

EFFECT OF SUPPORT CONDITIONS ON FRACTURE ENERGY MEASUREMENTS FOR CONCRETE BEAMS

S. E. Swartz,\* and Y.-C. Kan\*

Support settlement--or punching--for TPB concrete beams is estimated using plasticity concepts and assumptions of either constant, plastic pressure,  $f_p$  or linear pressure varying from zero to  $f_p$ . The second assumption gives support deformation four times the first assumption. Results based on the second assumption using notched beams with span/depth equal 3.75 and different depths show errors in determining the measured load point displacement (LPD) at peak load varying from 1.1% to 5.8%. Using the kinematics of the beam during softening, it is seen that the LPD measurements conform to those of the CMOD.

INTRODUCTION

In fracture energy measurements for beams in four-point-bending (4PB), Carpinteri and Ballatore (1) discuss the influence of support punching on these results and attempt to eliminate this effect with a special measurement system following the ASTM test method for fiber reinforced concrete beams (2). They state that the fracture energy results obtained are quite comparable to those obtained in three-point-bending (TPB) tests.

An extensive discussion of the effects of spurious sources of error on fracture energy measurements is given by Planas and Elices (3). Note that support settlement includes two errors: erroneous measurement of displacement and localized energy dissipation. This has also been discussed by Malvar and Warren (4) who suggest measuring the fracture energy from only the softening branch of the load-displacement curve.

The work presented here attempts to quantify the influence of the settlement effects on the measurement

\*Department of Civil Engineering, Kansas State University, Manhattan, Kansas, U.S.A.

of the energy released during the fracture process for TPB beams tested at Kansas State University with a view toward providing practical procedures for estimating these errors.

LOAD POINT DISPLACEMENT (LPD)

In Fig. 1 different approaches to measuring the LPD are given. The first approach, termed here "direct measurement" involves measuring  $\delta_1$ , the actual movement of the load point -- i.e. the activator movement. Other approaches measure  $\delta_2$  or  $\delta_3$  or something similar which is the movement of the beam in the vicinity of the load point. These approaches are termed here "indirect measurement."

PUNCHING ERRORS IN DIRECT MEASUREMENT

The total measurement error is  $e_1 + 1/2 (e_2 + e_3) + e_4$ . The term  $e_4$  represents movement of the entire support system and is  $e_4 = P/k_s$ . This term is neglected in the following. From symmetry of loading it is assumed  $e_2 = e_3$ . Thus, the total punching- or settlement-error is  $e_T = e_1 + e_2$ .

The punching is clarified in Fig. 2 in which the concrete is assumed to deform plastically while the roller remains rigid.

Constant Reaction Pressure  $f_p$

For this case

$$\theta = \sin^{-1} \frac{F}{2 BR f_p} \approx \frac{F}{2 BR f_p} \quad (1)$$

$$e = R(1 - \cos \theta) \approx \frac{1}{2} R\theta^2 = \frac{1}{8R} \left( \frac{F}{B f_p} \right)^2 \quad (2)$$

Linear Varying Pressure

For this case it is assumed that

$$f = \left( 1 - \frac{\xi}{\theta} \right) f_p, \quad 0 \leq \xi \leq \theta \quad (3)$$

This results in

$$e = \frac{1}{2R} \left( \frac{F}{B f_p} \right)^2 \quad (4)$$

It is seen that  $e$  given by Eq. (4) is four times that given by Eq. (2). Henceforth, the error will be calculated by Eq. (4).

Since the value of  $F$  at each support is one half that of  $P$  at the load point, then  $e_2 = e_3 = 1/4 e_1$ . So, the total punching error is, for  $F = P$ ,

$$e_T = e_1 + e_2 = \frac{1.25}{2R} \left( \frac{F}{B f_p} \right)^2 \quad (5)$$

#### SOFTENING KINEMATICS

One way to estimate the validity of the direct measurement of the LPD is to compare it with the CMOD during crack growth and softening. The kinematics in this situation are presented in Fig. 3. This gives  $\sin \alpha \approx 2v/s$  and

$$\frac{dP}{du} = \frac{S}{4W} \frac{dP}{dv} \quad (6)$$

where  $u = \text{CMOD}$ ,  $v = \delta_1 = \text{LPD}$ .

