A RE-ASSESSMENT OF DUCTILE TEARING RESISTANCE. PART I: THE GEOMETRY DEPENDENCE OF J-R CURVES IN FULLY PLASTIC BENDING.

C.E. Turner †

The question addressed is whether a common curve of tearing toughness as a function of crack growth, Δa, can be separated out from conventional J-R curves measured in fully plastic bending tests on a given material. In Part I, the experimental evidence in the literature is shown to give a complex pattern. Some fully plastic J-R-curves are higher for wider pieces, some lower; some show a change for geometrically similar pieces whilst other cases do not.
In Part II, analysis in terms of energy dissipation rate, dU_{dis}/Bda, gives a term that reduces with growth but scales with $(\Delta a/b_o)(S/b_o)$, where S is span and b_o is ligament, leading to a rising R-curve that is $f(\Delta a/b_o)$. A consistent pattern for the geometry dependence of dissipation rate is seen as a function of thickness, ligament and crack growth in terms of the ratio between the maximum possible plastic zone size for the material and the dimensions of the test piece. For some patterns of behaviour, this allows a lower bound R-curve relevant to tearing in contained yield to be deduced. For others, areal and volumetric components of the dissipation rate can be found allowing the effect of changes in size and proportion to be better understood.

INTRODUCTION

In the 1960's and 70's elastic-plastic fracture mechanics grew from the concepts of linear mechanics, itself clearly established in the previous decade by Irwin from the classic ideas of Griffith. It will be argued here that in respect of ductile tearing in the plastic regime current concepts are not adequate and that plasticity and conventional mechanics offers a more satisfactory treatment of tearing on the macro-scale.

The traditional presentation of fracture mechanics is maintained for conditions of lefm and indeed for the 'initiation' of a crack (from a pre-existing sharp defect) in the presence of plasticity. It is where conditions include an element of crack advance with appreciable plasticity, perhaps better referred to

† Mechanical Engineering Department,
Imperial College of Science, Technology & Medicine, London.

as tearing, that alternative views are proposed. When full plasticity is reached, the new viewpoint proves to be radically different.

The motivation for the re-assessment, which becomes the objective of the present paper, is to offer an answer to the question,

i) 'can tearing toughness as a function of crack advance be separated out from J-R test data as conventionally presented?'

That poses the underlying question, to which the foregoing is but one possible answer,

ii) 'can J-R-curve data be translated from the laboratory piece to another configuration?'

That in turn implies the yet more fundamental question,

iii) 'is the rising J-R-curve a reasonable measure of tearing toughness or should some other measure be adopted?

In the first part of the paper some aspects of conventional fracture mechanics are reviewed, sufficient to highlight the features that have driven the existing developments and the factors that point towards the need for an alternative interpretation. In the second part of the paper the plasticity view is examined in more detail, particularly for the fully plastic tearing regime. The writer sees the characterisation of fracture in terms of the global or far-field energy as at least one main stream of fracture mechanics and that is the model persued here. The arguments presented cover only static fracture in geometries of high constraint bending type stress systems, ductile in the micromode. That other views exist is evident from the literature. They are left aside as not directly relevant to the present arguments which are here limited to changes of geometry, not of constraint. In retrospect the present characterisation of initiation might also be challenged for highly plastic conditions but again that is not entered into here.

LEFM AND R-CURVES IN CONTAINED YIELD

Small Scale Plasticity in the Conventional LEFM Model

The present concept of fracture toughness expressed through the energy release rate per unit area of crack advance, G, is attributable to Griffith, [1]. The crack tip stress state was later described by Irwin, [2], through the 'stress field intensity factor', K, which is a measure of the inverse square root singularity of stress and strain close to a crack tip in a linear elastic situation. The two concepts were related long since by

$$K^2 = EG \tag{1}$$

where E is the tensile modulus, modified by $1/1-\nu^2$ for plane strain.

It is then argued that, to within practical limitations, 'fracture occurs' at a critical value of K or G, denoted K_c or G_c. In the 1950's it was recognised that a material property, unique for a given environment (including strain rate), was approached only in the limit of plane strain, when the term was denoted K_{Ic} or G_{Ic}, the plane strain fracture toughness.

