CRACK SPACING AND CRACK WIDTH IN REINFORCED CONCRETE MEMBERS

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Anchorage lengths, crack spacings and crack widths in a reinforced concrete structure can be determined by means of methods which are based on the τ-s-relation of a reinforcement bar in concrete. In this research programme, the τ-s - relation is mathematically approximated by the expression :

$$\tau = \tau_u(1 - \mu e^{-\lambda s}).$$

This relation holds both for small and great values of s. Calculations regarding crack spacing and crack width have been carried out for centrically as well as for eccentrically loaded reinforced concrete tensile bars. At present a test programme is going on to check the proposed model.

INTRODUCTION

Because of the small tensile strength of concrete, cracks are likely to occur in reinforced concrete members. However, this cracking greatly influences the stiffness (deformations) and the durability (carbonation of concrete, corrosion of the reinforcement, ...) of the structure.

In order to get a good idea of the cracking behaviour of a reinforced concrete member, it is necessary to have an insight into the way a tensile force, exerted on a rebar, is transferred, by way of bond stresses, to the enveloping concrete. The bond between the reinforcement bar and the concrete is, consequently, one of the basic properties which make reinforced concrete possible.

FROM τ-s-RELATION TO CRACKING

The bond between the reinforcement bar and the concrete may be

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described in an idealized way as a shear stress between the surface of the reinforcement bar and the surrounding concrete. The bonding mechanism may be expressed by the relation between the shear stress τ (= bond stress) and the relative displacement s between the reinforcement bar and the concrete.

On the basis of an extensive test programme of beam tests, executed at the Department of Civil Engineering of the K.U.Leuven (1), the τ-s-relation is approximated by the expression :

$$\tau = \tau_u(1 - \mu e^{-\lambda s}) \qquad (1)$$

with μ = 0,78 en λ = 9,78. From calculations and tests (1), it follows that the mean value of the ultimate bond strength τ_u can be obtained from :

$$\frac{d}{\phi} \leq 3 \qquad \frac{\tau_u}{f'_c} = \frac{\sqrt{K}}{2} \left[1 + (1 - K)\, 0{,}353 \frac{\frac{\phi}{2} + d}{\frac{\phi}{2}}\right] \qquad (2a)$$

$$\frac{d}{\phi} > 3 \qquad \frac{\tau_u}{f'_c} = \frac{\sqrt{K}}{2} \left[1 + (1 - K)\, 2{,}473\right] \qquad (2b)$$

with $K = f_{ct}/f'_c$. The expression (1) has the merit of describing the τ-s-course up to the bond fracture.

For a centrically loaded reinforced concrete tensile bar (Fig. 1) the transfer of the tensile force N in the bar to the surrounding concrete is, for an elementary part dx, described by the following differential equation :

$$\frac{\phi\, E_s}{4(1 + m\omega)} \frac{d^2 s}{dx^2} = \tau_x \ . \qquad (3)$$

The length, needed for the transfer of the tensile force N, is called the anchorage length. At the end of the anchorage length (in U, see Fig. 1) the concrete and the steel strain are equal. If the force N is increased in such a way that the concrete stress at U becomes equal to the concrete tensile strength, i.e. $N = N_{cr}$, the length OU is equal to the anchorage length ℓ_d.

After substituting (1) in (3) and numerically solving the differential equation (3), one obtains the anchorage length ℓ_d, belonging to the force N_{cr}. In Fig. 2 the value ℓ_d/a can be read as a function of $(\sigma_{s,cr}\, a)/E_s$ with

$$a = \sqrt{\frac{\phi\, E_s}{4(1 + m\omega)\, \tau_u\, \lambda}} \tag{4}$$

and

$$\sigma_{s,cr} = f_{ct}\left(\frac{1}{\omega} + m\right). \tag{5}$$

The slip (= s_{cr}) at 0 and corresponding to the force N_{cr} is determined from :

$$\sigma_{s,cr} = \frac{E_s}{a\,\lambda}\sqrt{2(\lambda\, s_{cr} - \mu + \mu e^{-\lambda\, s_{cr}})}\ . \tag{6}$$

When subjecting a reinforced concrete tensile bar to a force N_{cr}, one assumes that in the first instance all "first order cracks" are formed. A first order crack is by definition a crack at such a distance from the nearest crack that the transfer zones of both cracks do not influence each other (Fig. 3). This requires that the distance between the two cracks in question is greater than or equal to 2 ℓ_d. The crack width w_{cr}, immediately after cracking is then equal to :

$$w_{cr} = 2\, s_{cr}$$

in which s_{cr} is determined from (6).

