THE EFFECT OF SNUBBING FRICTION ON THE FIRST CRACK STRENGTH OF FLEXIBLE FIBER REINFORCED COMPOSITES

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For brittle matrix reinforced with flexible fibers, matrix cracking is generally accompanied by fiber pulled out at various inclined angles. This generates a snubbing effect, analogous to pulling a rope over a friction pulley. This paper analyzes such effect on the composite first crack strength and suggests that the reliability of such composites could be enhanced by exploiting the snubbing effect.

INTRODUCTION

The first crack strength is one of the most important tensile properties in fiber composites. In this paper, we attempt to relate the first crack strength to the bridging stress versus crack opening curve for a generic brittle matrix reinforced by discontinuous and randomly distributed flexible fibers. For such a fiber composite, recent works (Li et al, [1,2]) suggest that fibers inclined at an angle to the matrix crack plane may lead to a snubbing effect which increases the bridging force exerted by the fibers across the crack plane. This paper emphasizes the contribution of this snubbing effect to the first crack strength.

THE BRIDGING STRESS-CRACK OPENING RELATIONSHIP

The Bridging Stress-Crack Opening Relationship (hereafter abbreviated as the $\sigma_B-\delta$ curve) results from fibers bridging across a matrix crack. In this paper, without loss of generality, we adopt a simple model of fiber pull-out with only frictional bond in which the matrix is assumed to be relatively rigid. Furthermore, we assume that the

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fibers are short enough (or with low enough interface bond strength) that all fibers will be pulled out without rupture. Consider a single fiber with an embedded length of ℓ. During debonding, the fiber bridging load P versus displacement δ is given by

$$P(\delta) = \pi\sqrt{E_f d_f^3 \tau\delta / 2}\, e^{f\phi} \quad for \quad \delta \le \delta_o \tag{1}$$

where $\delta_o \equiv \frac{2\ell^2\tau}{E_f d_f}$ corresponds to the displacement at which debonding is completed for a fiber with embedment length of ℓ, diameter d_f, elastic modulus E_f and with an interface bond strength τ. The exponential function in (1) is introduced to account for the snubbing effect (Li et al, [1]) which amplifies the bridging force due to fiber/matrix snubbing when a fiber is pulled out at an angle of ϕ to the loading axis. The snubbing coefficient f is specific to a particular pairing of fiber and matrix. The value of f has been experimentally measured to have a value of close to unity for two fiber types in a cement matrix.

When the fiber is fully debonded, the load point displacement is mainly due to that of the fiber end slippage, and (for simplicity) we assume that the elastic stretching of the fiber can be neglected. This results in a fiber pull-out load versus displacement relation given by

$$P(\delta) = \pi\tau\ell d_f (1 - \delta / \ell) e^{f\phi} \quad for \quad L_f / 2 \ge \delta > \delta_o \tag{2}$$

For a composite with fiber volume fraction V_f, Li et al [2] show that the composite $\sigma_b - \delta$ curve can be predicted by integrating over the contributions of individual fibers crossing a matrix crack plane:

$$\sigma_B(\delta) = \frac{V_f}{\pi d_f^2 / 4} \int_{\phi=0}^{\pi} \int_{z=0}^{(L_f/2)\cos\phi} P(\delta) p(\phi) p(z)\, dz\, d\phi \tag{3}$$

where $p(\phi)$ and $p(z)$ are probability density functions of the origination angle and centroidal distance of fibers from the crack plane. For uniform random distributions, $p(\phi) = \sin\phi$, and $p(z) = 2/L_f$ (Li et al, [2]).

Using (2) in (3), we find, in normalized form:

$$\tilde{\sigma}_B(\tilde{\delta}) = g\left[\tilde{\delta}^2 - \frac{2}{3}\tilde{\delta}^*\tilde{\delta}^3 + 2(\tilde{\delta}/\tilde{\delta}^*)^{1/2} - \frac{4}{3}(\tilde{\delta}^3/\tilde{\delta}^*)^{1/2} - \tilde{\delta}/\tilde{\delta}^*\right] \quad for \quad \tilde{\delta} \le \tilde{\delta}^* \tag{4}$$

where $\tilde{\sigma}_B \equiv \sigma_B/\sigma_o$ and $\sigma_o \equiv V_f \tau (L_f/d_f)/2$, and $\tilde{\delta} \equiv \delta/(L_f/2)$. $\tilde{\delta}^* \equiv (\tau/E_f)(L_f/d_f)$ corresponds to the maximum attainable (normalized by $L_f/2$) value of δ_o for the fiber with the longest embedment length of $L_f/2$. The snubbing factor defined as

$$g \equiv \frac{2}{4+f^2}\left(1+e^{\pi f/2}\right) \tag{5}$$

essentially scales the bridging stresses in relation to the snubbing effect. A detail derivation of (4) and its post-peak tension-softening counterpart, together with comparisons with experimental data, is given in Li [3]. Equation (4) is used to compute the pre-peak part of the σ_B–δ curve, shown in Figure 1, for various snubbing coefficients. In general, the peaks of the σ_B–δ curves occur slightly prior to δ^*.