Plasticity on a micro-scale was introduced by Orowan, [3] and Irwin, [4], to account for the difference between true and effective surface enenergy on a quasi-macro-scale. The measure adopted by Irwin, [5], was

$$r_p = (K/\sigma_Y)^2/m\pi = (EG/\sigma_Y^2)/m\pi \qquad (2)$$

where r_p is the extent of the plastic zone ahead of the tip, with $m = 2$ for plane stress or 6 for plane strain. Even in 'strict engineering plane strain', that term itself is defined by the ratio of a plastic zone size to the smallest geometric dimension, usually one fiftieth of the thickness. Thus for lefm purposes plane strain is taken to exist when

$$B, b, a \geq 2.5\,(K_{Ic}/\sigma_Y)^2 \qquad (3)$$

Plane stress is not formally defined but is seen as

$$B \leq r_p \text{ (for plane stress, with } m = 2) \qquad (4)$$

so that the relative size of the plastic zone is the main distinction between plane strain and plane stress modes of fractures. As a convenience the quite appreciable plastic zone found in plane stress is treated by a correction to the lefm analysis so that plasticity is at once subsumed into fracture mechanics rather than fracture mechanics into plasticity.

In elasticity, meeting the energetic balance between potential energy release rate, G, and effective surface energy, G_{Ic}, is a necessary and sufficient condition for fracture since there is no other role for energy to assume. This view easily leads to the expectation that there will in general be an effective surface energy or fracture toughness, since lefm proclaims it as an entity in its own right. The physical fact that effective surface energy is just localised plasticity is recognised but not persued even though in the case of plane stress, the plastic zone is a macro term comparable to the thickness dimension of the component. Another view is that fracture initiates at some characteristic severity, $K = K_{Ic}$ for plane strain, and that the role of G relates to the stability or otherwise of the growth. Given that in the classic Griffith problem of a body with a small crack, held at fixed displacement remote from the crack, only two energetic terms are distinguished, recoverable strain energy and surface energy, then the two physical events of crack initiation and onset of unstable growth, coincide, but the controlling event is initiation at some characteristic severity. The energetics of stable or unstable growth rather than the applied intensity view of fracture were well brought out by Gurney, [6].

The key point is that either criterion, $K = K_{Ic}$ or $G = G_{Ic}$, can describe crack initiation but neither imply anything about the stability of the process. In most remotely loaded cases the derivatives, dK/da or dG/da, are positive even at fixed displacement so that for a 'neutral material' in which the toughness is constant with repect to any changes, including strain rate and temperature, the instabilty condition is automatically satisfied once the critical criterion for initiation is met. Stable growth at K_{Ic} or G_{Ic} is however quite feasible for a neutral material provided dK/da or dG/da is negative. A well known case is the 'crack line loading' of a large plate where a central crack is pulled open by opposing forces, P, applied at the crack face such that $K = P/\sqrt{\pi a}$ and dK/da is indeed negative.

Thus a statement of fracture should always include two conditions, a critical quantity at which a crack growth will initiate, be it expressed for lefm in terms of K or G, and a d/da rate, expressed in lefm by dK/da or dG/da, to show whether the crack will continue to grow in relation to a corresponding measure of toughness. It is convenient to refer to toughness as 'resistance to cracking or tearing', R, with the implication that in general R may be a function of crack growth expressed in units compatible with K or G as appropriate. Further complexities of whether a crack remains stable or unstable with yet further extension after the first burst of growth and the fact that some materials seem to experience a decrease in toughness with strain rate or crack speed are not persued here.

Although in strict lefm conditions the K and G concepts have the same implications, it can be argued that in more general circumstances a measure of applied severity (which reduces to K for linear elasticity) is appropriate to initiation whereas a measure of energy rate (which reduces to G for linear elasticity) is appropriate for unstable behaviour. That statement does not of course deny that energetics must be satisfied on the micro-scale even for initiation. That requirement is automatically satisfied just prior to initiation by the equilibrium, compatibility and stress-strain statements of continuum mechanics. However, the second derivative of energy, dG, rather than the first derivative, G, is the usual criterion for instability and is often expressed as

$$\partial G/\partial a \geq dR/da \qquad (5)$$

where ∂G is taken to be retricted by whatever physical conditions of end fixation apply, constant load, known compliance or fixed load as may be.

For reasons that are not clear, the plane stress lefm case is often illustrated at fixed load whereas in later epfm cases the role of external compliance is always emphasised. Compliance is always relevant in respect of instability for both lefm and epfm. As already remarked, if circumstances are such that dG is automatically more positive than dR then the first derivative of energy becomes the criterion. A similar point with respect to ductile tearing instability will be referred to later.