After the completion of this first order crack pattern, the so-called second order cracks will be formed. The distance between two cracks is now smaller than 2 ℓ_d and the transfer zones overlap partly. After completion of this second order crack pattern, the distance between two cracks is at least ℓ_d. Only at this distance taken from another crack does the concrete stress attain the concrete tensile strength again. The crack width of second order cracks varies as a function of the crack spacing between 75 % and 100 % from that of the first order crack.

Similar calculations are applied to a reinforced concrete beam, loaded by a constant moment and normal force along the axis of the member. Once the position of the neutral axis in the cross-section of the beam is determined, only the tensioned zone of the cross-section is considered in the further calculations concerning the crack spacing and the crack width. An analogous differential equation to that (1) for a centrically loaded tensile bar can be derived. However, an important difference between a centrically loaded tensile bar on the one hand and an eccentrically loaded one (= beam) on the other, is the existence of a deformation gradient in the cross-section of the beam (see Fig. 4) which is not the case for a centrically loaded bar.

In reality the distribution of cracks is very irregular because it is determined by stochastic effects. The concrete tensile strength is indeed a quantity which is liable to a relatively great spread. Therefore, strictly speaking, only maximum and minimum values can be given. So it may be said that the crack spacing (L_{cr}) and crack width (w_{cr}) to be expected is situated between two extreme values. A mean value for those quantities, obtained by making the calculations with the mean concrete tensile strength or another intermediate value derived from calculations or probabilities, is, consequently, to be interpreted with caution.

EXPERIMENTAL INVESTIGATION

At present, a test programme is going on to verify the above-mentioned calculations. The influence of the following parameters on the cracking behaviour of a reinforced concrete member is examined :

- diameter of the reinforcement bar (ϕ = 16 - 20 - 25 mm)
- number of bars in the section (1 or 2)
- ratio between the concrete cover on the bar (d) and the diameter of the bar (d/ϕ = 1,5 - 2)
- distance between the rebars
- position of the rebar in the section (centrical -eccentrical).

The beam has a rectangular cross-section. A schematic picture of the test set-up is shown in figure 5. At both ends, the beam is loaded by a tensile force N, exerted on the rebar(s).

If there should be a good correlation between the theoretically derived results and those obtained from the tests, the proposed theory will be used afterwards to work out calculations with regard to the deflection of a reinforced concrete beam.

SYMBOLS USED

d = concrete cover on the reinforcement bar (mm)

E_c = Young's modulus of concrete (N/mm²)

E_s = Young's modulus of steel (N/mm²)

ϕ = diameter of the reinforcement bar (mm)

f'_c = concrete compressive strength measured on cylinder (N/mm²)

f_{ct} = uniaxial concrete tensile strength (N/mm²)

m = E_s/E_c

s = slip (= relative displacement between the rebar and the concrete)(mm)

c = concrete stress (N/mm^2)

σ_s = steel stress (N/mm^2)

τ = bond stress (N/mm^2)

ω = A_s/A_c

REFERENCES

(1) Vandewalle, L., "Hechting tussen wapening met verbeterde hechting en beton bij gewone en cryogene omstandigheden", Ph.D.-thesis, K.U.Leuven, 1988.

(2) Rostasy, F.S. and Scheuermann, J., Deutscher Ausschuss für Stahlbeton, Heft 380, 1987, p. 43 - 105.

