REMARKS ON THE INTERPRETATION OF CRACKING IN CONCRETE STRUCTURES ACCORDING TO LINEAR ELASTIC FRACTURE MECHANICS

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Starting from previous studies, the writers deal with the problem of cracking of reinforced concrete structures subjected to bending moments. They consider different stress diagrams in the cross section and establish comparisons on results according to the different hypotheses.

1. Some previous papers (2) have dealt with cracking of reinforced concrete in terms of Linear Elastic Fracture Mechanics (LEFM). The present paper starts from the above results and carries on some further advancements. A rectangular cross section of a bent beam is composed by a concrete active section (delimited by the tip of a crack of depth a and subjected to a compression force) and by a steel section (subjected to a tensile force). We consider the compression force F located at the half depth of the beam and equal to the tensile force in reinforcement. On the active concrete section shall act also a bending moment M_1, where:

$$M = F\ (d/2 - h) + M_1 \qquad (1)$$

(M is the external bending moment: see fig.1).
At the crack tip, K_I is given by superposition:

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$$K_I = \frac{M_1 \, Y_m \, (a/d)}{W \, d^{3/2}} - \frac{F \, Y_p \, (a/d)}{W \, d^{1/2}} \qquad (2)$$

The shape functions Y_m and Y_p are given in Reference (2). From eqs. (1) and (2), without introducing supplementary hypotheses, it is possible to establish, for a given value of M, the separate values of M_1 and F(d/2-h), for each value of a/d, which produce at the crack tip $K_I = K_{IC}$. Carrying on the calculations for $a/d=0.3\div0.6$ and $M/(K_{IC} W d^{3/2})=0.5\div1.4$, we can verify with satisfactory approximation:

$$M_1 \cong F\,(d/2 - h) \cong M/2$$

In general, without the obove simplification, we remark that after yielding of reinforcement ($F = F_y$ = const.) M and a/d are univocally correlated (2). The same conclusion can be extended to the beams with unbonded prestressed tendons, where the stresses are not depending on the external moment.

2. In the general case, when the reinforcement is not yielded, we are not able to establish a relation between M and a/d for a known K_{IC}, because the two equations (1) and (2) involve three unknowns (M_1, F, a/d). So it is necessary to introduce a supplementary hypothesis, for instance assuming a given stress distribution law in the cracked section. As a first attempt we can choose an hypothesis according to the n-method. The position of neutral axis is given by ($\mu=A_s/Wd$):

$$x/d = n\mu \left[-1 + \sqrt{1 + 2\,(1 - h/d)/(n\mu)} \right] \qquad (3)$$

and is not intended to coincide with the crack tip. From eqs. (1), (2) and (3), being M=F(d-h-x/3), we obtain:

$$\frac{M}{K_{IC} \, W \, d^{3/2}} = \frac{1 - h/d - x/(3d)}{Y_m \left[1/2 - x/(3d) \right] - Y_p} \qquad (4)$$

Fig.2 (for different values of $n\mu$) shows the couples of values M and a/d for which the critical K_{IC} is attained at the crack tip. The validity range of such diagrams is limited by the yielding of reinforcement. We can calculate the limit value of $M=M_y=F_y(d-h-x/3)$, where $F_y=f_y\mu W\ d$:

$$\frac{M_y}{K_{IC}W\ d^{3/2}} = \frac{f_y\mu d^{1/2}}{K_{IC}}\left[1 - h/d - x/3d)\right] \quad (5)$$

The range of validity of each curve $n\mu$=const is limited by an horizontal line (fig.2).

3. A second model for stress distribution can be proposed from the application of σ-ε laws of concrete and steel, according to the standards on ultimate limit state (fig.3). With iterative procedure (assuming the planarity of the section) we obtain $M/(K_{IC}d^{3/2}W)$ versus a/d for different values of μ (fig.4). If we compare the results of models 1 and 2, we can remark that the greater complexity of the second model does not provide a substantial improvement (at least in our numerical example) with respect to the results of the first model.