Plasticity Limited by Thickness

For lefm plane stress testing G or K is used as the measure of toughness, invariably corrected for the size of plastic zone. For most pieces with remotely applied loads, dG/da is positive but for thin sheet the material toughness, dR/da, increases even more quickly so that growth is stable. Although there is evidence, Larsson & Carlsson,[7], that as plasticity increases the size of plastic zone is affected by the difference in transverse stresses in different geometries, at least some examples exist e.g. McCabe & Heyer,[8] where the resulting R-curves for two different configurations are closely similar. Plasticity is always invoked to explain the increase of toughness with crack growth. The conditions are such that the crack tip plastic zone, which of course starts vanishingly small, grows with increase of load according to Eqn.2 until comparable in diameter to the plate thickness. For materials of moderate toughness, tearing may lead to a final unstable fracture which occurs whilst the stresses on the ligament are well below yield so that lefm is still plausible.

The well known picture of the effect of thickness on 'toughness' in the lefm regime is shown Fig.1. It is clear that for the thinner sizes the toughness represented is not that at which tearing initiates but that for an arbitrary amount of tearing, perhaps maximum load or unstable growth in a particular piece. Such an lefm type treatment is again acceptable in practical terms but tends to hide the fact that the problem is physically one of contained plasticity, i.e. the plastic zone is restricted in size by the thickness of the sheet coupled with the inability of the material to resist tearing. The importance of restriction of the size of the plastic zone is crucial to the later arguments. The concept was used, Bluhm, [9], to explain the reduction in toughness of sheet below a certain thickness, the left hand limb of Fig.1. The explanation was simply that there seems to be a maximum size of plastic zone that any material can sustain at a sharp crack. If that size is not fully exploited because the zone size is restricted by thickness then the maximum possible plane stress toughness is not reached. The more subtle understanding of the second part of the argument, that as plane strain is induced in cracked pieces of thicker section then the capacity of the material to resist tearing is reduced, was, it seems appreciated before the first 'self evident' stage.

EPFM AND R-CURVES IN UNCONTAINED YIELD

Initiation

Two main epfm concepts for severity up to initiation, have emerged, COD, [10] [11], and J, [12], with the related HRR stress fields, [13][14]. Both show that lefm with the plastic zone correction is surprisingly useful up to some 0.8 of the limit load. Most evidence suggests that a single term description of fracture is not correct in the presence of plasticity but that it may be adequate for practical purposes for other than shallow notches. If a one parameter description of a crack tip fields is adequate then either term, J or COD will suffice, especially when restricted to deep notch bending, as here. They are related by

$$J = m\sigma_Y\delta \qquad (6)$$

where m is a number usually taken in the range $1 \leq m \leq 3$ depending on the geometry and the work hardening properties of the material in question. If a single parameter model is inadequate then neither term can suffice. A difference in the critical values or J or δ at initiation is often accepted as between tension and bending type loadings but evidence of which is the more appropriate is not convincing. That the value of m will depend on geometry is supported by slip line field, McClintock,[15], and computational studies, McMeeking & Parks,[16], whereas experimental evidence, Robinson, [17], Gibson *et al*, [18], points to a near identity over a number of configurations, at least for low strength structural steels.The J model is persued here without further comment but the data used will relate only to various sizes and proportions of bending or compact tension type pieces for which m is usually taken to be about 2. The relevance of the arguments that follow to a relationship between bending and tension stress states has not yet been studied.

In the J test methods the prime quantity measured is the work done. From that, J is derived as an implied severity using the relation

$$J = \eta A/Bb_o \qquad (7)$$

where A is the area of the load v load-point displacement diagram, B is thickness, b_o is ligament width and η is a geometry dependent constant, closely 2 for deep notch bend geometries and about 2.2 for compact tension pieces. The difference of intention between lefm and epfm tests must be recalled. For lefm, a close approach to unstable fracture is sought, whereas J_{Ic} data are found by extrapolation of an R-curve back to a blunting line (or to some agreed small growth such as 0.2mm averaged across the thickness, probably implying initiation only at the centre of the cross section) in a condition of stable tearing.

A schematic load displacement record extending into the plastic regime is shown Fig.2. It is notoriously difficult to determine the point of initiation from such a diagram without other data, now often found by use of the unloading compliance technique. This implies that at initiation (and as later seen, also after initiation) an increment of external work, Pdq, that actaully causes tearing, differs only in the second order from an increment that does not, the difference being the small 'wedge' of area *inm* on Fig.2.

