INSTABILITY OF CRACK PROPAGATION IN REINFORCED CONCRETE BEAMS WITH DOUBLE LAYER OF TENSILE REINFORCEMENT

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Strength and thoughness of concrete-like materials determine the failure mode of members. On the other hand, when the scale-effect is considered, the failure mode can turn from ductile to brittle simply by changing structure size as well as steel content.

In the present paper, the crack growth stability in a reinforced concrete element with double layer of tensile reinforcement is analised. The failure mechanism, even for these elements, rather appears brittle when large dimensions and/or low content of steel are considered.

Results obtained for different quantity of reinforcement are illustrated.

INTRODUCTION

The analysis of reinforced concrete elements has assumed more and more importance in the field of fracture mechanics, because of difficulties in interpreting some phenomena with the classical approach. One of the most relevant examples is the scale effect that cannot be explained without considering both strength and thoughness characteristics of concrete. Fracture mechanics allows in this case to reveal the transition from ductile to brittle failure mode when the depth of the structure increases, all other geometrical and mechanical characteristics being constant. On the other hand, the classical approaches could not be applied to explain this transition, which is experimentally confirmed even for reinforced concrete structures. In other words, we need to consider the surface energy dissipation, which explicitly refers to developing free surfaces, and not only the volume energy as in classical elasticity and plasticity.

After earlier applications of simple models to cracked reinforced concrete elements (Carpinteri (1), (2)), able to predict the

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transition between the two failure modes, some experimental confirmations (Bosco et al (3),(4)) suggested to continue the theoretical researches by using more refined models.

In the present paper the crack growth of a cracked r.c. beam with two layers of tensile reinforcement is analised, to catch the more relevant aspects of its failure mode and to compare the influence of the second layer of steel on the global brittleness characteristics, to the case where a unique tensile reinforcement is present in the element.

THEORETICAL MODEL

Let the cracked element (Figure 1) be loaded by the bending moment M and two concentrated loads P_1 e P_2 applied on the surfaces of the crack at distances c_1 and c_2 from the lower edge of the beam. If crack propagation occurs in Mode I (Okamura (5)), considering the superposition principle and taking into account that

$$K_{IM} = \frac{M}{bh^{3/2}} Y_M(\xi), \qquad K_{IPi} = \frac{P_i}{bh^{1/2}} Y_P(c_i/h,\xi), \quad (i=1,2) \quad ...(1),(2)$$

K_I being the stress intensity factor, the compliances, also called influence coefficients, are expressed as ($\lambda_{ij} = \lambda_{ji}$):

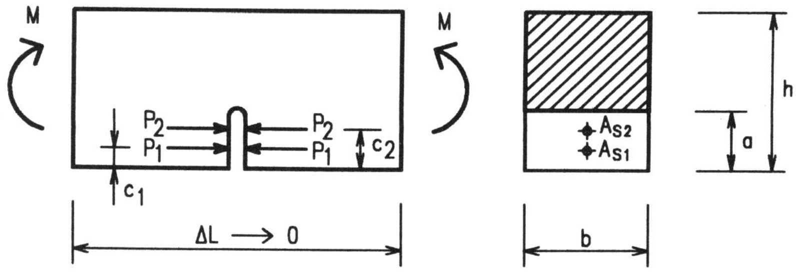

Figure 1 Cracked beam element and position of reinforcements

$$\lambda_{ij} = 2/bE \int_{c_i/h}^{\xi} Y_P(c_i/h,\xi) Y_P(c_j/h,\xi) d\xi \;, \quad (c_j > c_i,\; i>0, j>0) \quad \ldots\ldots (3)$$

$$\lambda_{i0} = 2/bhE \int_{c_i/h}^{\xi} Y_P(c_i/h,\xi) Y_M(\xi) d\xi \;, \quad (i=1,2) \quad \ldots\ldots (4)$$

$$\lambda_{00} = 2/(bh^2E) \int_{0}^{\xi} Y_M^2(\xi) d\xi \;, \quad \ldots\ldots (5)$$

Y_P ed Y_M being functions (Tada et al (6)) of the relative crack depth ξ = a/h and of the distance c_i of the layer of tensile reinforcement (i=1,2) from the lower edge of the beam.

