DETERMINING THE FRACTURE PARAMETERS AND LOAD CARRYING CAPACITY OF CONCRETE BEAMS USING ECM

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The authors' effective crack model (ECM) is first used to determine the fracture toughness and effective crack length for a variety of mixes and shown to be in excellent agreement with the predictions of the two parameter model (TPM). These are then used to estimate the additional material constants necessary for a full finite element description of the fracture process. It is shown that the predicted load carrying capacity of test beams is in good agreement with the test data. Finally, it is shown how the scaling law for the ECM established from laboratory-size specimens can be used to predict the load carrying capacity of large-size flexural members.

INTRODUCTION

Recent experience has shown that the fracture parameters of plain concrete (fracture toughness K^e_{Ic} and effective crack length a_e) determined using the authors' ECM are in excellent agreement with the predictions of other non-linear fracture models for concrete, notably with that of the TPM (1). This will be put beyond any doubt by presenting below a comparison of fracture parameters for a wide variety of concrete mixes, ranging in cylinder strength f_c from about 25 MPa to nearly 80 MPa. In the process we shall also show that the elastic modulus of the mix determined by direct testing is almost identical to that determined by indirect testing (load-deflection or load-CMOD plots in three-point bending) or to that predicted by the well-known ACI empirical relation.

The fracture parameters determined from the ECM on three-point bend specimens are then used to estimate the dependent material constants - flexural tensile strength f_t and critical crack opening displacement w_c - after appropriately approximating the post-peak tension softening diagram. The flexural tensile strength is suitably scaled (2) to reflect the direct tensile strength of the

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TABLE 1 - Mix Properties

Mix	Compressive Strength $f_c(MPa)$	Elastic Modulus (GPa) E^*	E_c^{**}	$E^\dagger$	$E^\ddagger$
C1	26.8	24.62	24.51	25.56(.35)	25.04(.29)
C2	39.0	33.80	29.56	29.87(.21)	31.56(.64)
C3	49.4	34.65	33.27	33.28(.22)	32.96(.24)
C4	67.5	37.20	38.89	37.13(.23)	38.39(.82)
C5	78.2	40.30	41.86	40.99(.60)	40.26(.99)

* Determined from separate cylinder tests (using strain gauges)
** Estimated from the relationship, $E_c = 4734\sqrt{f_c}MPa$ ($=57000\sqrt{f_c}$psi)
† Calculated from $P - \delta$ plot; ‡ Calculated from $P - CMOD$ plot

mix. The latter is required for a finite element implementation of the fracture process using the smeared crack model (3). It is shown that the additional material parameters determined from the two fracture parameters of ECM, together with a non-linear approximation to the tension softening diagram (4) provide reasonably accurate predictions of the load carrying capacity of the test beams, while the linear approximation dangerously overestimates it. Finally, the test data from laboratory specimens is used to establish a scaling law for the ECM which enables us to predict the load carrying capacity of large-size flexural members.

TEST RESULTS AND FRACTURE PARAMETERS

Notched beams from five mixes ranging in cylinder strength f_c between 25 and 80 MPa were tested in three-point bending in a servo-hydraulic machine with a view to determining fracture parameters according to ECM and TPM. Details of specimen geometry and testing procedure are described elsewhere(5). It should be mentioned that if ECM is being used to determine only the fracture parameters of a mix no information past the peak load is necessary. In such a case, a closed-loop testing system is not needed.

Fig 1 shows typical load-displacement (P-δ) and load-CMOD plots for a notched beam made from mix C1 with a compressive strength of 26.8 MPa. The plots have been displaced on both scales for clarity. The initial (linear) segments of the respective plots are used in ECM and TPM to determine the elastic modulus of the mix. The formulas for calculating E will be found in (1). Table 1 shows the calculated values of E for each mix, together with the values determined from direct testing and from the well-known empirical

TABLE 2 - Fracture Parameters for Various Mixes

Mix/ Data	C1 Mean(sd)	C2 Mean(sd)	C3 Mean(sd)	C4 Mean(sd)	C5 Mean(sd)
a_0/W^*	0.295(.001)	0.296(.000)	0.295(.001)	0.293(.003)	0.293(.003)
a_e/W	0.443(.005)	0.441(.001)	0.435(.004)	0.428(.002)	0.419(.006)
$\underline{a}/W$	0.443(.015)	0.442(.006)	0.436(.001)	0.430(.002)	0.413(.006)
$K_{Ic}^{e}{}^*$	0.992(.015)	1.265(.013)	1.376(.020)	1.502(.046)	1.881(.095)
$K_{Ic}^{s}{}^*$	0.993(.054)	1.269(.028)	1.381(.031)	1.509(.040)	1.847(.098)
$CTOD_c{}^*$	0.033(.010)	0.026(.001)	0.026(.001)	0.024(.001)	0.026(.001)

* $W \approx 200mm$ for all specimens; K_{Ic}^e, K_{Ic}^s in $MPa\sqrt{m}$; $CTOD_c$ in mm.

formula. It is clear from the results that all four values of E for each mix are in excellent agreement. Thus any differences in the fracture parameters calculated according to ECM, TPM or any other non-linear fracture model for concrete (or in the additional material constants estimated from these fracture parameters) cannot be attributed to the method of measuring E, as has so often been done in the past.