To investigate that argument, the split between the elastic and plastic components of initiation toughness is sought. Some J-R-curve data for a titanium alloy tested in 18 different deep notch bending sizes or configurations were given, John & Turner,[19], where the test conditions are described more fully. The scatter in initiation values from $J = 0.17 \pm 0.04$ MN/m reflects some unknown combination of experimental treatment and material variability. A rather similar scatter is detailed, Davis *et al,* [20], for A533B steel and a yet more extreme case, McCabe *et al*, [21], for A508. Despite such variations, a constant value of initiation toughness is often assumed in the plastic regime, with a value $E'J_{Ic} = K_{Ic}^2$. This can be viewed as a working approximation to some particular data or simply justified as a theoretical model of an idealised epfm. The mean value of 0.17MN/m initiation toughness, [19], has here been split into elastic and plastic components for one series of tests, B = 35mm, a/W = 0.55, $16mm \leq b_o \leq 28mm$, using the loading records to find the elastic component. The results, shown Fig.3, suggest a near linear variation of each component with size of ligament, such that extrapolation to very small width implies no elastic component and extrapolation to large width implies no plastic component. Such a large test would not be judged as plane strain according to Eqn.3 but would allow fracture to initiate well below net section yield where treatment by lefm is still acceptable.

Clearly, the lefm value can only be treated as 'all elastic' within the conventional framework that toughness is a surface energy. In the Orowan-Irwin physical model of effective toughness, [3][4],

$$2\gamma_{eff} = 2\gamma + U_{pl} = 2\gamma(1 + \beta) \qquad (8)$$

$$c = 0.2EJ/\sigma_Y{}^2 \tag{9}$$

When combined with the way in which J-R-curves are measured, this point becomes of crucial importance to the interpretation of tearing data.

Measurement of R-Curves

The present methods stem from Krafft *et al*, [24], where curves of rising toughness were derived in conditions of lefm plane stress. Such curves are usually derived by measuring load as a function of crack extension. K is then derived, probably with correction for plastic zone size, and plotted as such, or converted to G using Eqn.1. The abscissa may be just Δa, the physical crack extension, though some advocate correcting that for plastic zone size. Irrespective of the precise details, the toughness property, or R-curve, is inferred from the estimates of applied severity, K or G, on the grounds that for stable behaviour the applied and absorbed terms must be identical. No attempt is made to separate out the energy taken to cause the plastic zone from that conceptually causing the fracture. Indeed, an initiation toughness is often not quoted in the literature on lefm R-curves, perhaps because plane strain conditions as defined for lefm usage are clearly not met or perhaps because the whole process is directed more towards estimating the final instability of a sheet component rather than a condition for initiation. That difference in objective may well arise from the use of lefm R-curve methods for the residual strength of thin panels in aircraft or bridges where failure is usually approached by a continuing process of fatigue crack growth rather than by overload of a hitherto non-advancing crack.

The methods adopted in epfm are rather different, at least for J testing, in that the work done is the prime quantity measured. Crack growth is often measure by unloading compliance techniques although key-curve or multiple test piece methods are also used. From the work done, J is derived as an implied severity using an extension of Eqn.7. Various modifications have been proposed. Essentially all take the form

$$J_R = J_i + \Sigma dJ = \eta\,(A_i)/Bb_o + \Sigma\ f[\eta dA/Bb - (\eta A/Bb^2)db] \tag{10}$$

where A is an area taken from the loading diagram and suffix i implies the value at initiation, however defined. The particular functions adopted for area and for the last two terms has changed several times in the last few years according to fresh experimental evidence or analyses. In summary, the arguments depend firstly on whether the area needed to define J represents work done, U, or internal energy, w, and secondly on whether the value of b is left at its original value, b_o, giving a term, J_o, used in some early analyses, or is updated to its current value. Even then the final term in db (= -da) may be included, if dJ is formed by differentiating Eqn.7, or omitted in accord with [23].

Up to initiation U = w so that either may be used in Eqn.7 and for the first term on the right of Eqn.10. If A is taken as work, U, it is related in the increment to internal energy, w, whether w is recoverable or not, by

$$dU = dw + BRda \tag{11}$$

some large proportion of the toughness is admitted to be local plasticity, i.e $\beta >>> 1$ even for a rigorous lefm plane strain situation. The interpretation offered here is that initiation (and later, propagation) is a two stage process whereby preceding plasticity damages material just ahead of the crack tip until the elasticity causes a micro-instability for the actual separation. In the strict lefm case, a macro-instability occurs such that the damage and final separation are all one event, but even the lefm 5% offset procedure implies a division into the two stages. In the limit of vanishingly small width, the implication is that the damage or process zone covers the whole of the very small cross section so that no final area of separation exists on which micro-instability could occur. Such a two stage picture is very similar to that expressed by Wnuk, [22], as a 'final crack opening stretch' criterion. It is seen as compatible with the observation above that the contribution of external work to fracture is second order except through the preceding plastic damage.

