# FRACTURE PROPAGATION AND INSTABILITIES IN ELASTIC-COHESIVE CRACK MODELS: A BOUNDARY ELEMENT ANALYSIS

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A widely accepted idealization of fracture phenomena in concrete and concrete-like materials rests on the assumption of a 'cohesive-softening-crack' model for the 'craze' or 'process zone' and of linear elasticity elsewhere. On such basis the propagation of fracture was investigated by a multidomain boundary element (BE) approach. The following results are briefly presented herein: the BE method is ideally suited for this nonlinear analysis; bifurcations of equilibrium path may occur and prevent from considering one-half only of symmetric structures; the hypothesis on shear transmission in the crack zone may have a significant influence; overall instability thresholds can be characterized by criteria concerning small size matrices.

## INTRODUCTION

The problem to be discussed in this communication can be described as follows. The material is considered as linear-elastic and isotropic but a fracture process is modelled according to the cohesive-softening crack hypothesis. Namely, a discontinuity of normal displacements $\Delta u_n$ (normal to the discontinuity surface) arises when the tensile stress attains an ultimate value $\bar{\sigma}$ and is accompanied by a normal traction $p_n$ which is a decreasing function of $\Delta u_n$ and vanishes when $\Delta u_n \geq \bar{w}$, $\bar{w}$ being another material parameter. The surface where $0<\Delta u_n<\bar{w}$ is called the 'process zone' or 'craze', and, as the fracture propagates, is followed by the 'crack' where $\Delta u_n>\bar{w}$, $p_n=0$ (no interaction is assumed between the two faces). As for the transmission of shear force $p_t$ along the craze, two hypotheses can be assumed: (a) unlimited $p_t$ with no

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discontinuity in the tangential displacements ($\Delta u_t=0$); (b) no shear strength ($p_t=0$) with admissible displacement jump $\Delta u_t \neq 0$.

The above idealization of fracture processes is widely regarded as fairly realistic and practically acceptable with the shear assumption (a) for concrete and concrete-like materials (rocks, some kinds of ceramics, bricks and masonry); with shear hypothesis (b) for polymeric materials in various technological situations.

Two-laws constitutive models of this sort (elastic stable strain-stress law; unstable, softening law between relative displacement and stress) were originally proposed for the flexural behaviors of reinforced concrete beams in bending (1) (2), such as overall instability snap-back (catastrophic) instabilities and bifurcations of equilibrium paths. The cohesive two-law model was later adopted by various Authors, for continua of concrete-like materials (3) (4) (5) (6) (7) and experimentally investigations in (8) (9).

## METHODS AND RESULTS: AN OUTLINE

With reference to Fig. 1, the collocation BEM adopted can be summarized as follows. A standard Somigliana boundary integral equation (BIE) is written for each subdomain $\Omega^r$ (r=1,2). Displacements and tractions on the boundary $\Gamma^r$ and the interface $\Gamma_c$ are modelled by quadratic interpolations. The BIE is collocated in each node, the static and kinematic matching conditions are imposed for the nodal variables on $\Gamma_c$ and all other variables are condensed. The linear equations thus obtained are associated to the nonlinear, nonholonomic rate-relation $p_n$ vs. $\Delta u_n$ which characterize the current craze length $\Gamma_{cz}$. Assuming the location of the craze tip as the driving (input) variable and the load factor increment $\Delta\alpha$ as an unknown, the finite propagation-step problem is formulated (as a linear complementarity problem in the craze variables only). This problem is solved by an implicit, prediction-correction, algorithm. When the fracture propagation itinerary is not a priori known, before each step the direction of the craze tip advancement is determined as the direction of maximum principal stress and the interface $\Gamma_c$ ahead the tip is suitably adjusted whenever needed.

As an alternative to the conventional collocation BEM mentioned above, a symmetric BEM was also applied, resting on the Galerkin weighted residual double-integration approach developed in (10) and (11). The advantage of this alternative is that the second-order work involved by perturbations from a given state can be expressed by means of suitable quadratic forms in the craze variables only. This circumstance was used to establish overall stability and bifurcation criteria, conceptually similar to those earlier proposed for softening beams in (1) and (2).

The numerical results achieved by implementing in two computer programs the above outlined concepts and applying them to some typical situations are partly illustrated by Fig.2-4. These results corroborate the following conclusions.

1. For the same number of nodes along a fixed interface $\Gamma_c$, the collocation BEM and the FEM exhibit roughly the same accuracy and convergence properties at mesh refinement; these properties are generally better for the Galerkin symmetric BEM which, however, is more laborious to implement. The plots of Fig.2a obtained by the three different approaches turn out to be indistinguishable. Fig.1c visualizes the superiority of BEMs with respect to FEMs in problems like the present ones, where all nonlinearities are confined to the boundary or to an interface (while the domain is linear elastic). This superiority is further enhanced when the fracture itinerary is a priori unknown, so that frequent re-meshing is required by interface adjustments.