4. The two proposed models are not consistent with LEFM because for both the beginning of cracked zone, according to the stress distribution law, does not coincide with the location of the crack tip, according to Fracture Mechanics. In a proposed third model the stress diagram in concrete cross section is delimited by the crack tip.

We suppose (fig.5): (1) the planarity of the cross section (including reinforcement); (2) the elastic distribution of stresses in the uncracked concrete section.

This hypothesis is a very simple proposal, which can be improved with more elaborated stress distribution laws.

The position of neutral axis is obtained setting the static moment of active cross section equal to zero:

$$x = \frac{(d - a)^2 + 2\ n\mu d\ (d - h)}{2\ (d - a + n\mu d)} \quad (6)$$

Consequently we can calculate the moment of inertia, the stress in reinforcement $f = n\ M\ (b - h - x)/J$ and the tensile force $F = f\ A_s$. So we can write:

$$\frac{M}{K_{IC} W\, d^{3/2}} = \frac{J}{Y_m J - \psi n\mu (1 - h/d - x/d) W\, d^3} \qquad (7)$$

where $\psi = Y_p + (1/2-h/d)Y_m$. In fig.6 the limit curves ($n\mu$=0.75% and 7.5%) are drawn of the third model and also the same lines referring to the first model. We can observe that the first model shows considerably higher moment values in comparison with the third one. In fact, taking into account the tensile strength of concrete, leads to an increase of internal couple arm and therefore a decrease of compression in concrete, which originates a higher value of K_I. Further researches should be carried out toward more elaborated proposals for stress distribution laws.

SYMBOLS USED

d = beam depth
W = beam width
M = bending moment
F = tensile force in reinforcement
A_s = reinforcement cross section

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(2) Carpinteri, A. - J. of Structural Engineering (ASCE). Vol.110., 1984, pp.544-558.