The split into 'damage' and 'separation' energies may seem rather pedantic. If the split is made, the above arguments imply that the former is provided by external work whereas the latter is provided by release of internal elastic energy. Note however that the value of that elastic energy will only be G_{Ic} for the strict lefm case where there is no damage ahead of the fracture zone. For the epfm case, the more the prior damage, the less the elastic contribution needed to cause the final separation.

If the split is not made then there is no first order distinction between 'remote' and 'local' energy. The implication on the first objective (Introduction, Question i) then seems final. This picture will be persued in the second part of the paper.

Stable tearing in uncontained plasticity

The problem of tearing in full plasticity is generated by the desire to test as small a sample as feasible, to restrict failures under accident conditions to local rather than extensive rupture and to control problems of metal working in the fully plastic state where fracture has to be avoided. For these reasons a better understanding of the relation between conventional plasticity and fracture mechanics seems essential. Almost inevitably the measure of toughness is taken to increase with tearing. If it were not so, then how could the test remain stable with the tear advancing or stopping according to the whims of the operator of the testing machine? That the situation is stable is indisputable. Whether the toughness is increasing is central to the present discussion and depends on both circumstances and definition.

A complete solution to the general problem of ductile tearing is not known. The best available pictures are a continuation of the HRR fields for a hypothetical total theory plasticity/non-linear elastic material and the RDS model, [23], for non-hardening rigid plastic behaviour. In the RDS model the crack tip stresses are remarkably similar to the Prandtl field whilst the strain singularity becomes logarithmic. But this is a contained yield model and even if it is argued that the dominant terms are not altered until the boundaries interfere with the singular zone, the effect of uncontained yield is felt in another way. In [23], J-R-curves are presented on a normalised abscissa of $\Delta a/c$, not just Δa, where

where BRda is the 'effective surface energy' corresponding to BGda in lefm or BJda in a non-linear elastic (nle) model, shown dotted, Fig.2. The latter follows the strict nle or deformation plasticity derivation of J and is usually denoted J_d, but that does not represent the physical behaviour of real incremental plasticity materials. An example of the effect of the choice of dU or dw was shown, Etemad & Turner, [25], where the same data were analysed by both interpretations. According to the former, J must continue to increase, reflecting the continued dissipation of work. According to the latter, the value of J at first increases with the increase of U but then decreases as ever larger values of BRda are subtracted at each step whilst the value of the internal energy, w, decreases with falling load and smaller ligament.

Later a modified J, J_m was introduced, Ernst, [26], arranged such that when plasticity was negligible $J = J_{el} = G$ and when the elastic contribution was negligible, the term reduced to $dJ = \eta dU/Bb$ for non-hardening plasticity . A term corresponding to d(1/b)/da was not included in accordance with the conclusions of [23] that the characterising parameter should not be an explicit function of crack growth. Yet more recently, [27], the elastic component has been derived from the load, as for G, and the plastic component of has been taken more in accord with the J_o formulation.

There is no doubt that if rather severe restrictions on size and amount of crack growth are introduced then several of these formulations give results that often seem independent of geometry within one family of types, such as bending and CT, although none of the methods appear satisfactory for transferring data from bending to tension type loadings. In summary it can be said that the J based R-curve is essentially the summation of increments of work, normalised by a size term, Bb, and a shape term, η, sometimes with corrections to follow the original nle J model and sometimes to follow the RDS model of [23]. It must remain open to question whether the analysis now being advocated is but another turn of the correlation wheel or whether a more fundamental relationship is indeed obtained.

The geometry dependence of tearing toughness

It is universally accepted that toughness is a function of thickness since that affects the degree of triaxiality. It is the essence of fracture mechanics that for a given thickness, the effects of changes in other dimensions and loading patterns can then be accounted for but there is no absolute agreement on whether toughness beyond the lefm regime is indeed independent of geometry. Arguments related to initiation have already been cited, [16]-[21]. The difference of evidence is not fully resolved but is not persued here. In the case of ductile tearing there is even less agreement on the effects of geometry and loading than for initiation. As remarked earlier, the present analyses have only been explored for bending or CTS type loading, but even within that restriction there is a range of trends, apparently conflicting at first sight.

In an early study, Garwood *et al*, [28], it was not supposed that R-curves would be size independent. The suggestion made was that an R-curve component for flat fracture and a separate component for slant or shear-lip fracture might individually be geometry independent. Some evidence of that was presented by splitting R-curves for several cases, after the amounts of flat and shear-lip fracture had been observed. Prediction of the amount of each component beforehand, was not made. A more extensive study of the same idea was made, [18], and reached the same conclusion and stumbling block. The present most common interpretation seems to be that a small amount of growth will still be dominated by the pre-existing HRR field. This has led to the concept of 'J-controlled growth' extending, according to computational studies, Shih *et al*, [29], up to $\Delta a \leq 0.06b$.

