On Significance of Elastic-Plastic Crack Propagation Velocity in Asymptotic Solutions

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In the present paper it was generally confirmed that the effective stress must be singular on the new generated crack surfaces behind the growing crack tip, if there exists a plastic reloading region. The singularity of the stress on the new crack surface is at least so strong as it at the crack tip. A general formulation with respect to the crack propagating velocity is introduced for elastic-plastic materials with linear hardening, to investigate the influence of the crack propagating velocity on the solution of the stress and deformation field. It was shown that even a velocity of crack propagation e.g. *100 m/s* do not change the singularity and distributions of stresses and strains.

1. Introduction

Comparing with the asymptotic solution for stationary crack field under the deformation theory of plasticity [1, 2], the solution of the asymptotic field for a growing crack is much more complicated, due to difficulties in field equation formulations and inconsistency between elastic and plastic strain increments. While steady-state crack extension in elastic-perfectly plastic material has numerously been discussed [3-7], little progress has been archived in general elastic-plastic material with hardening, since Amazigo and Hutchinson [8] presented a solution with a power-law singularity for steady-state crack extension in linear-hardening materials. Recently Castañeda [9] introduced plastic reloading into Amazigo-Hutchinson's solution, similar to solutions in elastic-perfectly plastic materials, and predicted that some stress components (σ_{rz} for Mode III and σ_{rr} for Mode I) become infinite for $\vartheta \to \pi$. The numerical studies of ductile crack growth [10] confirm that in plane strain case a plastic region on the new crack surface exists, but not in plane stress calculations, as the asymptotic solution in [9] indicated.

In the present paper, we are going to discuss generally the existence of the singularity on the new crack surface and the influence of the crack propagation velocity on the stress and deformation field.

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2. Singularity on Crack Surface Behind the Growing Crack Tip

We consider a crack which grows steadily in an elastic-plastic material with isotropic hardening. It is assumed that the crack tip field consists from two or three angular sectors, *i. e.* the plastic loading zone, the elastic unloading zone and possibly the plastic reloading zone near to the crack surface, see Figure 1. We consider a material particle in the crack tip field, *P*, which moves along a line *L-L* with $h =$ *constant* during crack growing. Due to the steady-state crack growth the particle goes through the plastic loading zone and then the elastic unloading zone, at last to the plastic reloading zone, if it exists. This is the loading history of any material particle in the crack tip field discussed here since the crack grows in the homogeneous medium at a velocity w and the crack tip field does not change. Equivalently the loading history of each particle, which moves along the line *L-L* , can be described through the polar angle ϑ. The distance to the momentary crack tip

$$r=\frac{h}{\sin\vartheta}, \tag{1}$$

is defined through ϑ in the open interval $(0, \pi)$. However, the radius, r , and its derivative will be singular for $\vartheta \to 0$ and $\vartheta \to \pi$, i.e. on the uncracked ligament and on the crack surface. It is obvious that the particles on the ligament are irregular points in the whole crack tip field, which can not be written as a ϑ-function.

We introduce the power-law function for the coordinate r just as done in asymptotic analyses for a stationary crack [1,2] and a growing crack [8,9].

$$\sigma_e(r,\vartheta)=r^{-s}\tilde{\sigma}_e(\vartheta) \tag{2}$$

where $s > 0$ is the eigenvalue of the ordinary differential equation system and represents the intensity of the stress singularity at the crack tip. The material particle P which locates at (r, ϑ) at time t and after a time increment Δt supposed at $(r+\Delta r, \vartheta+\Delta\vartheta)$. In both the plastic loading zone and the plastic reloading zone the effective stress rate may not be negative, that is

$$(r+\Delta r)^{-s}\tilde{\sigma}_e(\vartheta+\Delta\vartheta)-r^{-s}\tilde{\sigma}_e(\vartheta)\geq 0. \tag{3}$$

The increment of the angular function $\sigma_e(\vartheta)$ in Equation (3) can be replaced asymptotically by its differential. For $\Delta\vartheta \to 0$ follows with help of Equation (1)

$$\tilde{\sigma}_e'(\vartheta)\geq -s\,\tilde{\sigma}_e(\vartheta)\cot\vartheta. \tag{4}$$

