DETERMINATION OF TENSION SOFTENING CURVES OF CONCRETE BY MEANS OF BENDING TESTS

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A simple method to determine the tension softening curves of concrete from bending tests on notched beams is proposed. Using test data among the load, the load-point displacement and the crack opening at the notch tip, a tension softening curve is roughly estimated through "New Method". Then the rough curve is corrected by multiplying the factor, which is drawn from analysis. The validity of the proposed method is certified through the comparison between the experimental and the numerical results. This proposed method can be combined with the RILEM testing method for the fracture energy.

INTRODUCTION

The shape of the tension softening curve plays a decisive role in the analysis of concrete fracture behavior. It is important to establish the testing method for determining the tension softening curve of concrete. According to the original meaning, the tension softening curve ought to be measured by uniaxial direct tension tests. But it is not easy to measure the stable tension softening curve without any bump [1] or sudden jump by uniaxial tension tests. The method to obtain tension softening curves from bending tests was originally proposed by Li et al. [2] and improved by the authors [3] (named "New Method").

In this paper, the "New Method" for determination of the tension softening curve of concrete by means of bending tests on notched beams is outlined and then further improved. The bending tests were performed for plain concrete to determine the tension softening curve by the proposed method.

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ESTIMATION OF TENSION SOFTENING CURVE

Assumptions

To obtain the tension softening curve by bending tests through simple procedures, the following assumptions are applied:

i. A main crack exists through a ligament. A rotational axis exists on the top of the ligament. The distribution of the crack opening is linear, as shown in Fig. 1.

ii. The mean crack opening in the ligament is represented by a half of the crack opening displacement at the notch tip.

iii. The applied energy to a specimen is transformed into the energy used for the crack development. The effect of the elastic strain energy is ignored.

iv. The shape of the uniaxial tension softening curve of concrete is similar to the following curve, which was proposed by Hillerborg [1].

$$\sigma(\omega) = f_t(1 + 0.5\frac{f_t}{G_F}\omega)^{-3} \tag{1}$$

where σ is the tensile stress, ω is the crack opening, f_t is the tensile strength and G_F is the fracture energy of concrete. Generally, this curve agrees well with experimental data for normal concrete.

Concept of "New Method"

The total applied energy to a specimen, E_{ap}, during bending test is given as:

$$E_{ap} = \int P(D)\, dD \tag{2}$$

where P is the load and D is the displacement of the loading point. The energy used for the crack development, E_{cr} , can be written as:

$$E_{cr} = A_{lig} \int s(w) dw \tag{3}$$

where A_{lig} is the ligament area, s is the mean stress of the crack and w is the mean crack opening in the ligament. With the second assumption, the mean crack opening can be written as:

$$w = COD/2 \tag{4}$$

where COD is the crack opening displacement at the notch tip. With the third assumption stated above, $E_{ap} = E_{cr}$, then the softening curve s(w) is estimated as follows:

$$\mathrm{s(w)} = \frac{1}{A_{lig}} \frac{\partial E_{ap}}{\partial \mathrm{w}} \tag{5}$$

Correction of "New Method"

FE-analysis for a notched beam specimen using fictitious crack model was carried out. The tension softening curve defined by eq.(1) was adopted as the constitutive law of the fictitious crack model. Eq.(1) is shown by the solid line in Fig. 2. From the result of FE-analysis, s(w) was calculated through the "New Method" and shown by the broken line in Fig. 2. The shape of s(w) is similar to the uniaxial tension softening curve $\sigma(\omega)$, but s(w) is smaller than $\sigma(\omega)$. The difference between s(w) and $\sigma(\omega)$ is caused by the second assumption given above regarding the mean crack opening. That is, when the distribution of crack opening is not uniform, the calculation of the tension softening curve using the mean crack opening makes some error [1].