FIRST CRACK STRENGTH

The first crack strength represents the tensile stress at which a matrix crack spreads throughout a cross section of a uniaxial tensile specimen. In a fiber composite, propagation of a matrix crack is resisted by matrix toughness as well as by bridging fibers. Aveston et al [4] considered the energy balance in the propagation of a matrix crack in a fiber composite under the steady state condition, and yields a lower bound of the first crack strength. Marshall and Cox [5] reanalyzed the problem based on balancing of the stress intensity factor due to loading, bridging and matrix toughness resistance, to include pre-steady state cracking for aligned fiber composites. Leung and Li [6] extended the analysis to discontinuous aligned fiber systems.

The analysis of first crack strength in short random flexible fiber composite follows the procedure of [5, 6]. Balancing of the combined stress intensity factor due to applied remote loading K_L and that due to fiber bridging behind the crack tip K_B, with the crack tip fracture toughness K_c requires:

$$K_L + K_B = K_c \tag{6}$$

For small fiber volume fraction K_c can be taken to be simply the matrix toughness.

For a penny shaped crack of radius c, the normalized stress intensity factor due to ambient tensile loading σ is given by

$$\tilde{K}_L \equiv \frac{K_L}{\sigma_o\sqrt{c_o}} = 2\sqrt{\frac{\tilde{c}}{\pi}}\tilde{\sigma} \tag{7}$$

where

$$c_o \equiv \left(\frac{E_c L_f}{K_c}\right)^2 \frac{\pi}{16(1-\nu^2)^2} \tag{8}$$

and the normalized crack length $\tilde{c} \equiv c / c_o$. In (8), E_c and ν are the composite Young's modulus and Poisson's ratio respectively.

The normalized stress intensity factor due to fiber bridging may be obtained by integrating the solution for a penny shaped crack loaded by concentric distributed pressure (see, e.g. Tada et al, 1973) over the crack plane,

$$\tilde{K}_B \equiv \frac{K_B}{\sigma_o \sqrt{c_o}} = -2\sqrt{\frac{\tilde{c}}{\pi}} \int_0^1 \tilde{\sigma}_B(\tilde{\delta}) \frac{X dX}{\sqrt{1-X^2}} \tag{9}$$

The negative sign in (7) is due to the crack closing effect of the fiber bridging stress. In general, the crack opening $\tilde{\delta}$ is not known *a priori* as a function of position X along the crack line, and evaluation of K_B typically requires an iterative process. For simplicity, the crack profile is assumed to take on the same parabolic shape as if the bridging stresses on the crack flanks are uniform. This is clearly not the case in reality. Nonetheless, this assumption allows us to extract rough estimates on the first crack strength without resorting to a full-scale numerical analysis. For the parabolic shape crack, the normalized crack opening is given by:

$$\tilde{\delta} = \sqrt{\tilde{c}}\,(1 - X^2) \tag{10}$$

The first crack strength σ_{fc} may be obtained when equation (6) is met, and making use of (7), (9) and (10):

$$\tilde{\sigma}_{fc} / g = \frac{-15\tilde{c} + 40\tilde{c}^{3/4} - 16\tilde{c}^{5/4}\tilde{\delta}^{*1/2} + 10\tilde{c}^{3/2}\tilde{\delta}^{*} - 5\tilde{c}^{2}\tilde{\delta}^{*2} + 15\sqrt{\pi}\tilde{\delta}^{*}(\tilde{K}_c / g)}{30\tilde{c}^{1/2}\tilde{\delta}^{*}} \tag{11}$$

where (4) has been used to evaluate the integral in (11), for the purpose of analyzing steady state cracking. For steady state cracking to occur, two conditions must be met: (1) The stress at the mid-point of the crack σ_m must equal the first crack stress σ_{fc}, and (2) The crack opening displacement at the mid-point of the crack δ_m must be less than the displacement δ_p corresponding to the maxima of the bridging stress as expressed in (4).