The procedure for determining the two fracture parameters (fracture toughness K_{Ic}^e and effective crack length a_e) according to ECM and (fracture toughness K_{Ic}^s and effective Griffith crack length $\underline{a}$) according to TPM is described in (1) and will not be repeated here. The results are given in Table 2. Note that only two of the three parameters K_{Ic}^s, $\underline{a}$ and $CTOD_c$ are independent.

ADDITIONAL MATERIAL CONSTANTS FROM ECM

For a full finite element implementation of the fracture process, as well as for predicting the load carrying capacity of flexural members, one needs, besides K_{Ic}^e and a_e, the flexural tensile strength f_t, the critical crack opening displacement w_c and of course the shape of the post-peak tension softening diagram. It should however be pointed out that these additional material constants are not independent. They are derived from K_{Ic}^e, a_e and the assumed shape of the tension softening curve. The latter is usually assumed for simplicity to be linear, although as we shall subsequently see a non-linear approximation proposed by the authors (4) not only better describes the observed behaviour but also better predicts the load carrying capacity than the linear approximation. Linear, bilinear, quasi-exponential, and exponential strain-softening approximations all have one major drawback; they do not cater for the observed smooth transition from positive to negative stiffness at the peak load. The non-linear approximation proposed by the authors and for which an analytical solution is available removes this drawback.

In order to estimate the additional material constants f_t and w_c from K^e_{Ic} and a_e, as well as to see the effect of shape of the tension softening curve on these constants, we assume

$$\begin{aligned} \frac{\sigma}{f_t} &= 1 - \frac{w}{w_c} \quad \textit{(Lin)}, \\ \frac{\sigma}{f_t} &= \left[1 - 9.243\alpha^2 + 33.826\alpha^3 - 59.425\alpha^4 + 49.300\alpha^5 - 15.472\alpha^6\right], \quad (1) \end{aligned}$$

where $\alpha = w/w_c$. Then it may be shown (4) that

$$K^e_{Ic} = 0.7071\sqrt{E' w_c f_t} \textit{ (Lin)}, \quad K^e_{Ic} = 0.7043\sqrt{E' w_c f_t} \textit{ (Non-lin)}. \quad (2)$$

The work of fracture $W_d = \int_0^{\ell_{pc}} \int_0^{w_s} f^{-1}(w)\, dw\, ds$ is given by

$$W_d = 0.1050E' w_c^2 \textit{ (Lin)}, \quad W_d = 0.0521E' w_c^2 \textit{ (Non-lin)}, \quad (3)$$

where $f^{-1}(w)$ refers to the right hand side of Eqn (1), $E' = E$ (plane stress) or $E/(1\text{-}\nu^2)$ (plane strain) and ℓ_{pc} the length of the process zone (not to be confused with a_e) is

$$\ell_{pc} = 0.366E' w_c/f_t \textit{ (Lin)}, \quad \ell_{pc} = 0.359E' w_c/f_t \textit{ (Non-lin)}. \quad (4)$$

The work of fracture W_d may also be equated to the energy required for creating a hypothetical supplementary traction-free crack of length $\Delta a_e = a_e - a_0$ in a material with toughness $G^e_{Ic} = (K^e_{Ic})^2/E'$, thereby putting the ECM on a firm physical footing

$$\Delta a_e = 0.210E' w_c/f_t \textit{ (Lin)}, \quad \Delta a_e = 0.105E' w_c/f_t \textit{ (Non-lin)}. \quad (5)$$

We note *en passant* that whereas K^e_{Ic} increases with increasing f_t (Eqn 2), Δa_e depicts the opposite trend. Thus, it is Δa_e rather than K^e_{Ic} which defines the brittleness of the material; the higher the strength f_t the lower Δa_e. This situation is unlike that in metals for which the fracture toughness itself decreases with increasing tensile strength.

We now use the fracture parameters (K^e_{Ic} and Δa_e) calculated according to ECM (Table 2) and determine w_c and f_t from Eqns (2,5) for the two assumed approximations to the tension softening curve. The results are given in Table 3, which also shows the tensile strength according to the well-known ACI empirical formula, the length of the process zone, as well as the $CTOD_c$ from Table 2.

