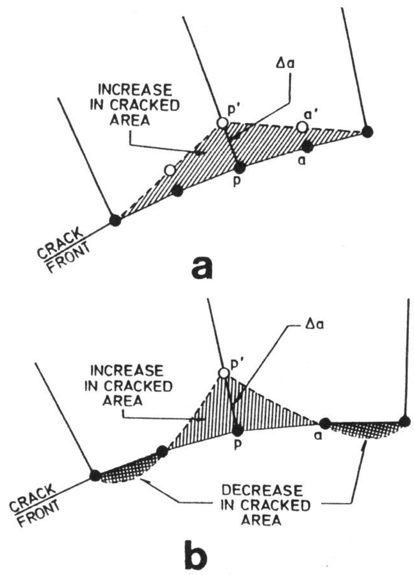


Figure 6. Local crack advances in the plane of the crack: (a) without decrease in cracked area, (b) with decrease in cracked area.

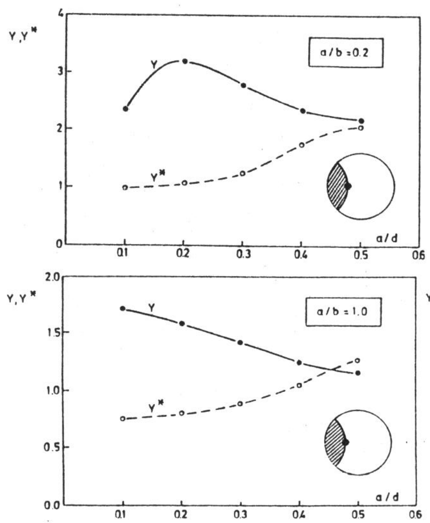


Figure 7. S.I.F. vs. crack depth (center; Y for nut loading, Y* for tension loading)

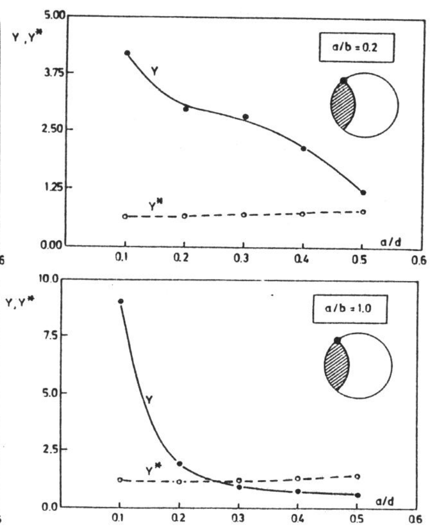


Figure 8. S.I.F. vs. crack depth (surface; Y for nut loading, Y* for tension loading)