PREDICTION OF CRACK PROPAGATION IN REINFORCED CONCRETE STRUCTURES

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ABSTRACT Fracture mechanics solutions are developed to predict crack propagation behaviour in reinforced concrete elements. These solutions take into consideration those factors that can control the susceptibility of concrete elements to fracture, such as: concrete cover, crack size, percentage of steel reinforcement and strength and fracture properties of steel and concrete materials. Based on these solutions, cracking moments at various crack sizes as well as the ultimate fracture moment can be predicted. Expressions for surface crack width and rotation of cracked sections have also been obtained. A close agreement is obtained between experimental and fracture mechanics solutions.

INTRODUCTION

In the conventional analysis approach to concrete structures, concrete is assumed not to be working in tension. However, such analysis does not take into consideration the stiffness variation and stress concentration due to the presence of cracks. These effects can be considered if fracture mechanics techniques are employed to determine whether existing cracks pose a problem to the integrity of concrete structures. Most of the work on fracture mechanics has so far only dealt with metals [1]. Some of the fracture control methods used for metals are however not suitable for concrete, the fracture behaviour of the two materials being different [1]. Although a large amount of work has been carried out in order to investigate crack propagation behaviour in plain concrete , limited analytical and experimental studies [2-4] have been carried out in order to predict crack propagation behaviour in reinforced concrete structural elements. Fracture mechanics solutions are developed, in this paper, to predict crack propagation behaviour in reinforced concrete elements. These solutions cover the full range of relative crack depth ratio. Based on these solutions, simple expressions have been obtained to predict cracking moments, ultimate fracture moment, crack width and rotation of cracked sections.

STRESS INTENSITY FACTOR SOLUTION

Solutions for K for cracks propagating in concrete members have been developed recently [2-5], which are only valid for small crack sizes. Solutions for K which cover the full range of relative crack depth ratio a/W are developed in the next section.

The effective stress intensity factor for a through crack in a reinforced concrete element shown in Fig.1 subjected to bending moment value M can be obtained by substracting the stress intensity value due to the force in steel bars T_s from the stress intensity factor due to M[2] as follows:

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$$K=6M\sqrt{a}.Y_M / (BW^2) - T_s\sqrt{a}.Y_T/(B.W) \quad (1)$$

where a is the crack depth, B and W are the width and depth of concrete section respectively. Y_M is a geometrical bending correction factor dependent on the relative crack depth ratio a/W [6] and Y_T is a correction factor dependent on crack depth ratio and concrete cover b [7]. Expressions for Y_M and Y_T are given below.

Short Crack (0<a/w <0.60)

$$Y_M = 1.99 - 2.47 (a/W)+ 12.97 (a/W)^2 - 23.17 (a/W)^3 +24.8(a/W)^4 \quad (2)$$

$$Y_T = Y_1 Y_2 W / a \quad (3)$$

where,

$$Y_1 = 3.52 / (1 - a/W)^{3/2} - 4.35 / (1 - a/W)^{1/2} + 2.13 (1 - a/W) \quad (4)$$

and

$$Y_2 =1.12+0.9(b/a) - 9.1(b/a)^2 + 33 (b/a)^3 - 48(b/a)^4+25 (b/a)^5 \quad (5)$$

Equation (3) is simplified using a least square fitting as follows:

$$Y_T=\alpha+\beta(a/W- \gamma)^2 \quad (6)$$

where α,β and γ are constants equal to 8,60 and 0.30 respectively for the case of b/w less than 0.10

Long Crack (0.6<a/W<1)