DISPLACEMENT CONGRUENCE CONDITIONS

Let the stress-strain constitutive law for steel be linear elastic-perfectly plastic while the concrete behaves linearly until crushing collapse. Consequently, the relationships between M, P_1 and P_2 are represented by congruence conditions that correspond to nil relative displacements w of the points of application of the loads on the crack surface. Displacements greater than zero are then only allowed when the yielding limit of steel is reached, or slippage due to local debonding between steel and concrete occurs. Then it follows:

$$w_1 = \lambda_{10} M - \lambda_{11} P_1 - \lambda_{12} P_2 = 0$$
$$w_2 = \lambda_{20} M - \lambda_{21} P_1 - \lambda_{22} P_2 = 0 \quad \ldots\ldots (6)$$

Up to the yielding limit in the most external layer of tensile reinforcement ($P_1 = P_{P1} = A_{s1} f_y$) and when the applied moment does not produce crack propagation, the reactions of the reinforcements are obtained from eqs (6). When the yielding limit has been reached the second of eqs (6) contains P_{P1} instead of P_1, while $w_1 > 0$ is given by the first of eqs (6) substituting $P_1 = P_{P1}$. Then, when both layers of steel are yielded ($P_1 = P_{P1}$ and $P_2 = P_{P2}$), both w_1 and w_2 are greater than zero.

CRACK PROPAGATION

Crack propagation occurs when the critical value of the stress intensity factor is reached ($K_I = K_{IC}$). As a consequence $M = M_F$ and, from eqs (1) and (2), it descends:

$$M_F = [K_{IC} bh^{3/2} + Y_P(c_1/h,\xi) P_1 h + Y_P(c_2/h,\xi) P_2 h] / Y_M(\xi) \quad \ldots\ldots (7)$$

Taken into account that in the elastic range and until $M \le M_P$, it is $P_i/P_{Pi} = M/M_{Pi}$ with $P_i = T_i P_{Pi}$ and $T_i = P_i/P_P < 1$, eq.(7), in non dimensional form becomes:

$$\frac{M_F}{K_{IC} bh^{3/2}} = \frac{1}{Y_M(\xi)} + \frac{N_P}{Y_M(\xi)} [Y_P(c_1/h,\xi) T_1 \rho_1 + Y_P(c_2/h,\xi) T_2 \rho_2] \quad \ldots\ldots (8)$$

where $N_P = f_y \rho h^{1/2}/K_{IC}$ is the brittleness number (Carpinteri (1), (2)), in which yielding limit of steel f_y, critical value of the stress intensity factor K_{IC} and total percentage of steel in the cross section $\rho = \Sigma \rho_i = (\Sigma A_{si})/bh$ are involved. In eq. (8) $\rho_i = A_{si}/bh$ represents the percentage of each layer of steel, always referred to the whole area bh of concrete and T_i (i = 1, 2) assumes the value 1 if $M_F > M_{Pi}$ and M_F/M_{Pi} if $M_F \leq M_{Pi}$.

Functions (8) are represented in Figure 2 against the variation of the initial relative crack depth ξ ($\xi > c_1/h = 0.1$, while $c_2/h = 0.2$) and varying the brittleness number N_P, in the case $A_{s1} = A_{s2} = A_s$ and then $\rho = 2A_s/bh$.

