NUMERICAL MODELLING OF THE DYNAMIC FRACTURE OF COMPOSITES UNDER IMPACT LOADING

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The behaviour of Kevlar plates impacted by lead projectiles is investigated both analytically and numerically. Firstly, a simplified unidimensional model for the resisting action of the target plate when impacted by the projectile is developed. Such model is applied to problems which have been previously studied experimentally. Then, a full numerical analysis of those problems is performed. It is concluded that the unidimensional model retains in its formulation the main aspects of the behaviour of the target plate, since a remarkable agreement with both the full numerical solution and experiment is obtained.

INTRODUCTION

The great importance of composite materials in modern industry has given rise to a large amount of research to study the behaviour of such materials under different loading conditions. One aspect of such work deals with the determination of the impact behaviour of composite materials. In this line of work, attention has been directed to the analytical or numerical determination of the dynamic response of composites under impact loading. Full numerical analyses of impact problems are usually too costly to be employed in an ordinary manner. On the contrary, analytical models can be used very advantageously from the point of view of computational cost, giving at the same time a greater understanding of the main factors which control the impact behaviour of the system studied.

In this paper we present an analytical model to study the impact behaviour of fabric and composite plates at medium impact velocities. We compare the predictions of such a model with the full numerical analysis of the same problem, as well as with experimental observations, showing that good agreement exists between them.

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ANALYTICAL MODEL

The model considers that as the fabric is impacted by the projectile, a transversal and a longitudinal wave propagate along the yarns, deforming them and giving rise to a force which tends to arrest the projectile. This situation is illustrated in Figure 1, where z corresponds to the projectile advance, θ is the angle between the projectile and the deformed yarn, l is the length of the yarn affected by the transversal wave, c_s the propagation velocity of the transversal wave, c_t the propagation velocity of the longitudinal wave, and x_o the projection of length l in the horizontal direction. The equations governing the propagation of both longitudinal and transversal motion correspond to the coupled system [1]:

$$\frac{\partial N d(x+u)/ds_1}{\partial s} - m_0 \frac{\partial^2 u}{\partial t^2} = 0 \qquad (1)$$

and:

$$\frac{\partial N d(z+w)/ds_1}{\partial s} - m_0 \frac{\partial^2 w}{\partial t^2} = 0 \qquad (2)$$

respectively. Here N is the tensile force acting on the yarn, which depends on time, m_0 is the mass per unit length of the yarn, u and w are the longitudinal and transversal displacements, and, s and s_1 are, respectively, the undeformed and deformed arc length. The transversal wave propagates with a velocity equal to:

$$c_s = \sqrt{N/m_0} = \sqrt{E\varepsilon/\rho} \qquad (3)$$

where E is the Young modulus, ρ the yarn density and ε the yarn strain. This means that the zone affected by the transversal deformation will be much less than that affected by the tensile wave (by a factor of $\sqrt{\varepsilon}$), and obtained as if equations (1) and (2) were uncoupled. This is the main difference to a previous model presented by Parga [2], who assumed that the transversal motion of the yarns took place within a zone defined by the propagation velocity of the compression waves(that is $\sqrt{E/\rho}$).

After some manipulation, the governing equation for the projectile motion comes to be:

$$\frac{d^2 z}{dt^2} = -2\frac{E S_f n_f n_l}{m_p + m_f}\varepsilon \cos\theta \qquad (4)$$

where m_p is the projectile mass, m_f is the mass of fabric pushed ahead by the projectile, n_f the number of fibres per layer resisting the projectile advance, n_l the number of layers of the fabric plate and S_f the transversal area of each fibre. The above equation can be integrated numerically to give the motion history of the projectile. A further refinement has been incorporated in the numerical model to take into account the fact that the number of fibres, opposing the projectile motion as the transversal wave propagates, increases with time. So, as a fibre is reached by the transversal wave, a resisting force is generated proportional to the elongation of such fibre, which is also supposed to contribute to projectile deceleration. In the case of a composite material [3], the presence of resin can be taken into account in the model in an approximate way by increasing the target material density, thus leading to a decrease of the transversal wave propagation velocity. In the model, a rupture criterion of the fibres is considered and it is based on a critical value of the strain, which may be a function of the strain rate.

APPLICATIONS

The above model was applied to the analysis of the impact of 9 mm Parabellum lead projectiles on 3.75 mm thickness Kevlar plates composed by six fabric layers, at velocities of 200 m/s, 275 m/s, 300 m/s and 375 m/s. Experimental observations have previously shown that projectile arrest took place for all such impact velocities. A modified form of equation (4) taking into account the contribution of all the fibres reached by the transversal wave, was then numerically integrated for the aforementioned cases, giving projectile arrest in all such situations. Full numerical analysis of such problems were also performed, by employing a finite difference code for the dynamic analysis of problems involving contact or impact. In this case each of the six fabric layers was modelled independently, and the contact between any one of them and its neighbour was also represented numerically. Thus, two adjacent layers of fabric were allowed either to remain in contact or to separate. Each layer was modelled supposing that the medium was homogeneus and isotropic and its properties were taken as equal to the corresponding ones of the material of the fibres. These computer runs gave results which compared very well with those corresponding to the analytical model. Figure 2 shows the histories of linear momentum of the projectile in the impact direction for an initial impact velocity of 300 m/s, obtained by both the analytical model and the full numerical analysis. A remarkable agreement between both results is observed. Moreover, in Figure 3 the deformed meshes corresponding to the full numerical analysis, at different times after initial contact, are shown. In this figure the position of the transversal wave as derived from the analytical model is also indicated, thus demonstrating the good correspondence between both types of computations. Finally, in Figure 4, time histories are shown of longitudinal strain of the uppermost fibre at a point located at a distance of the axis of symmetry equal to the projectile, and calculated from the analytical model and the full numerical model . The good correspondence between the two estimates is evident from this latter figure.