#### NUMERICAL RESULTS

In Fig. 4 are presented traces of  $P$  versus  $\delta_1 = \text{LPD}$  for beams in TPB tested with and without bearing plates. Clearly the measured differences are negligible. For these tests  $S/W = 3.75$ ,  $W = 102$  mm and,  $R = 25.4$  mm, and  $f_p = f'_c = 42.5$  MPa. Using  $P_{\max} = 2.58$  kN it is seen that  $e_T = 1.56 \times 10^{-2}$  mm or an error of about 12% in displacement at peak load. The steel bearing plates were 9.5 mm thick, 25.4 mm wide and length =  $B = 76.2$  mm. Thus, the contact stress at  $P_{\max}$  was 1.33 MPa and for an elastic modulus of 31.8 GPa the elastic strain was  $4.18 \times 10^{-5}$  m/m and total settlement using the full beam depth, was  $0.64 \times 10^{-2}$  mm for the beam with bearing plates. This is an error of about 5% of displacement at peak load. This

estimate is obviously much greater than the true error. In fact, the displacement of the beam with the bearing plates is slightly higher than the other--undoubtedly due to differences in materials, finishes, and testing.

Also shown in Fig. 4 is P versus CMOD for the beam with no bearing plates. For this case the slope relationship in Eq. 6 is 0.938. An examination of the softening slopes of the CMOD and LPD curves reveals this to be reasonably the case, hence the LPD response correlates very well with the CMOD response.

Finally, the influence of size effects on the punching error is presented for three beams in Table 1. In this all beams have  $a_0/W = 0.5$  and  $S/W = 3.75$ . The LPD at  $P_{max}$  is denoted by  $\bar{\delta}_1$ . It is seen that a size effect does exist but all errors are fairly small.

TABLE 1 - Support Punching Errors for Notched Beams With  $a_0 = 0.5 W$  and  $B = 76$  mm

No.	$f_p$ MPa	W mm	F kN	Eq. (5) $e_T$ mm	$\bar{\delta}_1$ mm	Error %
C18	55.8	102	1.29	0.0023	0.206	1.1
B34	53.1	203	2.49	0.0093	0.254	3.8
C33	54.4	305	3.69	0.0195	0.356	5.8

Note:  $R = 25.4$  mm,  $f_p = f'_c$ ,  $F = P_{max}$ ,  $S/W = 3.75$ .

#### CORRECTION TO ENERGY MEASUREMENT

In Fig. 5 is shown schematically the effect of support punching on the P-LPD response. The softening branch of the measured response will be virtually identical to the true response if support punching is indeed a plastic phenomenon, i.e.,  $e = e_T$  (curve B). On the other hand, if support punching is elastic then the consequences are that the error will be much smaller than given by Eq. (5) and the error will be zero when  $P = 0$ , i.e.,  $e = 0$  (curve C) in Fig. 5. The major source of error is in the ascending portion of the curve and is about  $1/2 P_{max} e_T$ .

### CONCLUSIONS

Estimates of support punching errors presented here show these to be relatively minor for the beams tested—even when no bearing plates are used. However, these can be corrected to obtain more accurate energy values. Of course, these settlements can also be measured directly but this adds complexity to the data acquisition. In any event, the direct method does measure the energy input to the beam. The indirect method does not measure the energy input to the beam but does measure relative beam displacement. When the relatively low span/depth ratios commonly used are considered one must be careful in extrapolating this measurement to the actual movement of the load point—it will always be smaller. These errors form the subject of another paper.

### ACKNOWLEDGEMENTS

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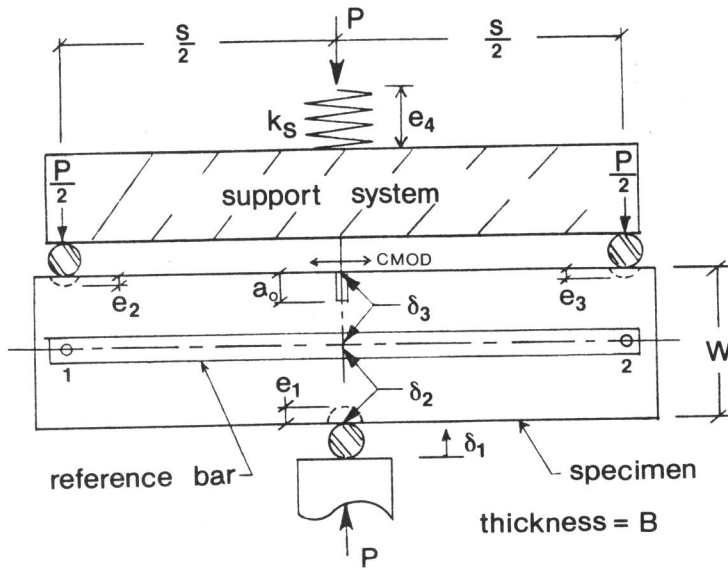


Figure 1. Schematic of Test Setup(s).