First of all, it is evident that for N_P greater than ≈ 0.15, the second layer increases the stiffness of the cross section, since relative crack propagation from $\xi = 0.2 - \varepsilon$ to $\xi = 0.2 + \varepsilon$ is possible only by increasing the applied bending moment to a value greater than that provoking crack initiation. As regards the steel conditions, by observing the lower part of the diagram it appears that crack propagation always occurs when both the layers of steel are yielded (i.e. low N_P corresponding to small percentages of steel or deep cross sections). On the contrary, if the crack depth is sufficiently limited ($\xi < 0.25$), crack propagation can occur with reinforcement in elastic conditions for N_P greater than ≈ 0.30. Moreover it is necessary to point out that, for small values of N_P ($N_P < 0.2$), the phenomenon of crack propagation is in any case unstable being the slope of the curves N_P = cost., always negative up to the value $\xi = 0.7$.

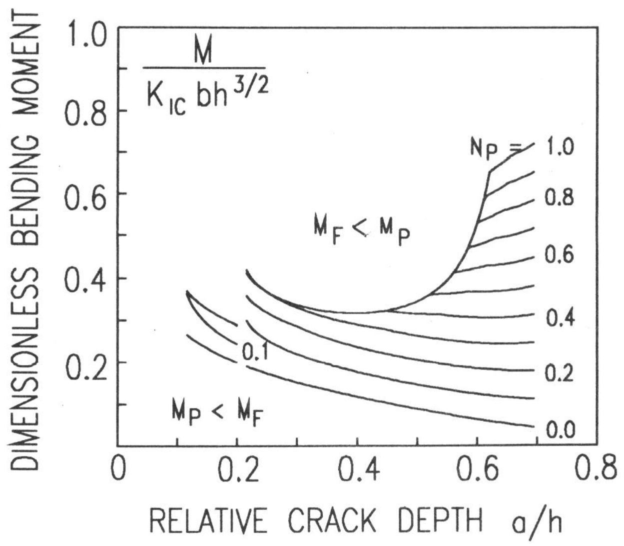

Figure 2 Bending moment of crack propagation against relative crack depth

On the other hand, if the various values of crack depth are interpreted as an evolutive phenomenon of cracking increase, it appears that is necessary to reduce the bending moment to avoid a very fast propagation and then a sudden failure of the element.

For N_P greater than ≈ 0.2 (see Figure 2), the crack propagation develops in unstable manner until the relative crack depth corresponds to the value for which the curve N_P = cost. presents a minimum (horizontal tangent). Beyond that point it is again necessary to supply energy to determine further increase of cracking. The model then predicts that the crack propagation phenomenon is stable only for sufficiently high value of crack depth ($\xi = 0.5 \div 0.6$) or that it may become stable only for N_P greater than ≈ 0.2.

MOMENT-ROTATION RELATIONSHIP FOR THE CRACKED ELEMENT

Local rotation due to the presence of crack may be obtained by superposition as follows:

$$\phi = \lambda_{00} M - \lambda_{10} P_1 - \lambda_{20} P_2 = \lambda_{00} M - \lambda_{10} P_P T_1 - \lambda_{20} P_P T_2 \quad \text{..........(9)}$$

where λ_{00}, λ_{10} e λ_{20} are expressed by eqs (5) and (4).

Let ϕ be the rotation obtained by eq. (9) for a defined relative crack depth ξ, while ϕ_0 be the rotation obtained for the initial relative crack depth ξ_0, both when $M = M_F$, according to eq. (8). It is then possible to obtain the normalized moment-rotation diagram ϕ/ϕ_0, against relative crack depth increase, in the range $\xi = \xi_0 \div \xi = 0.7$, maximum value for which $Y_M(\xi)$ is defined. In the present paper $\xi_0 = c_1/h = 0.1$ has been assumed.

In Figures 3a-3e the moment-rotation diagrams are reported for N_P = 0 (no reinforcement), 0.1, 0.4, 0.7, 1.0. The same figures also show the ultimate carrying capacity, when the cross-section is completely cracked (dotted horizontal lines) and both the reinforcements are yielded. To this purpose, since the resultant tensile force of the two layers of steel (having the same area A_s) is located at $d = [1-(c_1+c_2)/2h]h = \delta h$ from the upper edge of the beam, the ultimate bending moment results to be $M_u = 2P_P h\delta$.