It was also analysed the problem of a 9 mm Parabellum lead projectile impacting a 15 layer Kevlar fabric with a surface density of 4.2 kg/m^2, the ballistic limit predicted by the analytical model being 370 m/s. This value compares well with the experimental value of 413 m/s [see Ref. 4]. In the case of a 15 layer Kevlar fabric impregnated with resin with a surface density of 5.71 kg/m^2, the ballistic limit obtained with the model was 330 m/s, whereas the value observed in experiments was 371 m/s [4].

CONCLUSIONS

A simple analytical model for the study of impact problems on fabric plates is developed. This model gives results which for the case of impact on Kevlar at medium velocities, coincide both with a full numerical analysis of such problems and with the available experimental results.

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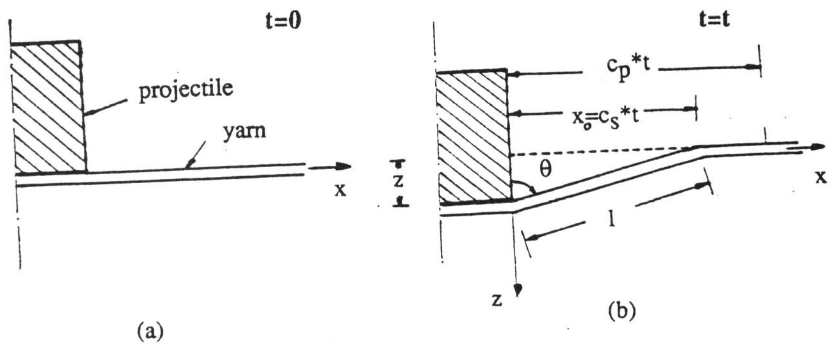

Figure 1 Projectile-yarn configurations at different times.

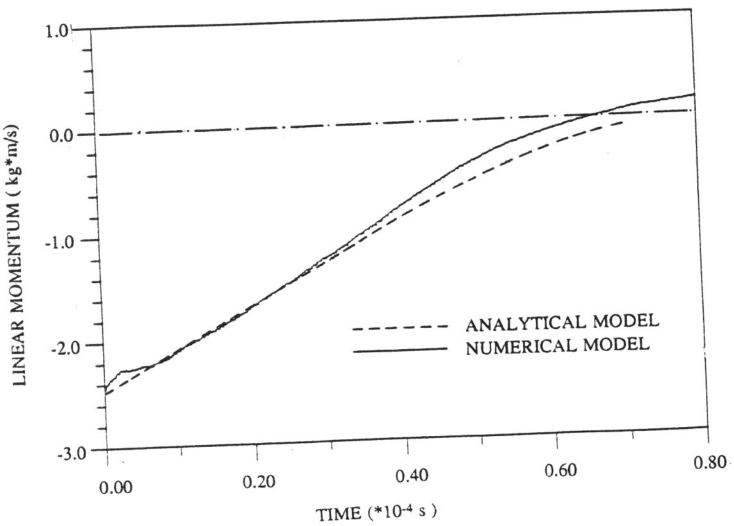

Figure 2 Time histories of linear momentum of the projectile

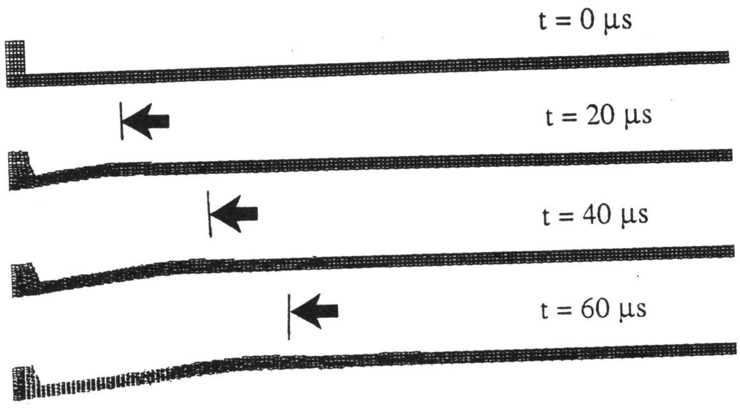

Figure 3 Different deformed projectile-target configurations at different times

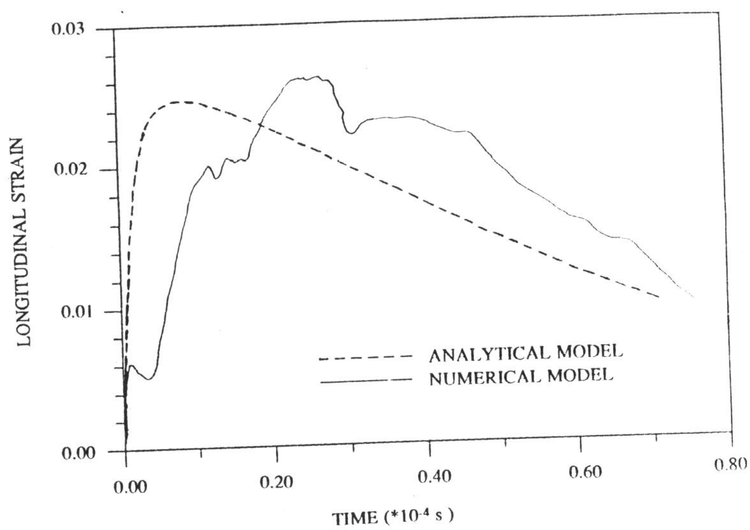

Figure 4 Time histories of longitudinal strain of the uppermost fibre