Expressions for Y_M and Y_T can be obtained using the solution for a long edge crack approaching the edge of the cracked section. This solution, which is derived from Neuaber's work on deep notches [8], is given by Paris and Sih [9] for the case of pure bending as follows:

$$K_M= 4.33 M / \{B (W-a)^{3/2}\} \quad (7)$$

Comparing this equation with the first term given in Equation 1, an expression for Y_M can be derived as follows:

$$Y_M=0.722 /\{(1-a/W)^{3/2} (a/W)^{1/2}\} \quad (8)$$

Stress intensity factor due to T_s can also be obtained based on Paris & Sih [9] solution as follows:

$$K_s=4.33 M_s/\{B(W-a)^{3/2}\} + 0.537 T_s/\{B(W-a)^{1/2})\} \quad (9)$$

Where M_s is the moment acting on the middle of the uncracked ligament due to T_s which is equal to :

$$M_s=T_s \{(W+a)/2 -b\} \quad (10)$$

Substituting Equation (10) into Equation (9) and rewriting the results in a form similar to the second term given in Equation (1) ,an expression for Y_T can be obtained as follows:

$$Y_T=2.7\{1+0.6(a/W)- 1.6(b/W)\}/\{(a/w)^{1/2} (1-a/W)^{3/2}\} \quad (11)$$

Values of T_s given in equation (1),can be determined by analysing the concrete section subjected to a known value of M.In the elastic case,the position of the neutral axis can be obtained by applying conditions of equilibrium and neglecting the cracked concrete as shown in Fig.(1) which leads to the following expression for T_s:

$$T_s=M/\{d-z/3 + B(w-a).(W-a-z) / (3nA_s. (d-z))\} \quad (12)$$

where z is the depth of the neutral axis measured from the top fibers and is given by:

$$z=\{B(W-a)^2/2+n.A_s.d\}/\{(n.A_s+B(W-a)\} \tag{13}$$

where $n = E_S / E_C$ and E_S and E_C are the elastic modulus of steel and concrete respectively.Once steel has yielded, the force T_S is assumed equal to $A_S.f_y$ where f_y is the yield stress of the steel. This assumption does not take into consideration the strain hardening of steel as strains increase.

ROTATION AND CRACK WIDTH

The rotation,ϕ, of cracked concrete section and the surface crack width,δ shown in Fig. (1) can be obtained using principles of super position as follows:

$$\phi=\phi_M-\phi_T \tag{14}$$

$$\delta=\delta_M-\delta_T \tag{15}$$

where ϕ_M and ϕ_T are the rotations due to M and T_S respectively.δ_M and δ_T are the crack widths due to M and T_s respectively. These values can be obtained based on Castiglianos theorem as suggested by Paris [7] according to the following integration:

$$\Delta_F=2/E\int_o^a K_p\,\partial.K_F/\partial F\ \ dA \tag{16}$$

where K_p is the stress intensity factor corresponding to the case of loading, K_F is the stress intensity factor due to load, F, applied in the direction of deformation Δ_F and A is the area of the cracked surface. Based on Equation (16) and the solutions for stress intensity factor given above, expressions for $\phi$and δ are derived below.

Short Crack (0 < a/W < 0.60)

Substituting for Y_T and Y_M from expressions given above an expression for ϕ is obtained based on Equations (1),(14) and (16)as follows:

$$\begin{aligned}\phi=&72/(E_cBW^2)\{M-T_s(W/2-b)\}\{1.98(a/W)^2-3.28(a/W)^3+14.4(a/W)^4-31.3(a/W)^5+63.6(a/W)^6\\&-103.4(a/W)^7+147.5(a/W)^8-127(a/W)^9+61.5(a/W)^{10}\}-\\&12T_s/E_cBW\{1.98(a/W)^2-1.91(a/W)^3+16(a/W)^4-34.8(a/W)^5\\&+83.9(a/W)^6-153.7(a/W)^7+256.7(a/W)^8-244.7(a/W)^9+133.5(a/W)^{10}\}\end{aligned} \tag{17}$$

To obtain δ as given by Equation (15), an approximate solution for K_F corresponding to force F applied at the bottom surface of the cracked section and valid for the case of short crack [7,10] is given below:

$$K_F=2F/(B\sqrt{\pi a}) \tag{18}$$

Based on equations 2,6,15,16 and 18 an expression for δ is obtained as follows:

$$\begin{aligned}\delta=&24M/E_cBW|\ \ \{1.99\ a/W-1.24(a/W)^2+4.32\ (a/W)^3-5.8(a/W)^4+5(a/W)^5\}\\&-4T_s/E_cBW|\ \ \{\alpha a+W\beta/3\ (a/W-o)^3+\beta W\delta^3/3\}\end{aligned} \tag{19}$$