As a further precaution against over-estimating toughness, side grooved data are often used, with the claim that such curves give rather similar initiation values but are 'lower bound' thereafter. Whilst it is true that side grooved R-curve data fall below non-side grooved data, the degree of grooving and the configuration (i.e. absolute size and B/b ratio) to give a truly lower bound curve are not known, Etemad & Turner, [30].

Cases where R-curves are found to be independent of size and/or configuration have been reported in the literature. In [18], J-R-curves measured for several bend and tension geometries on a low strength structural steel were geometry independent for growth up to some 30% of the ligament. The crack opening angle (COA) was also measured during stable tearing and that was similarly independent of the configurations tested. In Ingham *et al,* [31], the R-curves for A533B steel over a range of geometrically similar sizes from 10mm thick to 100mm thick were reported to be the same. In Etemad & Turner, [32], pieces of HY130 gave lower R-curves for wider ligaments; in Garwood, [33], pieces of A533B gave higher R-curves for wider ligaments. In fact quite cursory study of the literature shows J-R-curves following at least four different patterns, sometimes within a given material. In the following, data on A533B are marked # and on HY130 *. Curves for geometrically scaled pieces are the same, [31]#, as are some in [21][32]* whilst others in [21] and [32]* are not the same. For a given thickness, some R-curves fall lower as width increases, [19][20Fig.14]#[21][32]*, some rise higher as width increases, [28] [20Fig.13]#[33]#, whilst others are independent of width, [20Fig.12]#[20Figs.4-6]*[34]*

Examples of these four trends all for one material, A533B, are shown Figs. 4 a,b,c,d.

DISCUSSION

It is not apparent how much these different effects are caused by the differences in the definition of the J term used i.e. the differences of interpretation of Eqn.10. It must be recalled that standard or tentative standard test procedures restrict sizes and amounts of tearing such that J-controlled growth is expected whereas much of the data cited here falls outside at least the restriction on the amount of growth. Certainly some of the data cited are not derived by current standard methods. Nevertheless, for data to which the writer has direct access, and for other cases where some reconstruction of the raw data can be made, the writer has formed the impression that the definition of J used is not of major concern. There are too many cases, some contradictory for the same definition, some similar for different definitions, to be reconciled by use of a common definition of J in Eqn.10.

The effect of side grooving must also be recalled. Of the data with various trends referenced above, that for geometrically similar pieces giving identical curves from [31][32] were not side grooved but from [20] were side grooved. For the 'wider-lower' behaviour at constant thickness, pieces in [19] and [32] were not side grooved whereas those in [20] and [21] were. For the 'wider-higher' behaviour at constant thickness, pieces in [20] were side grooved but in [33], were not. For 'no trend' at constant thickness, pieces in [20] were side grooved but in Dagbasi, [34], were not.

Thus the answer to Question ii, 'Can J-R-curves be translated from one test-piece configuration to another' is 'No, not in terms of the conventional J-R-curves, even within the limited field of different widths of bending pieces'. It was recognition of that state of affairs that in due course led to the re-analysis introduced in John, [35], Etemad & Turner, [36] and John & Turner, [37], described in more detail in the second part of this paper. However, a partial solution of geometry dependence had meanwhile been seen. It was noted [32] that R-curves for a particular set of data on HY130 steel fell in the 'wider-lower' pattern in such a way that if the abscissa was scaled to $\Delta a/b_o$ then the curves were brought together. Other cases from the literature were noted, Etemad & Turner, [38][39], where the same scaling brought some data together although some other data did not require scaling and yet other cases seemed to scale on the ordinate. In Wei *et al*, [40], it was suggested that the scaling of the abscissa of R-curves was consistent with the RDS model in that Δa should be scaled to that factor that limited further disspation of work. The difference was that in the contained yield theory of [23] the factor was the material toughness as represented by c, Eqn.9, whereas in uncontained yield data the factor might be either ligament, b, or thickness, B. The 'wider-lower' data are controlled by ligament and it will be argued that if the ligament is increased sufficiently, lefm behaviour will be reached albeit not necessarily in lefm valid plane strain.