2. It was proved in this study that a sufficient condition for overall instability due to softening in the process zone $\Gamma_{cz}$ is the fact that a matrix of the same dimension as the number of nodes on the craze ceases to be positive definite. This finding is illustrated by a comparison between fig.2a and fig.2b, where the least eigenvalue of that matrix is plotted versus the displacement under load. The onset of internal (or catastrophic or snap-back) instability first studied in (1), turns out to be marked by a similar circumstance concerning another (much "stiffer") matrix.

3. The instabilizing effect of the cohesive-softening-crack model may cause bifurcation of the structural response. Namely, for the same tip advancement there may be more solutions: as in the four-point bending test of fig.3, a symmetric solution which can be singled out by dealing with only one-half of the system (say on the r.h.s. of its axis symmetry); two nonsymmetric solutions (symmetric to each other). One of the latter nonsymmetric step solution and subsequent propagation branch (solid lines in fig.3a) will actually be followed by the system, since it corresponds to an algebraically lesser external work. Therefore the real behavior can be captured only by considering the whole system (i.e. not exploiting its possible original symmetry).

4. The somewhat controversial assumption (a) or (b) on shear transmission mentioned in the Introduction is immaterial in situations like those of fig.2 (symmetry) and fig.3a (relatively small shear). However, the shear hypothesis has non negligible influence on the overall response whenever shear traction along

the craze may become significant, as shown by fig.3b, especially if contrasted to fig.3a.

5. The case, by which the BE mesh can be adjusted, makes BE approaches particularly advantageous, when the fracture propagation itinerary is unknown, as in the four-point-shear tests whose analysis by BEM is illustrated by fig.4.

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## REFERENCES

(1) Maier, G.,Rendiconti Istituto Lombardo, Classe Sci., A, 102, (1968) 648-677. English translation in *'Studi e Ricerche'*, 8, Politecnico Milano, (1986) 85-117.

(2) Maier, G., Zavelani Rossi, A., Dotreppe, J.C., Proc. ASCE, *J. Struct. Div.*, 99, 4, (1973) 897-906.

(3) Hillerborg, A. et al., *Cement Concrete Res.*, 6, (1976) 773-782.

(4) Willam, K.J. et al., *Proc. Int. Conf. on Fracture Mechanics of Concrete* (ed. Wittman, F.H.), Amsterdam, (1986) 149-169.

(5) Bazant, Z.P., *Cracking and Damage*, Mazars, J. and Bazant, Z.P. (eds.), (1989) 183-206.

(6) Carpinteri, A.,*Int.J.Num.Meth.Engng.*, 28, 7, (1989) 1521-1538.

(7) Carpinteri, A., *Rend. Mat. Acc. Lincei.*, s.9, v.1, (1990) 59-79.

(8) Dei Poli, S., Iori, I., *Studi e Ricerche*, 8, (1986), Politecnico di Milano, 35-62

(9) Biolzi, L., *SM Archives* 13, 3, (1988) 177-191.

(10) Sirtori, S., *Meccanica*, 14, (1979) 210-218.

(11) Maier, G., Polizzotto, C., *Computer Methods in Appl. Mech. and Eng.*, 60, (1987) 175-194.

(12) Cen, Z., Maier, G.: Instability, bifurcation and snap-back phenomena by elastic-cohesive crack boundary element analysis. *To appear.*

(13) Maier, G., Cen, Z.: Instability criteria for elastic-cohesive-softening crack systems. *To appear.*

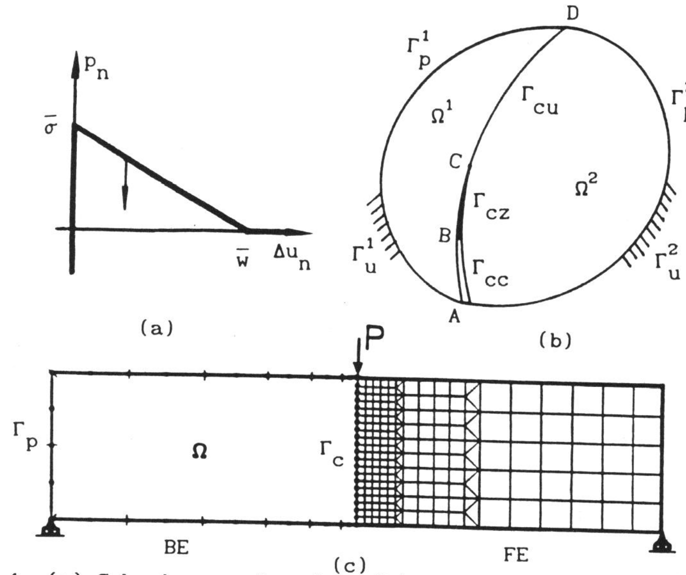


Fig. 1. (a) Cohesive-crack model, (b) Basic symbology, (c) FE vs. BE mesh for 21 nodes on $\Gamma_c$