Long Crack (0.6<a/W <1.0)

Based on Equations (7), (9),(14) and (16), an expression for ϕ is obtained as follows:

$$\begin{aligned}\phi=&18.75\ M/E_cBW^2\ \ \{(1-(1-a/W)^2)/(1-a/W)^2\}\\&-23.4\ T_s/E_cBW\ \{a/W(1-1.6\ b/W)+(a/W)^2(-0.2+0.8\ b/W)\}/\ (1-a/W)^2\end{aligned} \tag{20}$$

In order to obtain the crack width, δ, solution for K_F given in Equation (16) for the case of Long Crack corresponding to force F applied at the bottom surface of the cracked section, is obtained by substituting into Equation (11) for the case of b equals zero. Hence an expression for δ is obtained from equation (15) as follows:

$$\delta = 23.4\ M/E_cBW\ \{a/W(1-0.2\ a/W)/(1-a/W)^2\} - 7.29\ T_s/E_cB\ \{\{a/W(1.28-3.2\ b/W) + (a/W)^2.(1.28+0.64\ b/W)\}/(1-a/W)^2 - 0.72\ \ln(1-a/W)\} \qquad (21)$$

CRACK PROPAGATION PREDICTIONS

Incremental Crack Growth

While failure occurred suddenly in precracked plain concrete beams, incremental crack growth took place in the precracked reinforced concrete beams Fig.(2), until final failure occurred by crushing, failure of steel and fracture of concrete depending on the percentage of steel reinforcement [4]. The stable crack growth observed in the reinforced concrete beams was the result of the effect of reinforcing steel in these beams. This steel tended to close the crack and hence prevent sudden failure of such beams as simulated by the fracture mechanics model given by Equation (1).

A comparison was made between experimental [4] and theoretical results based on fracture mechanics (FM) criteria proposed above as shown in Figs.(3) and (4).The results were plotted in terms of applied load corresponding to cracking bending moment at mid span section versus crack size. Based on Fracture Mechanics criterion, the cracking bending moment values were calculated by substituting into Equation (1) for K equal to 0.6 MPa$\sqrt{m}$ which is the $K1_c$ value obtained from plain concrete tests. A close agreement was obtained between experimental and theoretical results as shown in the Figures.

Incremental crack growth in reinforced concrete beams can also be presented in terms of the non dimensional form $M_f/(K1_c.BW^{3/2})$, proposed by Carpinteri [2]. M_f is the cracking moment at specific crack size and percentage of steel reinforcement. This form can easily be derived from Equation (1) by substituting for K equal $K1_c$ as follows:

$$M_f = K1_c.BW^{3/2} / (6\sqrt{a/W}.Y_M) + A_s.f_s.W.Y_T/6.Y_M$$

$$\therefore M_f/(K1_c.BW^{3/2}) = 1/(6\ Y_M\sqrt{a/W}) + A_s.f_s.W.Y_T/(6.Y_M.K1_c.BW^{3/2}) \qquad (22)$$

Comparison between experimental and predicted results based on Equation (22) is shown in Fig.(5) at various percentages of steel. An increase in the cracking moment was noticed as the percentage of steel area increased. This effect was more pronounced as the crack size increased. It should be noticed that the moment required for propagation of pre-existing cracks decreases as crack size increases until it reaches a minimum and then increases as shown in Fig(5). This behaviour is typical of under-reinforced concrete sections where percentage of steel area is less than the balanced steel area [11]. However, for over reinforced concrete sections, the cracking moment is expected to increase with the increase of crack size [4].

Examining Equation (22) indicates that, at small crack sizes, the first term given at the right hand side of the equation is much larger than the second term. Hence the growth of small cracks is sensitive to the fracture toughness of concrete rather than the percentage of steel areas. However as the crack size increases, the first term represents a small proportion of M_f compared with the second term given at the right hand size of the equation. Therefore, the growth of large cracks is sensitive to the area and yield strength of steel reinforcement.

