

TWO PARAMETER CHARACTERISATION OF ELASTIC-PLASTIC CRACK  
TIP FIELDS AND ASSOCIATED FAILURE CRITERION

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Plane strain elastic-plastic tip fields have been modelled with modified boundary layer formulations based on the first two terms,  $K$  and  $T$  of the elastic fields. These formulations match the appropriate full field solutions. Compressive  $T$  stresses reduce the stresses by an amount independent of radial distance, corresponding to the introduction of a second term in addition to dominant plastic singularity. Geometries which maintain  $J$  dominance are characterised by zero or positive  $T$  stresses. Geometries with negative  $T$  stresses can be described by a two parameter characterisation using  $J$  and  $T$  into full plasticity. A two parameter fracture criterion based on these results is presented.

INTRODUCTION

In linear elastic fracture mechanics, the stress field can be expanded in a series, following the work of Williams (1). The first term in the series corresponds to the stress intensity factor, and the second one, a uniform stress parallel to the crack flanks and denoted  $T$  stress, has a significant effect on the shape and size of the plastic one, as being demonstrated by Larsson and Carlsson (2).

In elastic-plastic fracture mechanics, the stress field can also be regarded as a series expansion, in which the leading term can be characterised by the  $J$ -integral, being the stress fields given by Hutchinson (3) and Rice and Rosengen (4). In full plasticity it is known that the deformation field is not unique. The conditions under which the crack tip deformation maintains the character of the small scale yielding field has been defined as  $J$  dominance, and quantitatively studied by McMeeking and Parks (5) and Shih and German (6). In the present work, the effect of elastic  $T$  stresses on  $J$  dominance has

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been investigated. The crack tip field has been firstly modelled by plane strain elastic-plastic boundary layer formulations involving both the K and T terms. These have been correlated with full field solutions for geometries which exhibit positive and negative T stresses.

#### NUMERICAL METHODS

Plane strain crack tip deformation and modelled by boundary layer formulations using focused meshes displacement boundary conditions, corresponding to the displacements associated with a K field plus the displacement due to the T stress. J was determined by the virtual crack extension method of Parks (7) while T was calculated from the remote K field. The K and T terms were increased in a proportional way by an imposed biaxiality parameter B.

The three geometries examined with full field solutions were chosen to represent a wide range of T stresses. A centre cracked tensile panel,  $a/w=0.5$  features negative T stresses, in contrast, a deeply edge cracked bend bar,  $a/w=0.9$ , was chosen to be representative of geometries with positive T stresses. Finally, an edge cracked bend bar with  $a/W=0.3$  with a biaxiality parameter,  $B=0$ , was analysed.

The material response was described by a Ramberg-Osgood power law with the exponent  $n=13$  and  $\alpha=3/7$ . The analyses were based on small strain theory flow plasticity, adopting an incremental form of the Prandtl-Reuss flow rule. Further details are given by Betegón and Hancock (8).

#### RESULTS

The stresses were normalised by the initial yield stress  $\sigma_0$ , and distances non-dimensionalised by  $J/\sigma_0$ . With these normalisations the stress fields obtained from a boundary layer formulations with  $B=0$  are self similar, in the sense that data obtained for a given applied K field falls on the same curve as that for a higher K, but falls below the HRR field. When the biaxiality parameter, B, is positive, the stresses are comparable to the  $B=0$  field early in the deformation, but with increased loading the stresses rise slightly. A negative biaxiality parameter produces stresses initially close to the  $B=0$  field for small T stresses, but as the T stress becomes more negative, the stress fall and depart significantly from the HRR field. The data obtained from different B values are presented in figure 1 as a family of curves which are functions of the T stress but independent of the biaxiality. The boundary layer formulations are compared with a full field solution of a center crack panel geometry ( $B=-1.06$ ), by examining the stress at distances  $2J/\sigma_0$  and  $5J/\sigma_0$  from the crack tip, as shown in figure 2 for  $n=13$ . The two term boundary layer formulation is shown to match the full field solution even at deformation levels of  $\alpha\sigma_0J=50$  which corresponds to full plasticity. Similarly the full field solution

for the bend bar with  $a/w=0.3$  matches the corresponding boundary layer formulation in which only the K term is applied on the boundaries. Finally, the same match between the two term boundary layer formulation and the full field solution was observed for the deeply cracked bend bars ( $a/w>0.3$ ).

To examine the J dominance criteria suggested by Shih and German (6), the stresses may be compared with the HRR field with the criterion that J dominated flow fields have stresses within 90% of the HRR value at a distance  $x=2J/\sigma_0$ . On this basis J dominance is maintained for T stresses greater than  $-0.2\sigma_0$  for  $n=3$ . Using biaxiality values given in the literature, the size limitations for J dominated flow are given in figure 3 for four different geometries.

Finally, the two parameter characterisation has been applied to obtain an experimental J-T fracture locus by Betegón (9). The results obtained are shown in figure 4 for three point bend specimens with different  $a/W$  ratios. The smaller cracks have more negative B values, and the J value at cleavage instability is much larger than the J values obtained for the deep cracks.

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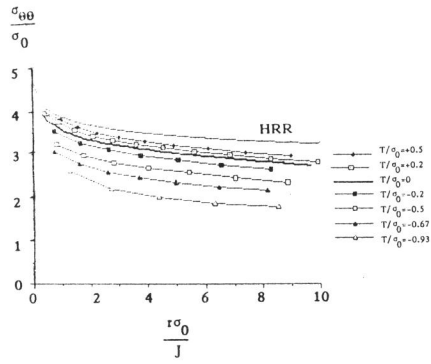


Figure 1 T-family curves

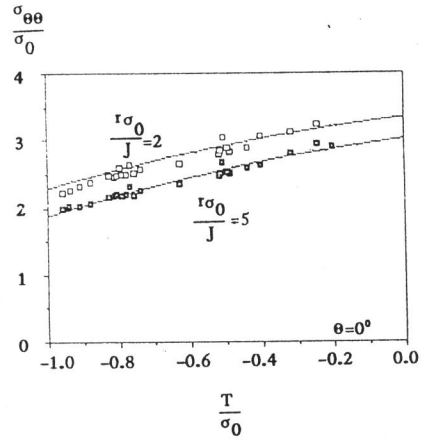


Figure 2 Boundary layer formulations and CCP full field solutions

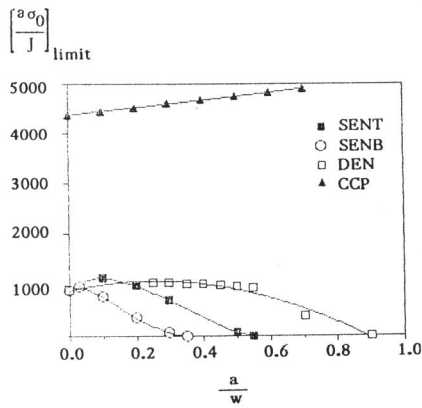


Figure 3 Size requirements for J dominance

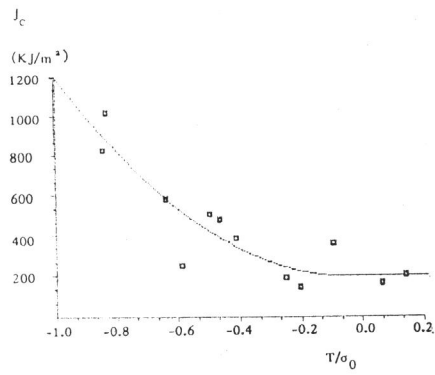


Figure 4  $J_C - T_C$  fracture locus