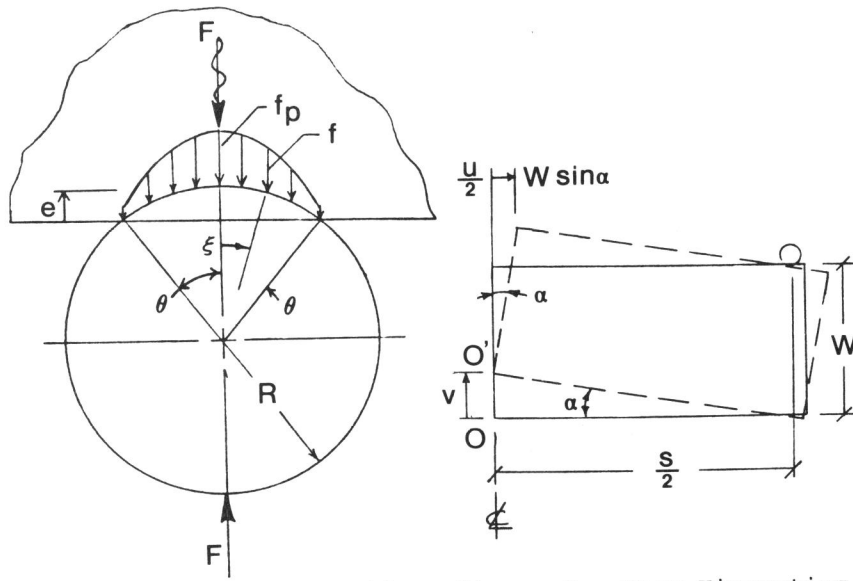


Figure 2. Support Punching. Figure 3. Beam Kinematics During Softening.

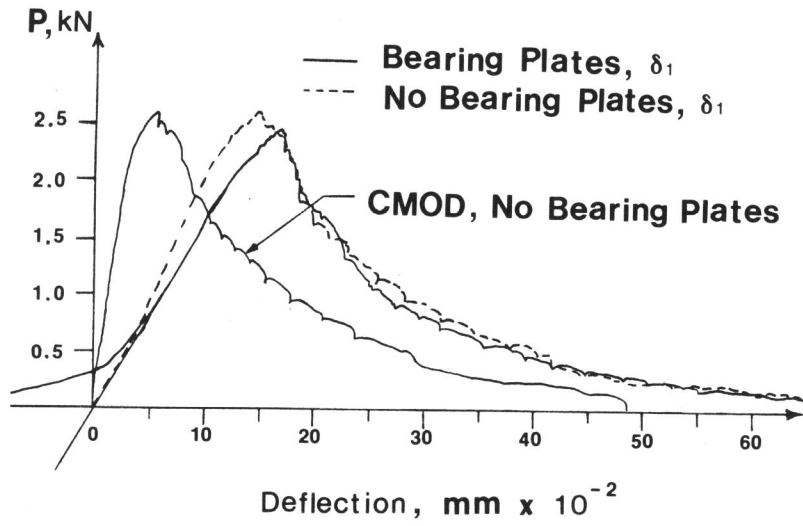


Figure 4. Load-Deflection Traces.

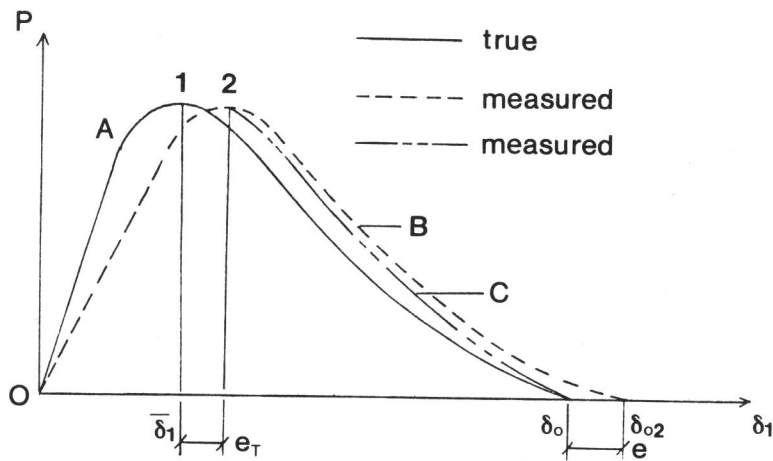


Figure 5. Effect of Support Punching on  $W_0$ .