It should also be noticed that the predicted cracking moment based on fracture mechanics criteria stays constant at long cracks as shown in Fig. 5 due to the fact that strain hardening of steel is neglected in the model. However, the advantage of the present model lies in its simplicity which avoids complicated iterative techniques and it can be employed in design calculations with reasonable accuracy as shown in the above figures.

ULTIMATE FRACTURE MOMENT

As noticed above, an incremental crack growth usually takes place in cracked reinforced concrete beams without sudden failure as long as no failure of steel bars has occurred [1-4]. However, as the moment increases, crack size increases until cracks become large and it is important to determine the cracking moment at large crack size which is considered as an ultimate fracture moment, M_{uf} and it is of interest to compare this moment with the ultimate moment calculated from the classical ultimate strength calculations. M_{uf} can easily be determined from Equation (22) as a/W reaches unity. At large crack sizes, stress in steel bars is expected to reach the steel yield stress. In addition, at large crage crack sizes the first term given at the right hand side of Equation (22) can be neglected as mentioned above. Hence M_{uf} can be derived from Equations 8 , 11 & 22 when a/W reaches one as follows:

$$M_{uf}=A_s.f_y.W.Y_T/6Y_M = W.A_s.f_y(1-b/W) \tag{23}$$

The above expression is of interest since it is simple and it shows that the ultimate fracture moment of concrete beams is only affected by the yield strength of steel, area of steel reinforcement and concrete cover b. Table 1 gives a comparison between Muf calculated via equation 23 and experimental ultimate moment capacity of reinforced concrete beams. It is noticed that for under reinforced beams, equation 23 results in moment values close to the ultimate capacity of these beams. Therefore equation 23 can be used in the ultimate design calculations of such beams.

CRACK WIDTH CALCULATIONS

For the appearance of and resistance to corrosion, service loadings should be controlled to limit the crack widths at the surface of damaged concrete members. Such crack widths are affected by the applied load level, amount of steel reinforcement, concrete cover, stress in steel bars and crack size as predicted by Equations (19) and (21) developed in the present paper. These equations can easily be employed to predict crack widths at surface load levels. A comparison is made between experimental and predicted crack widths at the surface of reinforced concrete beams as shown in Fig. 6. Hence, these equations become useful and can be employed in the design of reinforced concrete members where control of crack widths to specific limits is required.

SUMMARY

Simple expressions have been developed to predict the cracking moment at various sizes of cracks, ultimate fracture moment at large cracks, surface crack width and rotation of cracked sections subjected to bending moments. Close agreement was obtained between experimental and predicted results based on fracture mechanics solutions. It is shown that an increase in the cracking moment has occurred as the percentage of steel area has increased.. Although the cracking moment of small crack sizes was sensitive to the fracture toughness of concrete material, the ultimate fracture moment of concrete sections was shown to be only affected by the yield strength and area of steel reinforcement.

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Table 1

Experimental and Predicted Ultimate Moment Capacity of R.C. Beams.

Original Crack Depth,a,mm.	W mm	b mm	A_s mm^2	Ultimate Moment,KN.m	Predicted M_{uf},KN.m
0,45,75,90	150	40	14*	1.3,1.27,1.21,1.23	0.90
0	150	15	33**	3.1	3.2
0,45,75,90	150	40	33**	2,1.8,1.7,1.6	2.6
0,45,75,90	150	40	50**	3.9	3.9
0,45,75,90	165	15	50***	3.3,3.1,2.9,3	3
0,75,90	150	40	100***	4.6,5.1,4.6	4.4