The first condition implies

$$\tilde{\sigma}_m\left[\equiv \tilde{\sigma}_B\left(\tilde{\delta} = \tilde{\delta}_m \equiv \sqrt{\tilde{c}}\right)\right] = \tilde{\sigma}_{fc} \tag{12}$$

and results in a locus of $\tilde{c}$ for which (12) is satisfied. These values of $\tilde{c}$ are denoted $\tilde{c}_S$ in Figure 2 which illustrates the two conditions of steady state cracking schematically. In general, $\tilde{c}_S$ is a function of $(\tilde{K}_c / g)$ and $\tilde{\delta}^*$. For large value of $(\tilde{K}_c / g)$, no steady state cracking will be reached, and the first crack strength will continue to decrease with crack size. For small $(\tilde{K}_c / g)$, the first crack strength will plateau to a steady state value for crack size beyond $\tilde{c}_S$. The critical value of $(\tilde{K}_c / g)$ beyond which no steady state cracking occurs may be obtained from the second condition which translates into

$$\tilde{\delta}_m \leq \tilde{\delta}_p \tag{13}$$

from which $\tilde{c}_p$ can be determined (by setting $\tilde{\delta}_m = \tilde{\delta}_p$). That is, $(\tilde{K}_c / g)$ is at the critical value when $\tilde{c}_S = \tilde{c}_p$.

Figure 3 shows a specific example of these concepts for the case $\tilde{\delta}^*$=0.001. Using (4), $\tilde{\delta}_p$ was found to be .996 $\tilde{\delta}^*$. The critical value of $(\tilde{K}_c / g)$ was then found to be 1.87x10^{-4}. For $(\tilde{K}_c / g)$ smaller than this value, transition to steady state occurs at $\tilde{c}_S < \tilde{c}_p$ with $\tilde{c}_s$ marked by the black dots. It is clear that $\tilde{c}_S$ decreases with smaller $(\tilde{K}_c / g)$ or larger g. Figure 4 shows the transition crack size as a function of $(\tilde{K}_c / g)$. These results imply that the snubbing effect of angled fiber pull out enhances the tendency towards steady state cracking through the snubbing factor (5). Interestingly, it enters as an effective reduction of the normalized composite crack tip toughness, i.e. through $(\tilde{K}_c / g)$. The first crack strength at steady state may be obtained by substituting $\tilde{c}_s$ for $\tilde{c}$, together with the corresponding $(\tilde{K}_c / g)$ in (11). This results in a relationship between $(\tilde{\sigma}_{fc} / g)$ and $(\tilde{K}_c / g)$, as shown in Figure 5. To illustrate the direct effect of g on the first crack strength, we have converted this plot into Figure 6, using as an example $\tilde{K}_c$= 10^{-4}. For g increasing from 0.5 to 3, this example indicates an increase of the normalized first crack strength $\tilde{\sigma}_{fc}$ from about 0.5 to 1.9.

CONCLUSIONS

This paper presents a study of the influence of snubbing friction on the first crack strength of flexible fiber reinforced brittle matrix composites. From this study, it is concluded that (1) the snubbing effect enhances the tendency towards steady state cracking, at a smaller transition crack length, and (2) the value of steady state first crack strength increases with the snubbing coefficient f via the snubbing factor g. The first conclusion implies that a strong snubbing effect can lead to a composite with higher reliability. That is, the composite will never fail below the steady state first crack load level, no matter how large the flaw size the composite may contain. The second conclusion implies that a higher first crack strength, larger than the matrix strength, may be designed through the exploitation of the mechanism of snubbing. This mechanism is available only to randomly distributed fiber reinforcements, and may perhaps make up for the well known reduction of reinforcement efficiency when compared to aligned fiber composites. However, it should be emphasized that the present results are based on the assumption that the fibers are flexible enough to bend around the matrix crack during crack opening and fiber pull-out, and that the fibers are short enough or only moderately bonded so that their axial strength is never exceeded and fiber rupture is prevented.

