

## NUMERICAL SIMULATION OF BRITTLE FRACTURE IN DUCTILE MATERIALS

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Crack nucleation and propagation from a notch in a ductile material (polycarbonate) is simulated numerically on a microcomputer with a bidimensional non-linear dynamical finite differences lagrangian software. As it has been observed experimentally, numerical simulation shows that crack nucleates at the elastic-plastic interface and then propagates at high speed (near to the shear waves speed) through the specimen section. Fracture is brittle on a notched specimen and large plastic deformation occurs on a smooth specimen.

### INTRODUCTION

Polycarbonate (like mild steel) is considered to be a very tough material. Large plastic flow occurs in smooth tension tests but Garde and Weiss (1) showed that notched specimens may break in a brittle fashion. This has been explained by the existence of a critical normal stress criterion for brittle fracture. The stress triaxiality produced in the vicinity of notches in plane strain makes that the shear stress is small, though the normal stress may attain the fracture stress, at the elastic-plastic interface, as has been shown by Mills (2), using Hill or Green slip line field solution. The purpose of this paper is to simulate numerically the whole process (dynamic loading, plastic deformation, crack nucleation and propagation) using finite differences with a home-made software by Schaeffer (3,4) (finite strain and true stress, constant in a mesh, linear incremental constitutive law, plasticity and fracture criterion based on stress, numerical integration of Newton's laws of dynamics).

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## THEORY

The finite strain tensor (Hunter (5)) may be computed from the lengths of the sides of a triangle built on the diagonals of the quadrilateral meshes used to discretise the specimen (fig.1).

$$\epsilon_{ij} = \frac{1}{2} \left[ \frac{\partial x'_k}{\partial x_i} \frac{\partial x'_k}{\partial x_j} - \delta_{ij} \right] = a_{ij} + \sum_{k=1}^3 b_{ij} L_k^2 \quad (1)$$

The stresses are true stresses (relative to the instantaneous geometry) and not the nominal or engineering stresses (relative to the initial geometry). Indeed, laws of mechanics can be applied to the actual geometry only.

The constitutive law is a combination of hypoelastic and viscous behaviour:

$$\frac{d\sigma_{ij}}{dt} = K \frac{d\epsilon_{mm}}{dt} \delta_{ij} + 2\mu \frac{d}{dt} \left[ \epsilon_{ij} - \frac{\epsilon_{mm}}{3} \delta_{ij} \right] + 2\eta \frac{d^2}{dt^2} \left[ \epsilon_{ij} - \frac{\epsilon_{mm}}{3} \delta_{ij} \right] \quad (2)$$

Integration of this equation gives the resulting effort on an element, built by joining the four immediate neighbours of a node, and then the acceleration from Newton's law, integrated twice to obtain the deformed mesh.

The computing cycle is repeated for each node and each time step. The Courant-Friedrich-Lewy criterion states that, for stable computation, the propagation of the elastic wave has to be smaller than one mesh during one time step (Hirt(6)).

## NUMERICAL EXPERIMENT

The numerical experiment is a tensile test on a sample hold in a fixed grip at the bottom and in a moving grip at the top (fig. 1 and 2). Sample and grips are drawn with the mouse directly with the software (for precise drawing it is better to use a specialised software and then import the drawing). By clicking in the three contours (sample, fixed and moving grips), the corresponding regions are analysed by Deform2D<sup>®</sup>, the software used. The sample (dotted line) is meshed automatically. The direction of movement of the mobile grip is indicated by an arrow (fig. 2). The numerical values are introduced through a dialog window ( $E = 2100 \text{ MPa}$ ,  $\nu = 0.4$ ,  $\sigma_y = 69 \text{ MPa}$ ,  $\sigma_{TS} = 100 \text{ MPa}$ ,  $\eta = 1000 \text{ Pa.s}$ ).

$\rho = 1000 \text{ kg/m}^3$ ). The overall height and width are respectively of 10 and 5 cm, respectively. The tensile speed is 5 m/s. The duration of the computation (10,000 thousand degrees of freedom) is one day on a Macintosh II.

### RESULTS

The calculated plastic zone (hatched region) in a smooth specimen is shown on figure 2 (course grid) where striction is visible. A fine grid is used for notched specimens (fig. 3). The black regions on top and bottom correspond to built in nodes in the moving and fixed grips, respectively. At the beginning of the tensile test, the speed increases suddenly from zero to the prescribed value. This discontinuity generates waves propagating through the sample (fig. 4), damped by the high viscosity (fig. 5). A smooth specimen has a classical elastic-plastic load-elongation curve, slightly decreasing after the yield point, due to decrease in section. On the contrary, a notched sample shows brittle fracture with a higher maximum load (although the initial section is slightly smaller), but with a very small elongation (brittle fracture). The beginning of plastification is shown on figure 6a. The crack appears at the elastic-plastic interface (fig. 6b) and then propagates through the section (fig. 6c and 6d) at a speed found to be near to the shear wave speed.

### CONCLUSIONS

Crack nucleation and propagation near a notch in a ductile material has been simulated in a very realistic manner on a graphical microcomputer with an explicit dynamical finite difference method. Crack propagation may be simulated without using the concept of stress-intensity factor, the old criteria like Coulomb's may be used, provided one uses a fine grid. This is possible even on a microcomputer. Other non-linear problems of mechanics of continuous media have been solved with this software.

