AN INTERACTIVE CAUSTIC-BASED METHOD OF STRESS INTENSITY FACTOR DETERMINATION

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A technique for the determination of stress intensity factors from caustics by means of an interactive image processing system has been developed. For reliable results the use of a multipoint overdeterministic data reduction technique requires the selection of at least 10 points along the recorded caustic. The method pursued here needs little manual input from the analyst such as marking the approximate center of the shadow spot shown on the monitor and the number of data points. The selection of data points along the experimentally recorded caustic curve for the analysis is done automatically, the selected points are marked on the screen and if necessary interactive correction of the positions is possible. Final proof of the correctness of the result of the automatic data point selection is achieved by comparing for acceptable coincidence the numerically generated caustic determined on the basis of the results of the data reduction technique with the experimentally recorded caustic.

INTRODUCTION

Since the caustic or shadow-spot technique has been developed by Manogg [1] in the pioneering work on light deflection due to stress intensification at structural discontinuities this method has become a powerful tool in experimental mechanics. The method originally introduced by Manogg for transparent materials has been adapted by Theocaris and his co-workers [2, 3] for nontransparent materials. The general equations of caustics for plane static and dynamic elasticity theory may be found in the review article by Kalthoff and Beinert [4] and Rossmanith [5].

The physical principle of the method of caustics is the inhomogeneous deflection of parallel light rays during their passage through a plate specimen due to two effects: the reduction of the thickness of the specimen and the change of the refractive index of the material as a consequence of stress intensification. The transmitted and/or reflected light rays form a shadow space and the intersection of this shadow space with a screen produces a shadow area (shadow

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spot) surrounded by a bright curve - the caustic (Figure 1). For nontransparent materials with a mirrored surface the reflection-light method is utilized where qualitatively similar shadow patterns can be observed. In caustic analysis the cases of plane stress and plane strain, transmitted light and reflected light turn out to be basically similar and differ only by the values of the elasto-optical parameters.

CRACK TIP CAUSTICS

The classical mixed-mode crack problem is governed by the complex stress function

$$\Phi(z) = \frac{1}{2} K^* \frac{1}{\sqrt{2\pi z}} \qquad \text{with} \qquad K^* = K_1 + iK_2 \tag{1}$$

where K^* is the complex stress intensity factor. The associated parametric equations $Q'[x'(\theta), y'(\theta)]$ of the crack tip caustic on the screen plane in Cartesian coordinates read

$$x' = r_0 \cos\theta + C_{exp} r_0^{-3/2} \left[\cos\frac{3\theta}{2} - \mu \sin\frac{3\theta}{2}\right] \tag{2}$$

$$y' = r_0 \sin\theta + C_{exp} r_0^{-3/2} \left[\sin\frac{3\theta}{2} + \mu \cos\frac{3\theta}{2}\right] \tag{3}$$

$$\text{with} \qquad C_{exp} = K_1 \frac{z_0 dc}{\sqrt{2\pi}}, \qquad \mu = \frac{K_2}{K_1} \tag{4}$$

$$\text{and} \qquad r_0 = \left[\frac{3}{2} C_{exp}\right]^{\frac{2}{5}} \left[1 + \mu^2\right]^{\frac{1}{5}} \tag{5}$$

where θ is the polar angle, r_0 is the initial radius defined in the model plane, μ denotes the mixed-mode index and the constant C_{exp} depends on the geometrical set-up and material parameters (Figure 2), Kalthoff [6] and Rossmanith [7].

A multi-point overdeterministic method of data reduction for K-determination makes use of a large number of arbitrarily selected data points along the entire contour of the caustic. For data selection, a coordinate system (x_D, y_D) is placed parallel to the crack with the origin within the shadow area as shown in Figure 3.