This is a basic condition for plastic deformation in the whole crack tip field. As the angular function of effective stress $\tilde{\sigma}_e(\vartheta)$ may never be zero, its derivative must always be positive for $\vartheta \geq \pi/2$, and thus the angular function must be a strictly monotonically increasing function in the plastic reloading zone. By integrating both sides of the inequality (4) it follows

$$\tilde{\sigma}_e(\vartheta) \geq K(\pi-\vartheta)^{-s}, \qquad \text{for } \vartheta \to \pi, \tag{5}$$

where $K=\tilde{\sigma}_e(\vartheta_e)\sin^s\vartheta_e$ depends only on material properties and load configurations (bending or tension) and $\vartheta_e > \pi/2$ is the reloading angle. Equation (5) implies that under the postulate of power-law singularities at the crack tip the angular function in the plastic reloading zone is singular at least as strong as the stress at the crack tip, regardless of the velocity of crack extension and the material behaviours. It means, in Mode III the stress component σ_{rz} and in Mode I σ_{rr} (and σ_{zz} for plane strain) must be singular on the crack surface, as in [9] shown for static crack growth in material with linear plastic hardening.

3. Influences of Crack Velocity

In materials with linear hardening it was shown [8,9] that all stress and strain components obtain a uniform singularity at the crack tip, we can assume

$$\sigma_{ij}(r,\vartheta,\alpha,w) = Kr^{-s}\tilde{\sigma}_{ij}(\vartheta,\alpha,w) \tag{6}$$

where α, w and K denote the material hardening parameter, see [8], the crack propagation velocity and the plastic stress intensity factor, respectively. The eigenvalue of the differential equation system, s, represents the singularity of the stress and strain at the crack tip. Generally, the singularity is a function of material hardening and the crack growth velocity. In steady-state crack propagation the stress and strain rate can be substituted by their gradient in the coordinate x. It follows that the deformation velocities have the same singularity at the crack tip as the strain, regardless of the state of crack propagation, i.e. the deformation velocities are always infinite at the crack tip.

To investigate the influence of the crack propagation velocity on the asymptotic solution, we restrict our discussions to Prandtl-Reuss´s flow theory under the Mises yield condition and in plane stress case, much more detailed formulation of the equation systems and the boundary conditions and further discussions about the asymptotic analyses see [11].

Under the assumption of Equation (6) we have five homogeneous ordinary differential equations for the asymptotic analyses of the steady-state crack extension. Due to trivial boundary conditions on the uncracked ligament ($\vartheta=0$) and on the crack surface($\vartheta=\pi$) it becomes a eigenvalue problem with the eigenvalue s, see Equation (6), just like for static crack growth [8,9]. The equations for the elastic unloading zone can directly be integrated, see [11], *i. e.*, only the plastic zone(s) will be solved numerically. In the present work the multiple shooting method [12] combined with the continuation method was used since it is quite sensitive to initial values. The crack propagation velocity was taken as continuation parameter.

Our results for quasi-static crack extension have confirmed the solutions in [8,9]. Just as pointed out in the last section, the angular function of the stress component σ_{rr} is infinite on the free crack surface regardless of crack velocity, see Figure 2 a, and σ_{rr} becomes nearly constant behind the crack tip. it implies that the singularity of the angular function $\tilde{\sigma}_{rr}$ is equal to s . The crack velocity does not dramatically change the angular functions, see Figures 2 a and 2 b where all curves are normalized by the effective stress on the ligament ($\vartheta = 0$) and the elastic modulus $E = 210000\ MPa$, mass density $\rho = 7800\ kg/m^3$, Poisson value $\nu = 0.3$, linear hardening parameter $\alpha = 0.2$.

Figure 3 shows the changes of the singularity of the stresses at the crack tip. To give a direct physical imagination of crack extension we take the absolute crack velocity as the x-coordinate instead of Mach-value. The crack tip field for materials with much plastic hardening is little influenced by the crack velocity, while the singularity for small α is very sensitive to the crack velocity. Besides pure elastic material ($s = 0.5$) the singularities for all other elastic-plastic materials monotonically decrease with increasing of crack velocity. Especially, for material with a little plastic hardening it appears that the singularity for $w \geq 1500\ m/s$ will be reduced radically. Due to numerical difficulties of convergence the lower limits of the stress singularity were not obtained yet.

Contrary to distribution of the stress singularity, the elastic unloading angle is an increasing function of crack velocity at beginning, see Figure 4. However, changes of the angle seem not monotonic for all crack velocities. At least for materials with much plastic hardening there exists a limit of unloading angle. It can expected that for $w > 2000\ m/s$ the distribution of the elastic unloading angle over the plastic hardening parameter α will be other than that for low crack velocity. Just as in quasi-static analysis there is no plastic reloading zone for $\alpha \geq 0.1$.