It is clear that w_c according to authors' non-linear approximation agrees better with $CTOD_c$. Likewise, f_t according to this approximation agrees better with the empirical result. Further doubt on the appropriateness of linear approximation is cast by its inaccurate and non-conservative estimate of load carrying capacity. This is demonstrated below.

TABLE 3 - Calculated values of f_t, ℓ_{pc}, w_c and $CTOD_c$

	$f_t^{\dagger}$	$f_t(MPa)$		$\ell_{pc}(mm)$		$w_c(mm)$		$CTOD_c$
Mix	MPa	Lin	Non-lin	Lin	Non-lin	Lin	Non-lin	mm
C1	2.58	3.70	2.63	52.6	103.1	0.0208	0.0296	0.0332
C2	3.11	4.78	3.39	51.2	100.4	0.0213	0.0318	0.0263
C3	3.50	5.29	3.76	49.5	97.0	0.0219	0.0305	0.0261
C4	4.09	5.88	4.18	47.7	93.6	0.0202	0.0293	0.0242
C5	4.41	7.61	5.40	44.8	87.8	0.0221	0.0322	0.0261

† calculated from the empirical relationship $f_t = 0.4983\sqrt{f_c}MPa$

MAXIMUM FLEXURAL LOAD PREDICTION

For predicting the load carrying capacity of concrete beams two approaches can be taken based on the discrete crack and the smeared crack models (3,6). We take the second approach because of the ease of computations, but unlike most past works we do not arbitrarily vary the material constants to achieve a close agreement between the predicted and test load carrying capacities. We use strictly the fracture parameters determined from the ECM (Table 2) and the derived material constants (Table 3). However, since the local fracture in the smeared crack model is assumed to occur at the instant the principal (tensile) stress reaches the direct tensile strength f_t' of the mix, it is necessary to scale the flexural tensile strength f_t estimated from the ECM fracture parameters (Table 3) to reflect the direct tensile strength of the mix. There is unfortunately no unique factor available to make such scaling. A sound approach though is based on the highly-stressed volume concept (2) which results in $f_t' = 0.77\ f_t$. At the same time since K_{Ic}^e (and therefore $G_{Ic}^e = (K_{Ic}^e)^2/E')$ is a material constant, the scaling of f_t will necessitate a compensatory scaling of w_c. This is clear from an inspection of Eqn 2.

As in the preceding section so also in the finite element modelling, the post-peak tension softening diagram is replaced by a linear or non-linear approximation. Of course, in the smeared crack band model the post-peak load-displacement plot is converted to a stress-strain curve. A typical load-displacement diagram predicted by finite element modelling is compared in Fig 2 with the experimental curve for a mix, using the fracture parameters K_{Ic}^e, a_e and the material constants E, f_t and w_c determined from the ECM for this mix. It will be seen that a satisfactory prediction of the load carrying capacity is furnished by the material properties derived from the ECM, together with the authors' non-linear approximation to the tension softening diagram. The linear approximation on the other hand, overestimates the load carrying capacity by a dangerous 16%.

SCALING LAW ACCORDING TO ECM

In the previous section, we showed that the fracture parameters and additional material constants determined according to ECM quite adequately predict the load carrying capacity of laboratory size flexural members provided their post-peak tension softening behaviour is properly approximated. However, in view of the variation of a_e with the specimen size, it is obvious that the load carrying capacity will vary with the size of the flexural member. The question therefore arises, can one predict the load carrying capacity of large flexural members using the ECM fracture parameters from laboratory-size flexural specimens? If so, what scaling law does one use to make this extrapolation? The answer to both these questions is in affirmative, as explained in a recent paper by the authors (7). Here we present just enough information on the scaling law associated with ECM to complete the presentation.

Following the methodology proposed in (8) it may be shown (7) that the scaling law for extrapolation to large size structures according to ECM is

$$\left(\frac{K_{Ic}^e}{K_{IN}}\right)^2 = 1 + Q_1\left(\frac{\ell_c}{W}\right) + Q_2\left(\frac{\ell_c}{W}\right)^2 + Q_3\left(\frac{\ell_c}{W}\right)^3 + Q_4\left(\frac{\ell_c}{W}\right)^4, \quad (6)$$

where the characteristic length according to ECM is $\ell_c = (K_{Ic}^e/f_t)^2$ and the scaling factors $Q_1, \ldots, Q_4$ which depend on the mix properties as well as on the size W and a_0/W are given by $Q_1 = \lambda_1(\Delta a_e/\ell_c)$, $Q_2 = \lambda_2(\Delta a_e/\ell_c)^2$, $Q_3 = \lambda_3(\Delta a_e/\ell_c)^3$, $Q_4 = \lambda_4(\Delta a_e/\ell_c)^4$.