Considering that $N_P = 2P_P h/(K_{Ic} bh^{3/2})$, it is possible to obtain $M_u/(K_{Ic} bh^{3/2}) = N_P \delta$, which is the limit value to which the normalized moment-rotation curves tend, for every defined value of N_P. It is worth noting that for sufficiently high concrete strength and sufficiently low percentage of steel, this type of failure precedes the crushing of concrete. Figures 3a-3e illustrate the failure mechanisms given by the model, varying the brittleness number N_P.

A part from the evident case of not reinforced cross section, where no possibilities to increases the resistance are allowed after the crack propagation (Figure 3a), it is possible to observe:

(a) in Figure 3b (where N_P = 0.1 represents both very low percentage of steel or deep cross section) the contribution of the second layer do not involve any increase of load bearing capacity with regard to the moment of first crack propagation. Then the

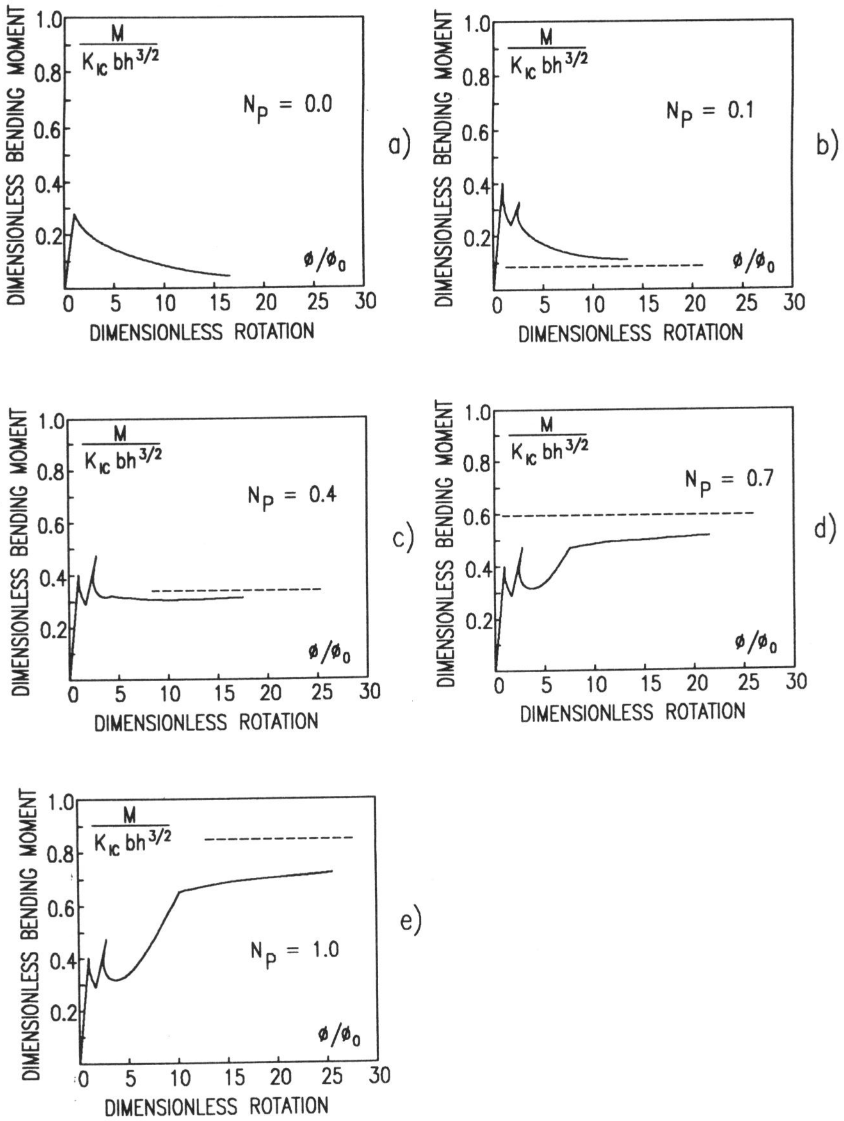

Figure 3 Normalized moment-rotation diagram varying the brittleness number N_P

phenomenon is unstable.