The transformation

$$x_D = x' + \Delta x \qquad y_D = y' + \Delta y \tag{6}$$

transforms the image equations of the method of caustics to the expressions

$$x_D = f(K_1, \mu, \Delta x, \Delta y; \theta) \tag{7}$$

$$y_D = g(K_1, \mu, \Delta x, \Delta y; \theta) \tag{8}$$

where the as yet unknown quantities K_1, μ, Δx and Δy will be determined by the method of minimizing the sum S

$$S = \sum_{i=1}^{n} \rho_i^2 = \sum_{i=1}^{n} \left\{ \sqrt{x_i^{*2} + y_i^{*2}} - \sqrt{x_i^{Q^2} + y_i^{Q^2}} \right\}^2 = min \tag{9}$$

of the squares of the differences ρ_i^2 (Knasmillner, [8]). The difference ρ_i is defined as the distance between the position of the point $Q_i^*(x_i^*, y_i^*)$ of the experimentally recorded caustic and the position of the corresponding point $Q_i(x_i^Q, y_i^Q)$ on the radial s_i determined by use of the estimates of K_1, μ, Δx and Δy (Figure 3).

INTERACTIVE MULTI-POINT METHOD

The hardware requirement for the technique introduced here consists of an image scanner (e.g. a CCD-camera with A/D-converter), an image storage device provided with a monitor and a "mouse". The digitized and stored caustic pattern displayed on the monitor is required to select interactively certain points on the screen.

First, a coordinate system (x_D, y_D) is oriented parallel to the crack and placed in such a way that its origin is located within the shadow area. Next, the region of the experimental caustic is selected where data points can easily be identified. In general, the part of the caustic adjacent to the faces of the crack is blurred and obscured (blurr zone **B**) and should therefore be discarded (Figure 4).

A total of n radials s_i, homogeneously distributed within the zone of the clearly visible caustic **A** is automatically chosen for point identification. This selection leaves open the proper position of the data-points Q_i^* on the finite width of the experimentally recorded light intensity distribution which makes up for the caustic. The proper positions of the Q_i^*'s are determined from an evaluation of the density distribution along the radials s_i within the caustic range (Figure 4). The experimentally recorded light intensity distribution (or film density distribution) across the caustic band is shown in Figure 5a. Low pass filtering produces the smooth density trace shown together with the ideal caustic density trace in Figure 5b. The correct K-value would obviously be obtained from the proper selection of the point Q_i^* at distance $r_i^* = r_i$.

The automatic point selection method bases on the fact, that a variable relative grey level serves as a degree of freedom. Upon interactive selection of an estimate relative grey level (0% and 100% correspond to grey minimum and grey maximum, respectively) radial positions r_i^* for the data points Q_i^* have been determined. The magnitude of the relative grey level depends on the experimental set-up and recording characteristics.

Once the data points Q_i^* have been identified the automatized method follows the multi-point overdeterministic data reduction technique outlined in the section before (Rossmanith and Knasmillner, [9]).

The last step in the K-determination procedure is the visual inspection of the result by plotting the caustic line and the associated experimental input points together with the stored image of the analyzed caustic on the monitor. If the coincidence of the analytically generated caustic line with the experimental recorded caustic is unsatisfactory, the filter characteristics, the relative grey level for the caustic line can be set to new values or some of the data points can be repositioned or deleted for a new analysis.

References

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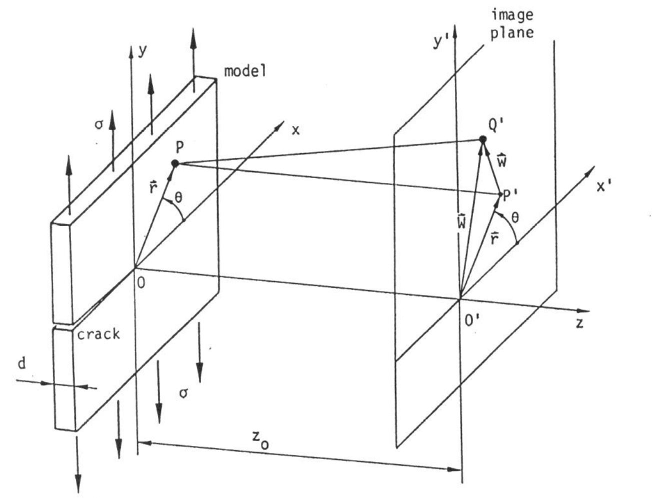