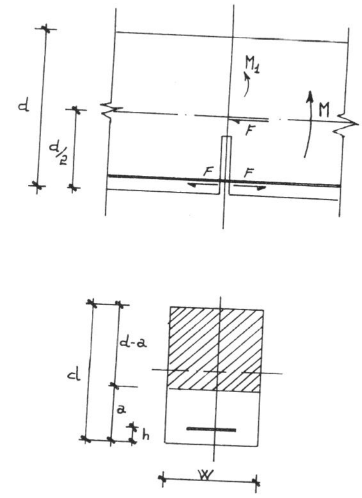


FIG.1

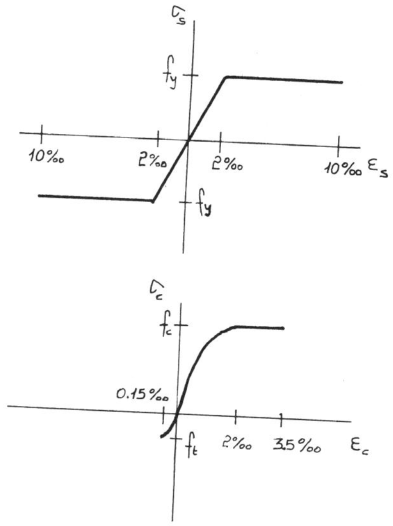


FIG.3

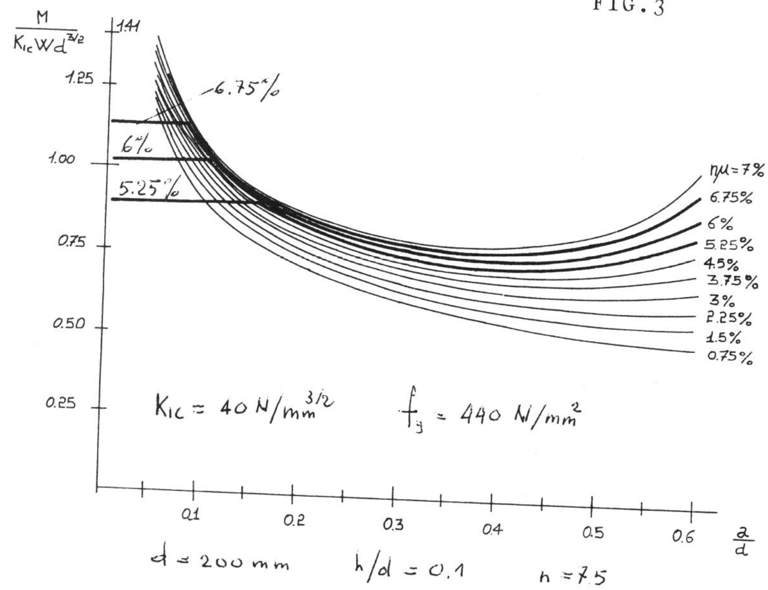


FIG.2

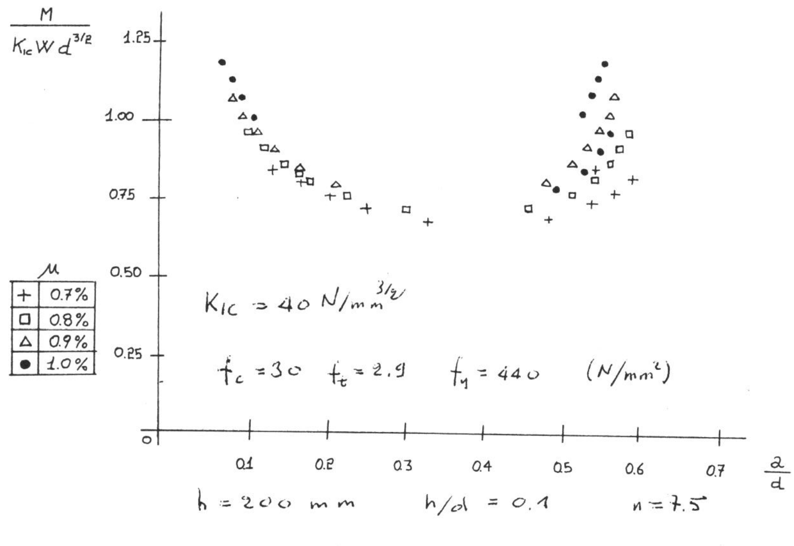
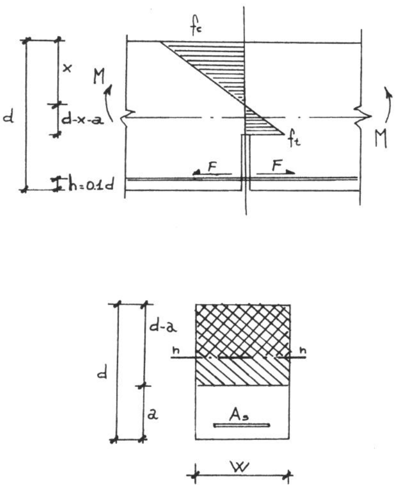


FIG.4



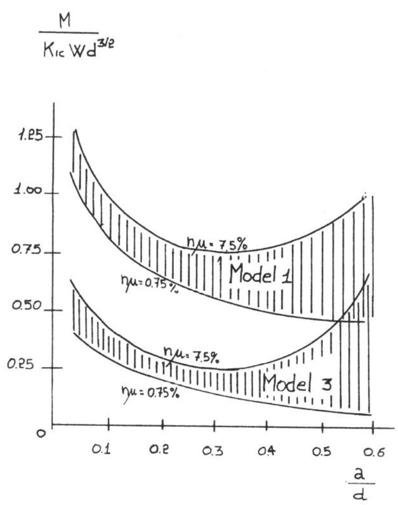


FIG.5

FIG.6