(b) in Figure 3c (where $N_P = 0.4$) it appears the benefits of the second layer of steel, being the second moment of crack propagation greater than the first. Nevertheless the ultimate resistant bending moment do not reach anyone of the peaks, then the phenomenon is still unstable.

(c) in Figures 3d ($N_P = 0.7$) and 3e ($N_P = 1.0$) on the contrary, M_u is greater than M_F. This means that the failure is not brittle and can only be reached by increasing the applied external action, after the second peak.

It is then possible to point out that even with two layers of reinforcement, extremely different failure mechanisms can occur and with regard to a definite strength of concrete and geometrical shape, they are well represented by the brittleness number N_P, containing the relevant parameters ρ (percentage of steel) and h (depth of cross section) that determine the failure mode.

As an example, for a concrete class C40 (f_{ck} = 40 N/mm²), having G_F = 0.09 N/mm according to Model Code 1990 (7) and E = 30000 N/mm², it results K_{IC} = 52 N/mm$^{3/2}$. If the yielding limit of steel is f_y = 430 N/mm² and the beam depth is h = 400 mm, the percentage of reinforcement assumes, respectively, the values 0.06%, 0.24%, 0.42%, 0.60% for each of the four considered N_P values from 0.1 to 1.0. The percentage of steel for which the transition from ductile to brittle failure occurs, results only just greater than 0.24%, see Fig. 3c and then of the same order of magnitude as the requirements of the most important codes concerning minimum percentage of steel in reinforced concrete structures.

CONCLUSIONS

The approach carried out by using a linear elastic fracture mechanics model, even though it is not possible to generalize its applicability to concrete-like materials (nevertheless for high strength concrete it seems acceptable), allows to predict the different failure mechanisms of cross sections, in function both of steel content and dimensional scale.

Moreover, the model provides information about the crack propagation mode that, in the cases herein analysed, appears unstable for low values of N_P (low content of steel or deep cross-sections) or for limited relative crack depth. More precisely, the crack propagation is stable only for relative crack depths greater than 0.5÷0.6 or it may become stable only for brittleness numbers greater than 0.2.

These conclusions are similar to those already found for reinforced concrete structures with a single layer of steel (Carpinteri (1), (2)).

It is worth noting that the failure mode predicted by using this model, approximately corresponds to the experimental behaviour of reinforced concrete with low percentage of steel (Bosco et al (3), (4)).

SYMBOLS USED

a = crack depth (mm)
A_s = area of one layer of reinforcement (mm^2)
b = beam thickness (mm)
c_i = distance of reinforcement from tensile edge of the beam
d = distance of the total reaction of the two layers of reinforcement from compressive edge of the beam (mm)
E = Young's modulus of concrete (N/mm^2)
f_y = yield strength of steel (N/mm^2)
G_F = fracture energy (N/mm)
h = beam depth (mm)
K_I = stress-intensity factor ($N/mm^{3/2}$)
M = external bending moment (Nmm)
M_F = bending moment of crack propagation (Nmm)
M_P = bending moment of reinforcement plastic flow (Nmm)
M_u = ultimate bending moment (Nmm)
N_P = brittleness number
P_i = force transmitted by reinforcement (N)
P_P = force of plastic flow for reinforcement (N)
w = relative displacement (mm)
λ = compliance due to the presence of crack
ξ = relative crack depth
ϕ = local rotation at cracked cross section (rad)
ρ = percentage of reinforcement

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