ON THE INFLUENCE OF THE BULK DISSIPATION ON THE AVERAGE SPECIFIC FRACTURE ENERGY OF CONCRETE

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This paper presents the first approach to evaluate bulk dissipation in concrete. A simple constitutive equation was derived for the bulk material, based on internal variable concepts and a modified Rankine criterion.To simplify the use of the model a first order perturbation approximation was developed. This method is applied, in conjunction with a cohesive crack model, to normal size specimens (using finite elements) and to very large sizes.The main conclusions are, as long as concrete is concerned, that the bulk dissipation may hardly account for the observed size dependence of the average specific fracture energy.

INTRODUCTION

Cohesive crack models describe the fracture process zone behavior with two basic parameters: the tensile strength and the specific fracture energy. For concrete and the usual structure sizes both parameters bear similar importance in structural design. A RILEM recommendation exists for the experimental determination of the fracture energy based on previous work by the group of Hillerborg at Lund University [1]. The method introduces the simplification that the energy dissipation occurs only at the cohesive crack, the bulk of the specimen remaining essentially elastic. In this case the fracture energy obtained in properly performed RILEM tests should be independent of the specimen size.

Unfortunately most of the published results of fracture energy tests show a strong dependence on specimen size, that make difficult to identify experimental values of fracture energy with a material property. The most remarkable example is a Round Robin promoted by RILEM that involved 14 laboratories, through 9 countries with more than 700 beams tested [2]. The possibility that this size effect come from the existence of bulk dissipation due to non elastic behaviour of the material surrounding the cohesive crack has been pointed out repeatedly [3, 4] and was even qualitatively analyzed in a recent RILEM state-of-art report by Elices and Planas [5]. However,

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probably due to the complexity of the necessary computations, no quantitative results have been reported up to now.

This paper presents the first results of a research on the influence of bulk dissipation around the cohesive zone on the variation of measured specific fracture energy with specimen size. To this end, a new simple constitutive equation has been developed for bulk concrete based on a modified Rankine criterion. In order to simplify the application of the model a first order perturbation method has been developed and applied both to usual laboratory sizes and to the limit of infinite size.

THE MODEL

To include bulk dissipation in the cohesive model we follow the general theory described in [5]. According to this, in a stable pure tensile test one would get the stress-average strain depicted in Fig. 1. The model is completely defined by a dissipative stress-*strain* relation in the loading branch previous to the peak, and by a stress- *crack opening* curve in the post-peak softening branch.

In this first approach to the problem, we consider a bulk stress-strain behaviour without stiffness degradation. During softening the bulk material close to the cohesive crack unloads —following a line parallel to the initial elastic branch— leaving a irrecoverable strain, which in the uniaxial case is ε^p, variable with the previously reached level of stress σ^p (Fig. 1). The relationship between ε^p and σ^p obtained from a uniaxial test will be the main input for our model, which, in order to be used in a non-homogeneous case, must be extended first to triaxial situations.This extension was developed in detail in [6] using an internal variable formulation together with a thermodynamic approach which is powerful enough for the model to be easily extended to include stiffness degradation as well as irrecoverable strains.

In its present simpler form, it happens to coincide with an elastoplastic model with a Rankine loading function and associated flow rule. The final equations are extremely simplified by introducing the *Supreme* functional of the maximum principal stress σ_I :

$$\mathrm{Sup}\,(\sigma_I) := \mathrm{Sup}\,[\sigma_I(\tau),t] = \max\,\{\sigma_I(\tau);\ \tau \in [0,t]\} \qquad (1)$$

With this definition, the governing equations are reduced to:

$$d\boldsymbol{\varepsilon} = \mathbb{C}\ d\boldsymbol{\sigma} + \mathbb{P}_I\ d\varepsilon^p \qquad (2)$$

$$\varepsilon^p = f(\sigma^p) \qquad \text{and} \qquad \sigma^p = \mathrm{Sup}\,(\sigma_I) \qquad (3)$$

Equation (2) is the classical split of the incremental strain into elastic and irrecoverable parts. The outlined fonts indicate second order tensors, and $\mathbb{C}$ is the elastic compliance fourth order tensor (constant). $\mathbb{P}_I$ defines de direction of plastic flow and is the projector tensor on the subspace associated to the largest eigenvalue of the stress tensor, σ_I, and ε^p is the (uniaxial) equivalent plastic strain. The two equations (3) are the hardening law and the integrated form of the Rankine load function $\sigma_I \leq \sigma^p$. The hardening law is the only material function of the model . It is the relationship between the equivalent plastic strain and the instantaneous yield limit, and it is directly obtained from a uniaxial tensile stress-strain curve in the pre-peak branch, as depicted in Fig. 1.