K_{IN} refers to the stress intensity factor that varies with the size of structure except when $W \to \infty$ where it coincides with the LEFM result for the critical stress intensity factor K_{Ic}. Coefficients $\lambda_1, \ldots, \lambda_4$ depend on only a_0/W (7). The scaling law (Eqn 6) is illustrated in Fig 3. For $W \to \infty$, $K_{Ic}^e \to K_{IN} = K_{Ic}$. It will be noted that the variation with a_0/W is rather insignificant.

For a concrete mix, the factors $Q_1, \ldots, Q_4$ in the scaling law for ECM (Eqn 6) can be established by testing laboratory-size specimens to determine a_e and K_{Ic}^e. It should however be noted that these factors will represent material constants (i.e. will be independent of geometry) only beyond a certain size (depth) of the specimen, which can be determined by plotting $\Delta a_e/\ell_c$ against W/ℓ_c, as has been shown in (7).

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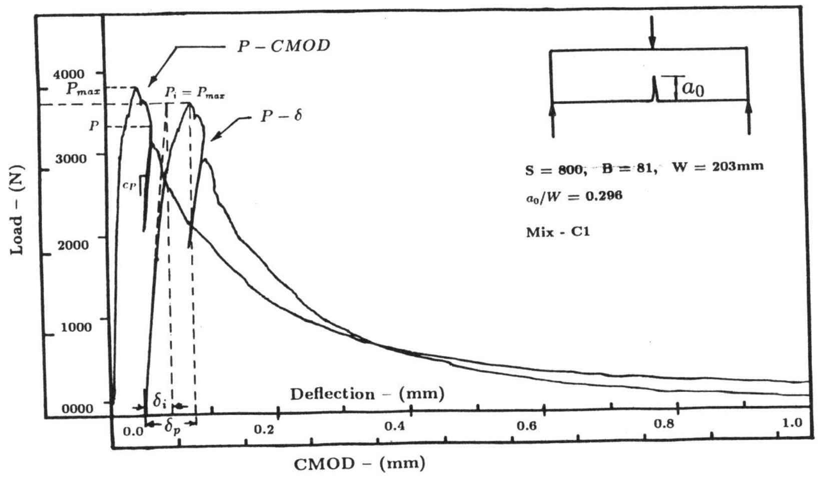

Figure 1: Typical $P - \delta$ and $P - CMOD$ plots for Mix C1.

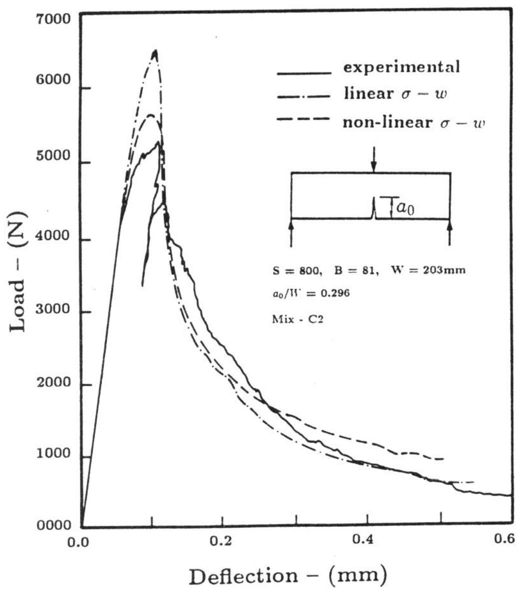

Figure 2: Comparison of load-displacement diagrams predicted by finite element modelling with the experimental curve

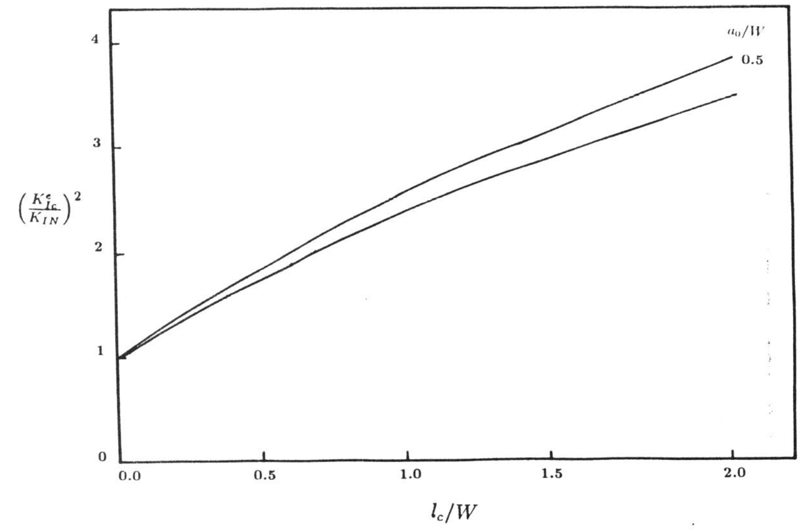

Figure 3: Scaling law according to ECM