Figure 1: Light deflection in the transmission method of caustics

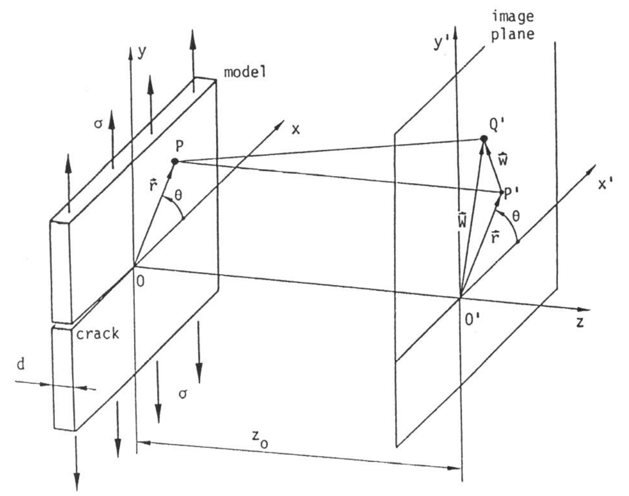

Figure 2: Stress induced light deflection in a transparent plate specimen

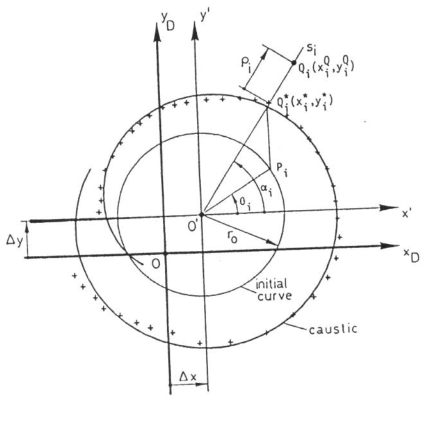

Figure 3: Coordinate systems

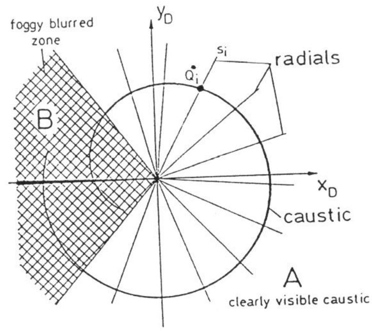

Figure 4: Data selection zones

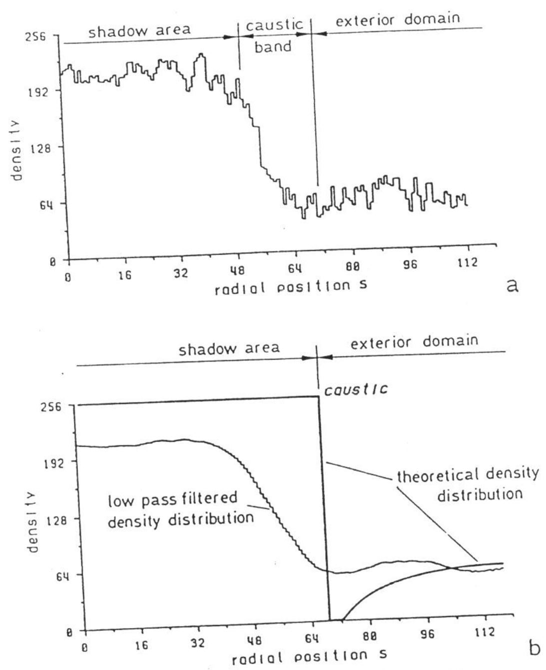

Figure 5: Density distributions for experimentally recorded crack tip caustics obtained from an analysis with an image processing system
a) experimentally recorded; b) smoothened (low pass filtered)