These equations may be easily handled to find a closed form equation relating the incremental strain tensor to the history of principal stresses:

$$d\varepsilon = \mathbb{C}\ d\boldsymbol{\sigma} + \mathbb{P}_I\ f'[Sup(\sigma_I)]\ d[Sup(\sigma_I)] \tag{4}$$

where f'(x) indicates fist derivative of function f(x) with respect of its argument.

A fundamental feature of the model is that the energy dissipated per unit volume w^D along any loading path, may be written in closed form as

$$w^D = D[Sup(\sigma_I)] = \int_0^{Sup(\sigma_I)} \sigma^p f'(\sigma^p)\ d\sigma^p \tag{5}$$

In this paper, the hardening law has been approximated by a parabolic equation of the second degree for stresses over 50 percent of the tensile strength:

$$\varepsilon^p = f(\sigma^p) = 0 \qquad \text{for} \qquad \frac{\sigma^p}{f_t} \leq 0.5$$

$$\varepsilon^p = f(\sigma^p) = \varepsilon_m \left(2\frac{\sigma^p}{f_t} - 1\right)^2 \qquad \text{for} \qquad \frac{\sigma^p}{f_t} \geq 0.5 \tag{6}$$

where f_t is the tensile strength and ε_m is the inelastic strain at the peak .

The softening function was taken to be the bilinear equation proposed by Hillerborg and coworkers which was exhaustively analyzed by Petersson [7], and has been used by many researchers to simulate concrete fracture.

PERTURBATION ANALYSIS

The study of the development of a cohesive crack inside an elastoplastic body is a complex task requiring very powerful special purpose numerical codes. However it is possible to reveal the dominant effects by means of a first order perturbation analysis set up considering a uniparametric family of elastoplastic bodies in which inelastic strain can be decreased uniformly to zero by writing the hardening function $f(\sigma^p)$ as $f(\sigma^p) = \lambda\ f^*(\sigma^p)$ which, according to Eq. (6), leads to the incremental constitutive equation

$$d\varepsilon = \underset{\#}{\mathbb{C}}\ d\boldsymbol{\sigma} + \lambda\ [P_I\ f^{*\prime}[Sup\ (\sigma_I)]\ d[Sup\ (\sigma_I)] \tag{7}$$

Obviously, when $\lambda \rightarrow 0$ the constitutive equation tends to the elastic form, so that at any loading step one can write the solution for the stress distribution as

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_0 + \mathbb{O}(\lambda) \quad \text{and} \quad Sup\ [\sigma_I] = Sup\ [\sigma_{0I}] + 0(\lambda) \tag{9}$$

where $\mathbb{O}(\lambda)$ and $0(\lambda)$ are, respectively, a tensor-valued function and a scalar-valued function vanishing for $\lambda \rightarrow 0$, and $\boldsymbol{\sigma}_0$ and σ_{0I} are the elastic solutions. The dissipation density may then be written, according to Eq. (6)

$$w^D = \int_0^{Sup(\sigma_{I0})} \sigma^p f'(\sigma^p)\ d\sigma^p + \lambda\ \ 0(\lambda) \tag{10}$$

The first order approach for the dissipated energy is obtained taking only the first term in equation (10). Under these circumstances we only need to know the distribution of principal maximum stresses computed with the hypothesis of linear elastic behavior. The smaller the plastic strains with respect to the elastic ones, the better the approach will work.

APPLICATION

When performing an ideal RILEM test on a bulk-dissipative material, the measured or experimental fracture energy, G_{FE}, will be the sum of two terms. One is due to the work of the cohesive stresses, which is independent of size and is equal to the area under the softening curve, G_F. The second term, G_D, comes from the energy dissipated in the bulk and it is size dependent because the size of the plastic zone changes with specimen size.