ADDITIONAL SYMBOLS USED

$a_{ij}, b_{ij}$	= coefficients relating $\epsilon_{ij}$ and $L_k$
$ds$	= length of vector $dx_i$
$x_i$	= coordinates before deformation
$x'_i$	= coordinates after deformation
$E$	= Young's modulus
$K$	= bulk modulus
$L_k$	= length of side k of a triangle
$\delta_{ij}$	= Kronecker symbol
$\epsilon_{ij}$	= components of the strain tensor
$\eta$	= dynamic viscosity
$\nu$	= Poisson's ratio
$\rho$	= specific mass
$\sigma_{ij}$	= components of the stress tensor

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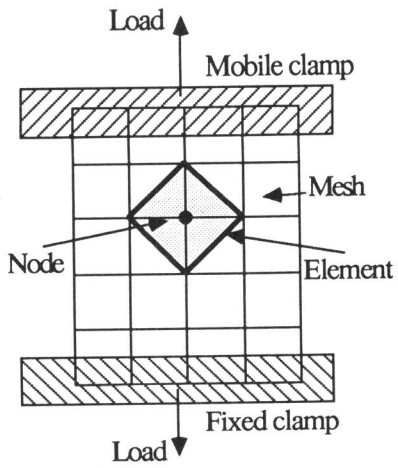


Figure 1 Geometry and grid for a schematic tension test.

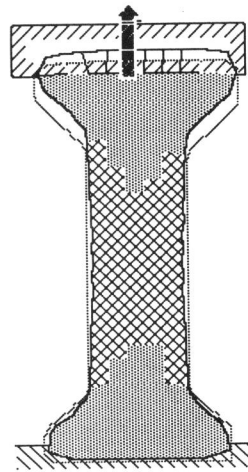


Figure 2 Plastic deformation of a smooth specimen

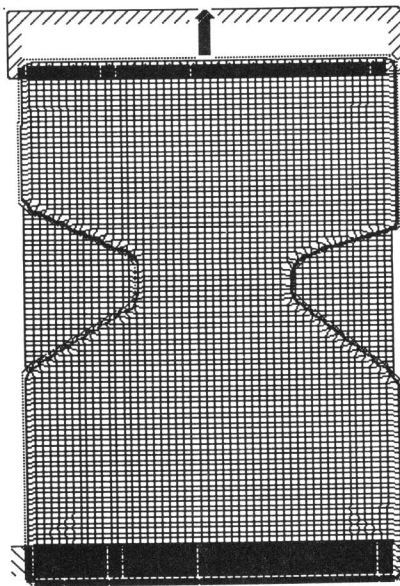


Figure 3 Lagrangian grid of a notched specimen (5000 nodes)

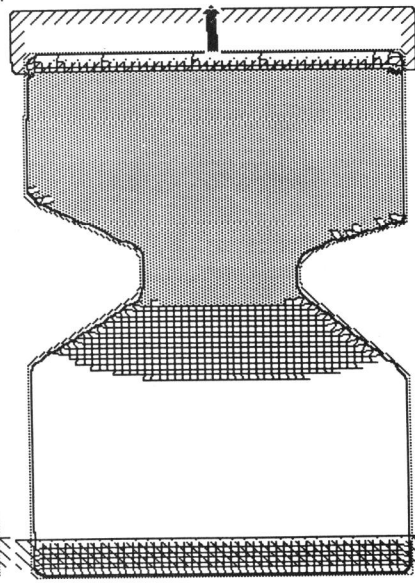


Figure 4 Elastic waves at the start of tension test of notched specimen

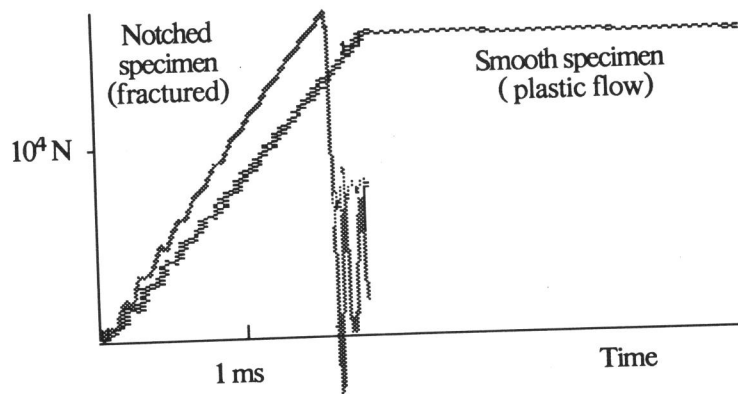


Figure 5 Tensile curve of notched and smooth specimens

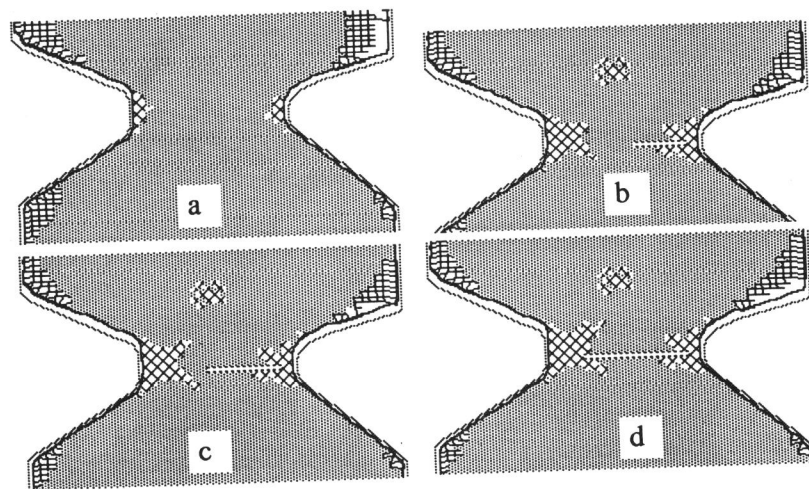


Figure 6 Plastic zone formation, crack nucleation at the elastic-plastic interface and propagation from a notch