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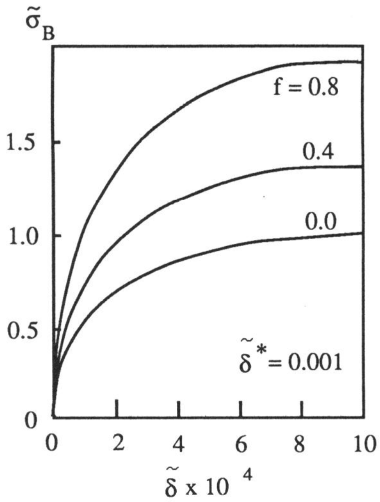

Fig.1: Pre-peak Bridging Stress vs. Crack Opening Relationship

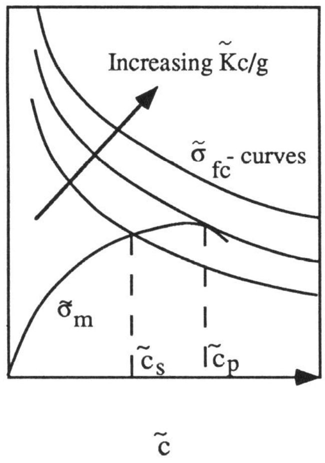

Fig. 2: Schematics Illustrating Steady State Cracking Concepts

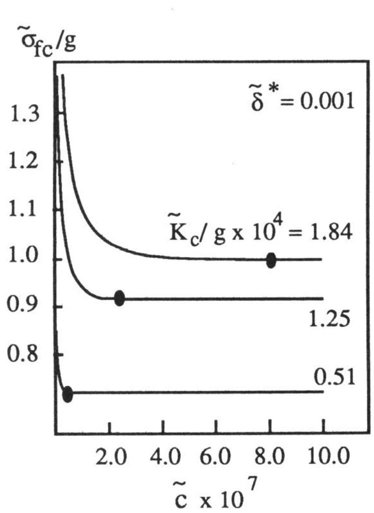

Fig. 3: First Crack Strength vs. Crack Size

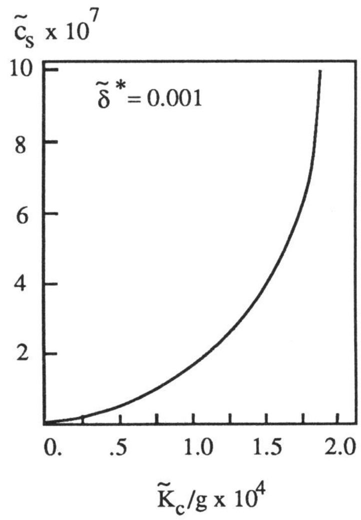

Fig. 4: Transion Crack Size vs. Normalized Crack Tip Toughness

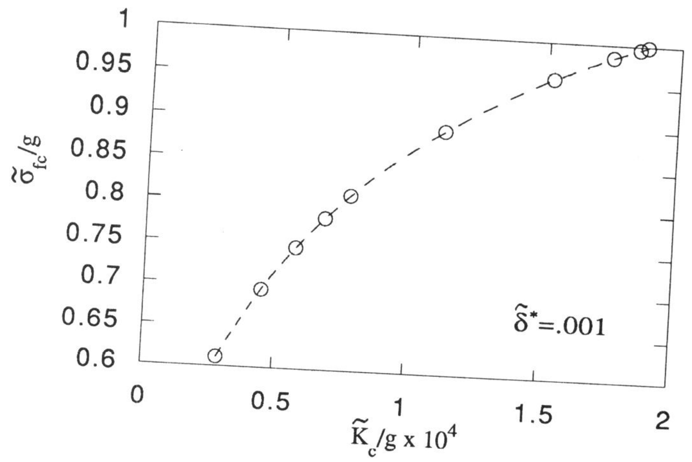

Fig. 5: Computed Normalized First Crack Strength to g Ratio as a Function of Normalized Crack Tip Toughness to g Ratio

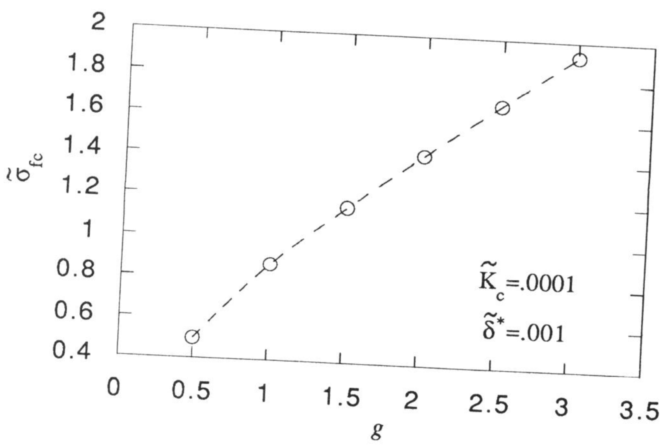

Fig. 6: Computed First Crack Strength as a Function of the Snubbing Factor g