Those arguments started independently of the rationale that is now emerging from [36] and [37]. In brief, tearing toughness is re-defined as the dissipation rate

$$D = dU_{dis}/Bda \tag{12}$$

That term includes the energy dissipated in 'remote' plasticity (provided it adjoins the fracture as distinct from a separate enclave) and in the fracture process itself. The source of the energy being dissipated is both external work and recovered internal elastic energy. The view that the former causes the plastic damage and the latter causes the actual separation has been argued for initiation, Fig.3. A similar distinction will arise in the tearing process with the further complication that additional elastic energy is converted directly into plastic work during the separation process in the fully plastic state.

In [36], data for A533B was taken from Mecklenburg *et al*, [41]. They termed the incremental rate dU_{pl}/Bda, and showed a series of separate curves for several a/W ratios, reducing against Δa. In [36] the scaling of those data to a common curve on an abscissa of $(\Delta a/b_o)(S/b_o)$ was noted. The concept of the combined energy dissipation, differentiated to give a d/Bda rate as in Eqn.12, and scaled against $(\Delta a/b_o)(S/b_o)$ was outlined [37] for the same data as in [19]. Watson & Jolles,[42] also proposed 'plastic energy dissipation' as a measure of 'toughness' in the fully plastic case. It was found to be very ligament size dependent for plain sided HY130 and independent of Δa

whereas for side grooved pieces it was a strongly reducing function of Δa. No scaling was noted and the concept was not apparently persued.

In [37] a relation between D, as defined by Eqn.12, and a rising R-curve defined in a particular way, J_{dis}, was also noted. That raised Question iii, 'If initiation causes plastic deformation and damage, should tearing toughness be regarded as increasing according to the conventional J-R analysis based on accumulated work done, or should it be regarded as decreasing, according to the increment of dissipated energy, Eqn.12?' At least in [36] and [37] there was the additional advantage that such a term seemed to scale more readily than the conventional one.These points will be discussed further in the second part of this paper but it is of interest to note that a reducing R-curve was advocated, Boyd,[43], before the rising R-curve came into vogue for lefm cases in plane stress.

INTERIM CONCLUSIONS

The contribution of external work to fracture is indirect. The work is absorbed as internal energy from which an elastic component is recoverable but from which the plastic component is not. Plasticity causes damage in the sense that after a certain amount of plastic deformation, less energy is needed to cause separation than before the damage. Thus the plastic component 'does the damage'; the elastic component 'causes the actual fracture'.

A 'fracture toughness' can be identified either as

a) as the elastic component of energy rate, dw_{el}/Bda, for the final separation; or

b) as the total dissipation rate, D, for plasticity and fracture combined.

The dissipation rate, D, includes all plasticity connected to the fracture surface but of course excludes any isolated from it. It is obviously geometry dependent but scales with the laws of plasticity, subject to reservations on the effects of constraint that have not been considered in the work so far.

No separation of energy into 'local' and 'remote' components, other than that implied by a), is apparent in the data at this stage, despite suggestions in [18] and [28] that R-curves might be split into flat and slant fracture components.

SYMBOLS USED

A = area under the load-displacement diagram

D = energy dissipation rate per unit area

q = displacement of loading point

r = plastic zone size

S = span (or arm of the bending moment for CT pieces)

U = external work done

w = internal energy, recoverable or not

η = geometric factor relating J to work per unit area.

suffix c; current size, (applied to the ligament)

suffix dis; dissipation

suffix el; linear elastic

suffix i; initiation

suffix pe; plastic, in plane strain, (applied to plastic zone size)

suffix ps; plastic in plane stress, (applied to plastic zone size)

suffix pl; plastic.

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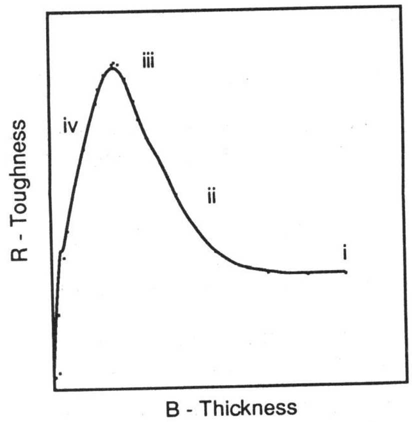
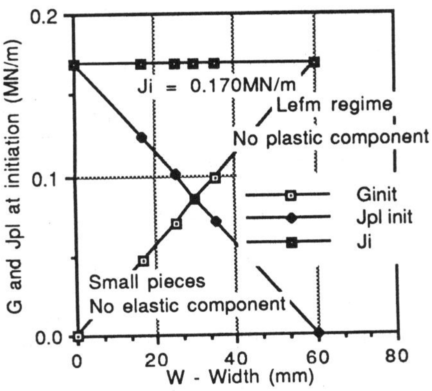