The analysis of the influence of the size on G_D has been performed for three point bend specimens with the geometry depicted in Fig. 2. The computations included an asymptotic analysis for infinite size The concrete properties are included in Fig. 2.

The computational method for small specimens was very simple in principle. The classical problem of the cohesive crack in a *linear elastic* medium was solved step by step with a commercial finite element code using 100 elements along the beam depth and special interface elements to simulate cohesive behaviour. The supreme of the major principal stress at each Gauss point was recorded along the process, and after complete fracture the energy dissipated per unit volume was found using Eq.(10). Volume integration (sum over elements) gave the total energy dissipation in the bulk, and division by the area of the initial ligament laid G_D.

For infinite size the elastic solution in steady-state crack propagation was obtained using the asymptotic method developed by Planas and Elices [8, 9]. The maximum principal stress was computed on a dense 2D grid around the cohesive zone, and the steady-state bulk energy dissipation rate was obtained using J-integral concepts. See [6] for details.

The essential results are displayed in Fig. 3, where the value of G_D relative to the cohesive fracture energy G_F has been plotted versus the size of the specimen. From this figure it becomes evident that the bulk dissipation gives a contribution to the experimental fracture energy which is strongly size dependent. It also appears that the usual specimen sizes (up to 40 cm depth) are very far from the asymptotic behaviour (infinite size). However, it is also apparent that the bulk dissipation is a small fraction of the total dissipation, affecting the experimental result in no more than 5 percent. Other sources of size effect must be operating to justify the variations of G_{FE} reported in [4] (up to a 50 percent variation in a three-fold increase in size).

CONCLUSIONS

1. The classical cohesive crack model has been extended to account for bulk dissipation by means of a reasonably simple model based on a Rankine loading function which fits well uniaxial tensile tests.

2. A perturbation theory has been developed to simplify making estimates of the contribution of bulk dissipation to the overall energy dissipation.

3. The results for three point bent notched specimens show that the contribution of bulk dissipation to fracture energy is strongly size dependent, and that the usual laboratory sizes are very far from the asymptotic limit.

4. For any size the bulk dissipation is a small fraction of the overall dissipation (less than 5 percent) and cannot explain the observed size effect on the fracture energy obtained by the RILEM procedure.

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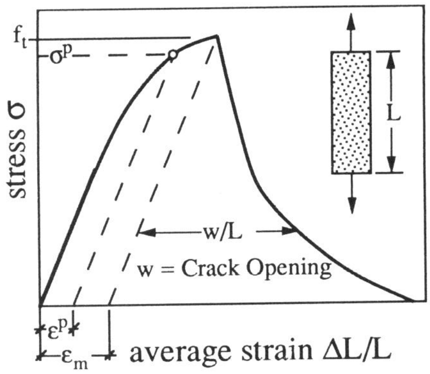
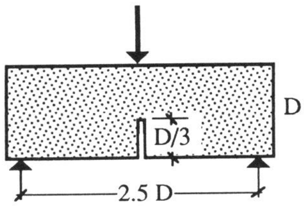


Figure 1. Schematic definition of the model in uniaxial tensile tests.



$\nu = 0.2$
$E = 27$ GPa
$f_t = 3$ MPa
$\varepsilon_m = 0.04$ ‰
$G_F = 100$ N/m
Bilinear softening

Figure 2. Definition of geometry and concrete properties.

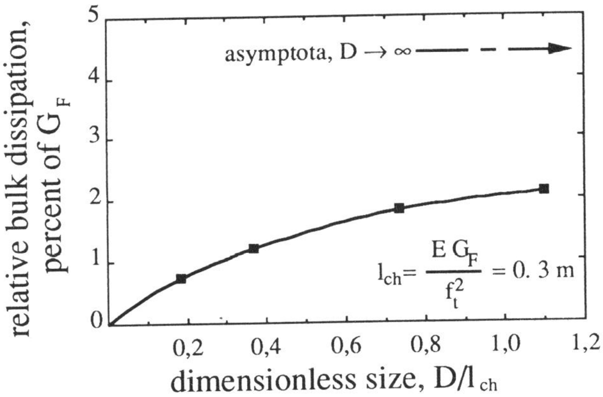


Figure 3. Contribution of bulk dissipation to the average fracture energy as a function of the specimen size