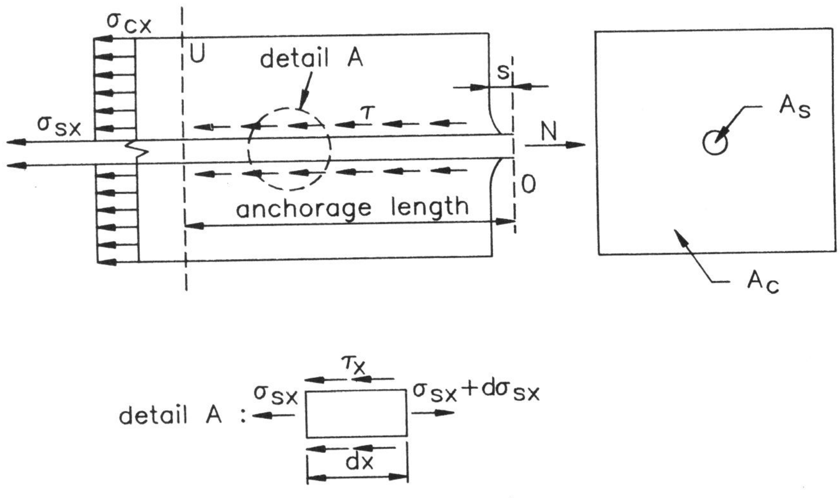

Figure 1 Centrically loaded reinforced concrete tensile bar

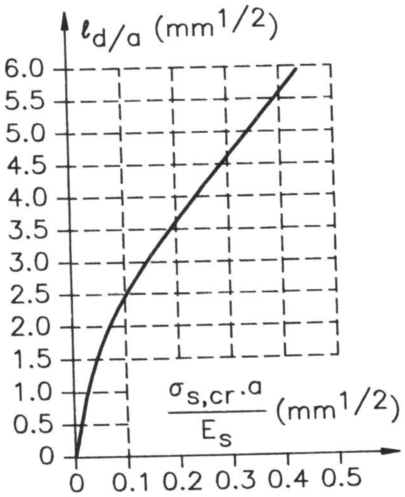

Figure 2 Anchorage length as a function of the applied stress

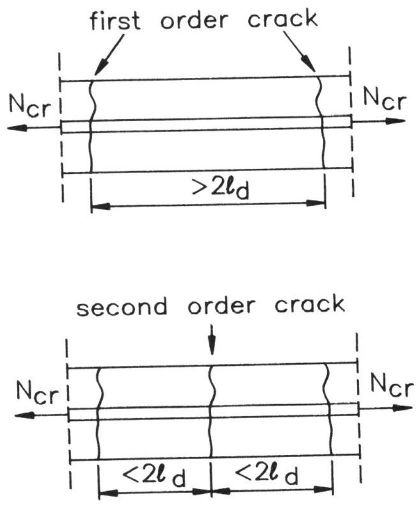

Figure 3 First order and second order cracks

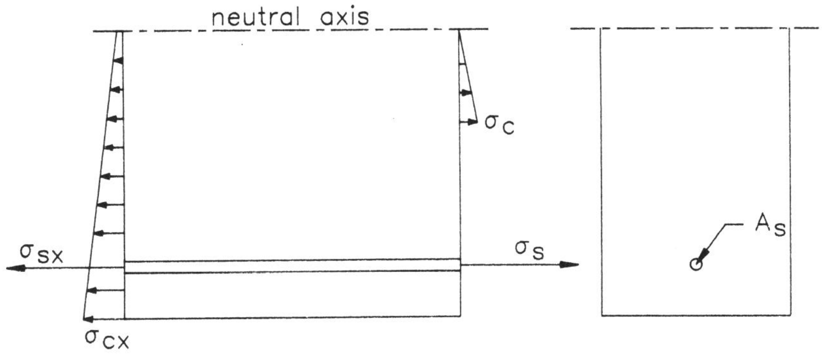

Figure 4 Eccentrically loaded reinforced concrete tensile bar

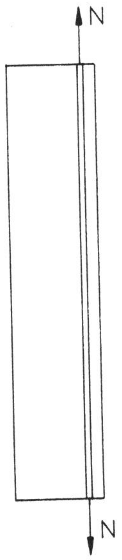

Figure 5 Test set-up