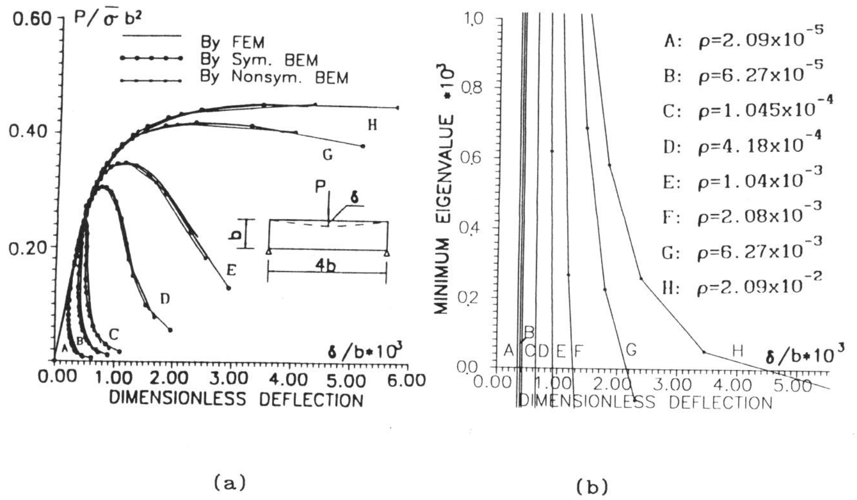


Fig. 2. Three-point test: (a) Load versus displacement for various brittleness ratios $\rho$, (b) Least eigenvalue of matrix H-Z

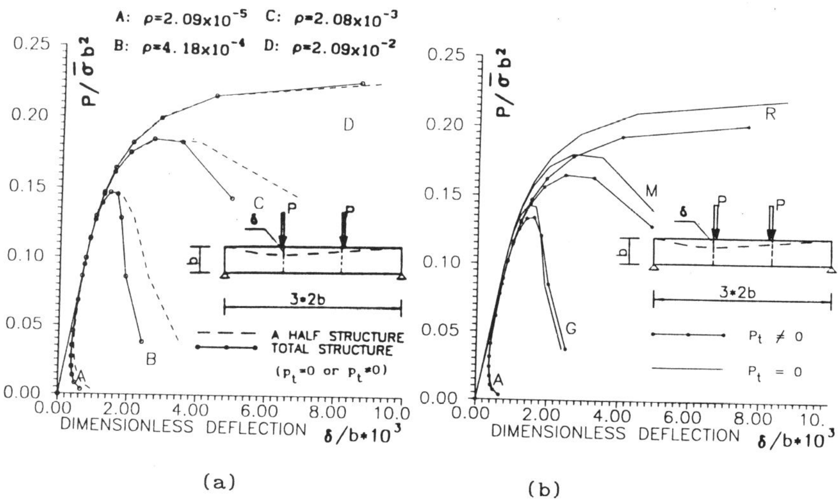


Fig.3 (a) Bifurcation: symmetric and nonsymmetric responces
(b) Influence of the shear transmission hypothesis

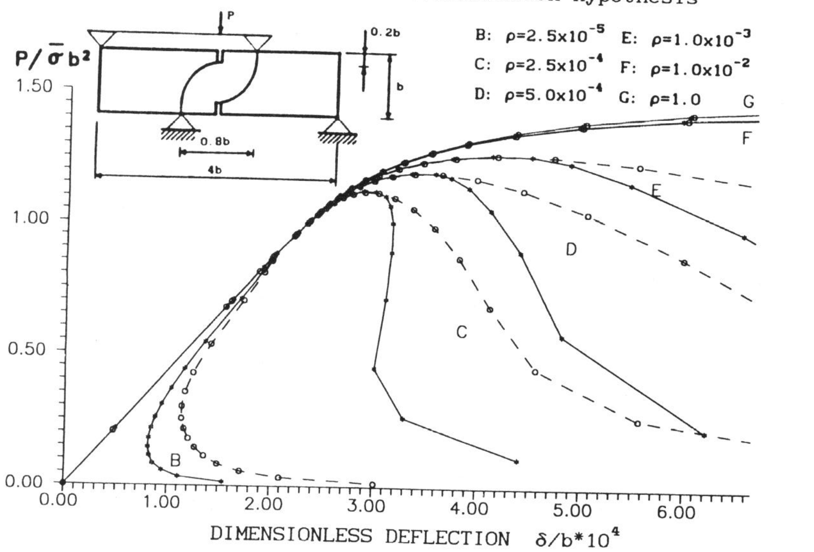


Fig.4 BE simulation of four-point shear model: nonsymmetric (solid lines) and symmetric (dashed lines) responses ($\bar{\sigma}$=16.5 kg/cm$^2$, E=270000 kg/cm$^2$, $\upsilon$=0.1, b=1.0cm)