*f_y=560 MPa , **f_y=720 MPa , ***f_y=400 MPa

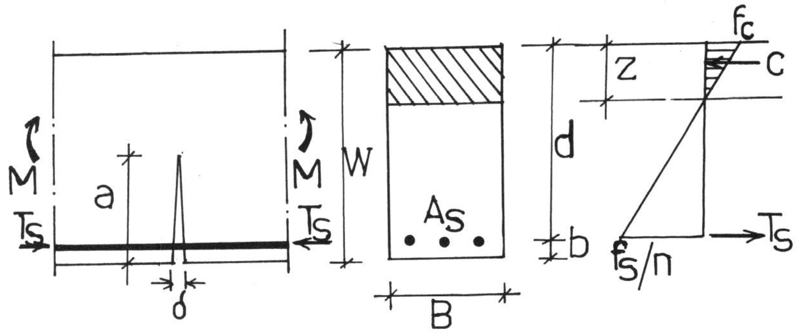

FIG.1 FORCES AND STRESSES IN CRACKED R.C MEMBERS

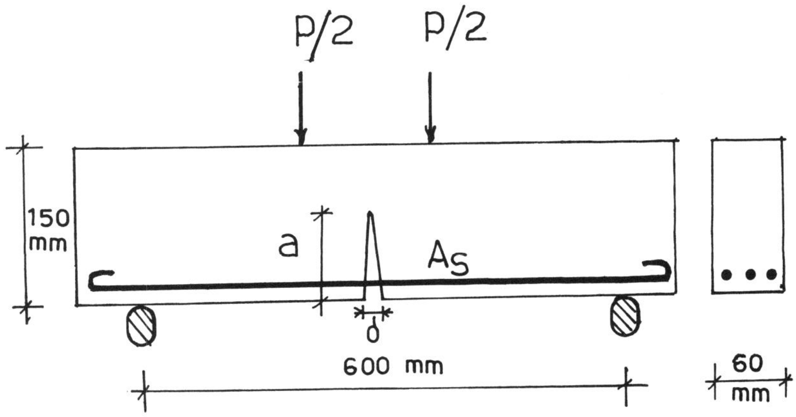

FIG.2 TESTED R.C BEAMS

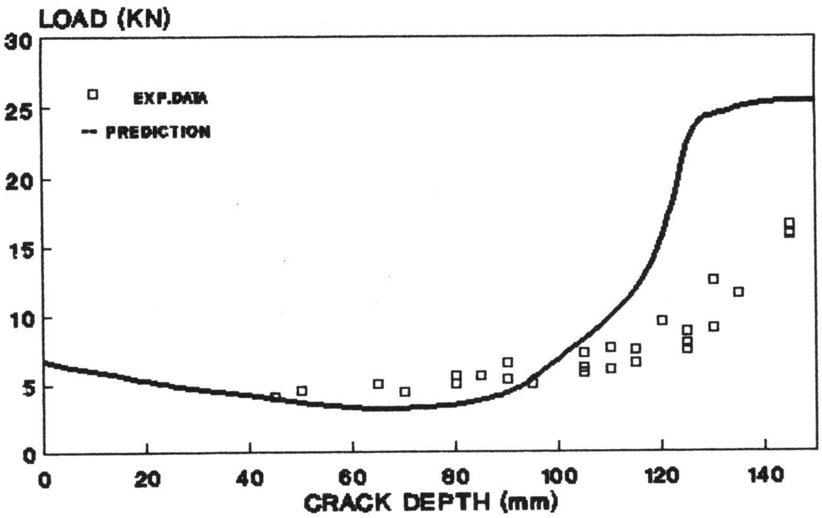

FIG.3 PREDICTION OF CRACK PROPAGATION IN R.C BEAMS(u=As/(B.W)=0.0037)