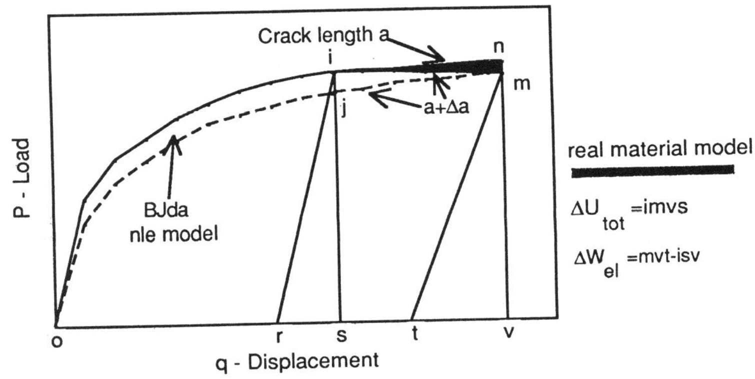
Note the ligament must be much larger than thickness if regimes ii, iii and iv are to remain elastic.

Figure 1. The variation of toughness with thickness in the lefm regime.



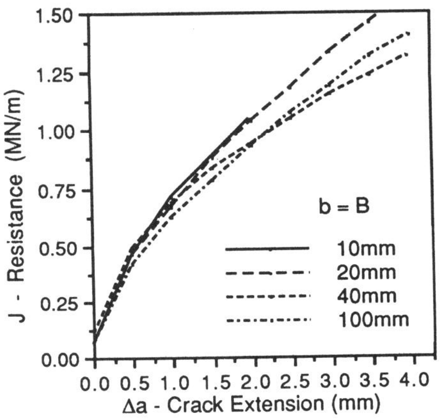
A value of $J_1 = 0.17$MN/m was assumed common for initiation in [19] for several series of tests for this tianium alloy.

Fig. 3. Elastic and plastic components of toughness for tests at various widths.

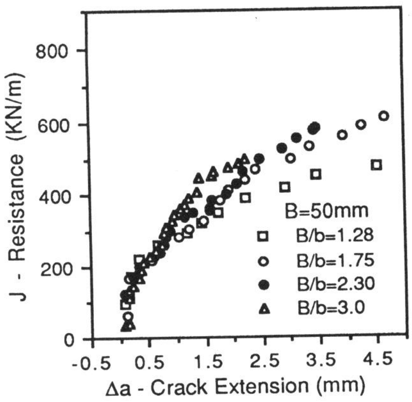


Oin is the loading curve for a crack length $a_o$with initiation at i and tearing Δa to point n. Note the second order difference between continued deformation and tearing. The relation between the increment of work, ΔU, the increment of internal energy, Δw, and the toughness according to the nle model of J, where $\Delta w = \Delta U - BJ\Delta a$, is shown dotted.

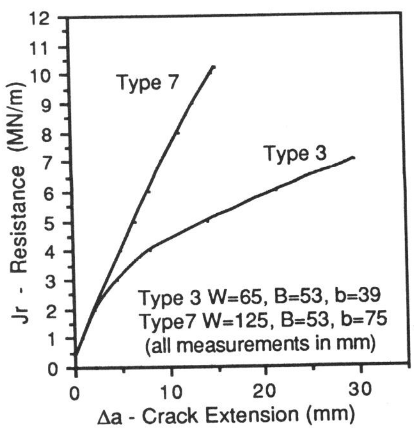
Figure 2. A schematic load-displacemen diagram for ductile tearing, when dw_{el} is +ive.



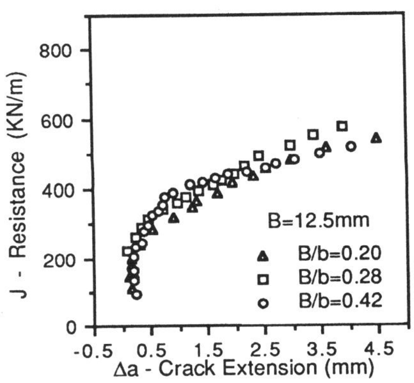
a) 'no-effect' pattern; geometrically similar
B = b = 10, 20, 40,100 mm from [31].



b) 'wider-lower' pattern;
side-grooved, 50mm thick, from [20].



c) 'wider-higher' pattern;
plain sided, 53mm thick, from [33].



d) 'wider-no-trend' pattern;
side-grooved, 12.5mm thick from [20].

Figure 4. Examples of R-curve patterns for A533B.