From this analysis it can be confirmed that the crack extension with a not too high velocity can be mathematically considered as a quasi-static problem and the crack velocity can only change the crack tip field for e. g. $w > 500\ m/s$, if other effects during crack extension can be neglected. Of course, in reality the dynamic crack propagation undergo a metallurgically different fracture mechanism compared with static crack extension, *i.e.* in dynamic crack propagations with not very high crack velocity we should concentrate to study the influences form the metallurgical mechanism rather than from the crack velocity.

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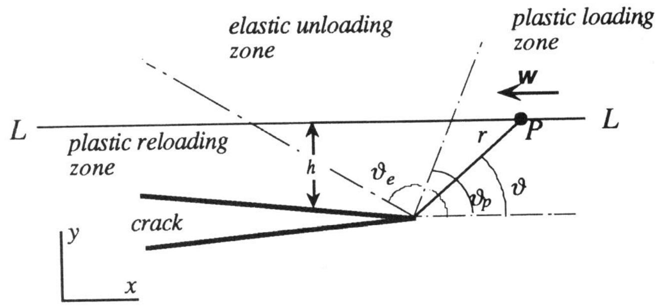
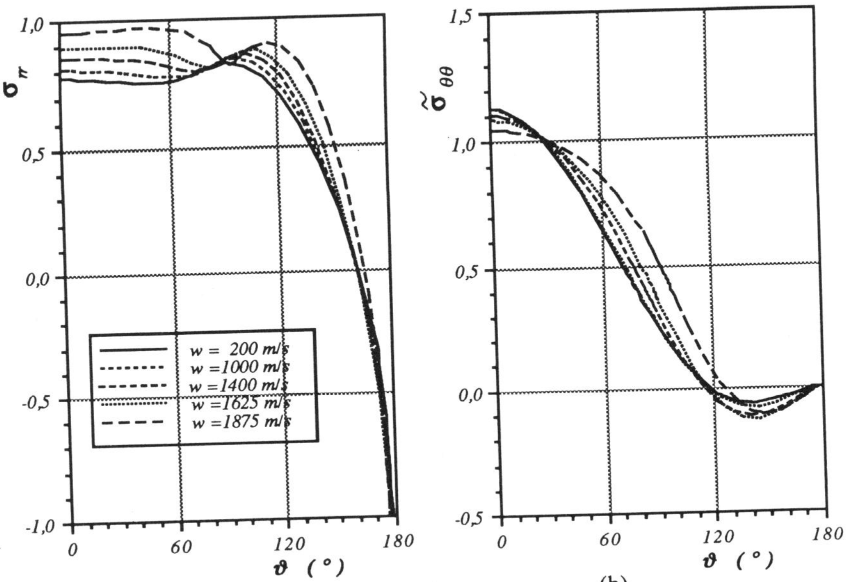


Figure 1 : angular zones at the crack tip



(a) (b)

Figure 2 : Influences of crack propagation velocity on the angular functions

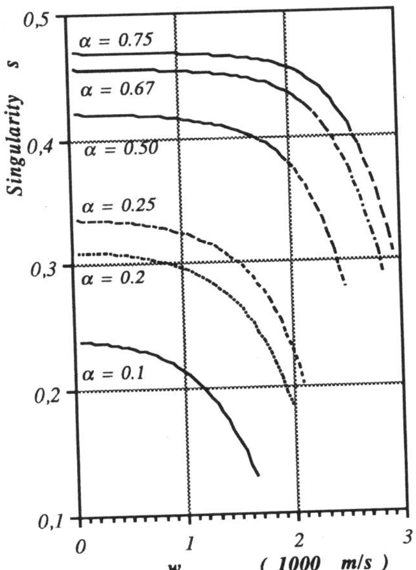


Figure 3: Distributions of singularities

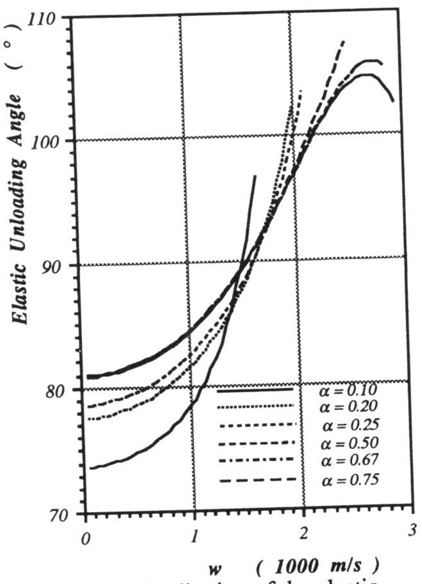


Figure 4: Distribution of the elastic unloading angle