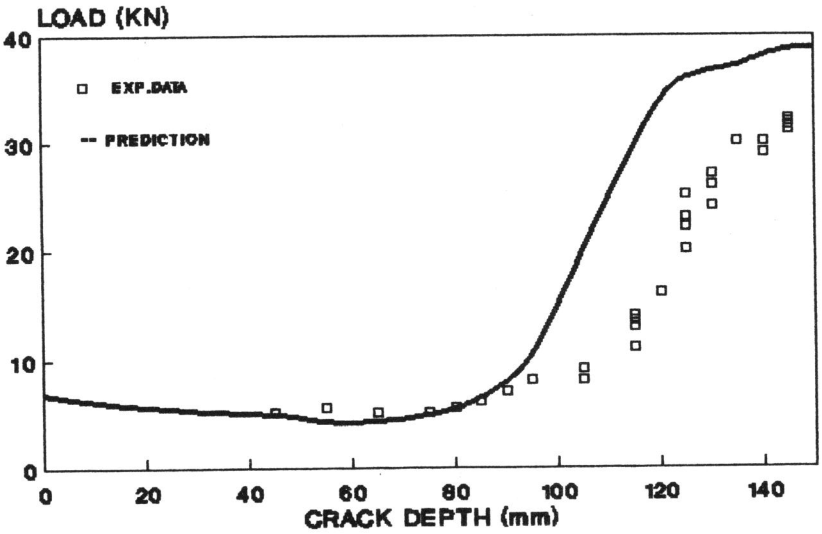

FIG.4 PREDICTION OF CRACK PROPAGATION IN R.C BEAMS(u=As/(B.W)=0.0055)

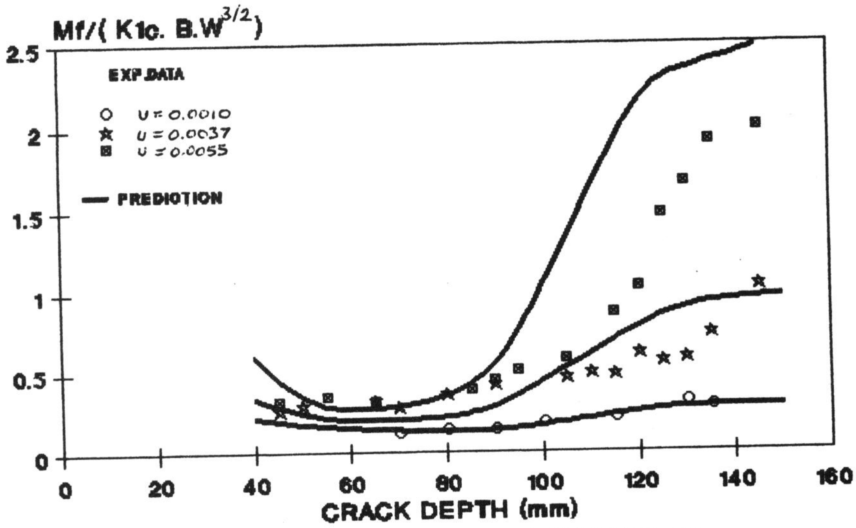

FIG.5 PREDICTION OF CRACKING MOMENTS IN REINFORCED CONCRETE BEAMS

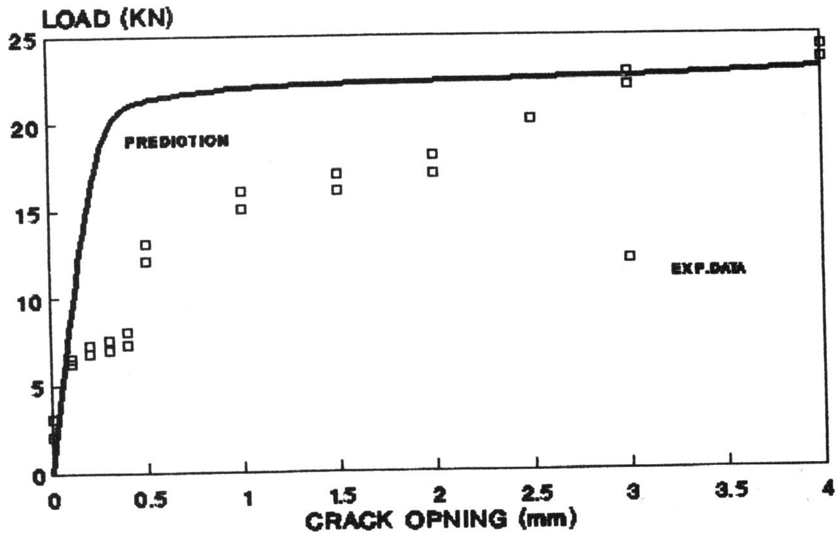

FIG.6 PREDICTION OF CRACK WIDTH IN R.C BEAMS(As/(B.W)=0.0055)