In order to correct the s(w) without changing the assumptions given above, the following procedures are made. If the uniaxial tension softening curve is defined by eq.(1), then integrating eq.(1), the energy used for the crack development, E_{cr}, during COD becomes 2w, i.e. the mean crack opening becomes w, is given as:

$$E_{cr}(= E_{ap}) = A_{lig}\mathrm{G}_F(1 - \frac{\mathrm{G}_F}{\mathrm{G}_F + \mathrm{f}_t\mathrm{w}}) \tag{6}$$

Substituting eq.(6) into eq.(5), s(w) is given as:

$$\mathrm{s(w)} = \mathrm{f}_t(1 + \frac{\mathrm{f}_t}{\mathrm{G}_F}\mathrm{w})^{-2} \tag{7}$$

It is noted that s(w) defined by eq.(7) is similar to the s(w), which is obtained by FE-analysis and shown in Fig. 2 by the broken line. The "New Method" tends to give a lower curve than the uniaxial tension softening curve. Therefore, with the fourth assumption given above and from eq.(1) and (7), the shape of uniaxial tension softening curve can be predicted by $S(\mathrm{w})$ given as follows :

$$S(\mathrm{w}) = \mathrm{s(w)}\frac{(1 + 0.5\frac{\mathrm{f}_t}{\mathrm{G}_F}\mathrm{w})^{-3}}{(1 + \frac{\mathrm{f}_t}{\mathrm{G}_F}\mathrm{w})^{-2}} \tag{8}$$

where s(w) is the softening curve calculated through the "New Method" (eq.(5)). The corrected tension softening curve $S(\mathrm{w})$ using eq.(8) is shown by the dotted line in Fig. 2. The estimated curve agrees well with the uniaxial tension softening curve (solid line) used for the FE-analysis.

EXPERIMENTAL EXAMPLE

Tension softening curve of plain concrete was determined form three-point bending tests on notched beam specimens through the proposed method in this study. The compressive strength and the split tensile strength were 39.9 and 3.36 Mpa, respectively. The size of specimens was 10 × 10 × 84 cm, and the span length was 80 cm, according to the RILEM recommendation [4]. The notch depth was 5 cm, a half of the specimen height. The relation among the load, the displacement (center deflection) and the COD at the notch tip was recorded during the loading test. After the loading test, fracture energy G_F was determined through the RILEM testing method. The fracture energy of concrete was 157 N/m. Measured load-displacement curves are shown in Fig. 3. Fig. 4 shows the tension softening curve determined through the proposed method. The mean load-displacement curve and the mean COD-displacement curve were used for the calculations.

To confirm the validity of the proposed method, the load-displacement curve was simulated by FE-analysis. The estimated tension softening curve shown in Fig. 4 was utilized as the input data for the fictitious crack behavior. The results of analysis are shown in Fig. 3. The results of analysis agree well with the experimental results.

CONCLUSION

Based on several assumptions for simplicity, a very simple and handy method for determining the tension softening curve from three-point bending tests on notched beams is proposed. A roughly estimated tension softening curve through our "New Method" is corrected by multiplying the factor given by eq.(8).This factor is obtained from the numerical and theoretical analysis.The tension softening curve for plain concrete is estimated through the proposed method. The load-displacement curve of notched beams is simulated by FE-analysis using the estimated tension softening curve. It can be concluded that the proposed method is valid because the simulated load-displacement curve agrees well with the experimental results. This proposed method can be combined with the RILEM testing method for determining the fracture energy of concrete.

SYMBOLS USED

A_{lig}	=	ligament area
E_{ap}	=	applied energy to specimen
E_{cr}	=	used energy for crack development
f_t	=	tensile strength of concrete
G_F	=	fracture energy of concrete

P(D)	=	load-displacement curve in bending test
s(w)	=	estimated tension softening curve by the "New Method"
S(w)	=	corrected tension softening curve of s(w)
$\sigma(\omega)$	=	uniaxial tension softening curve
w	=	mean crack opening in bending test (=COD/2)
ω	=	crack opening in uniaxial tension test

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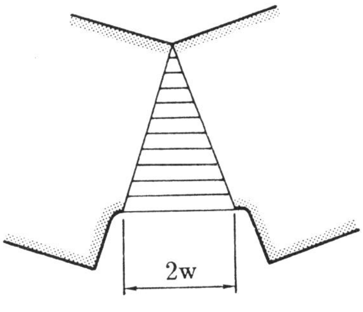

Fig. 1 Model of Ligament

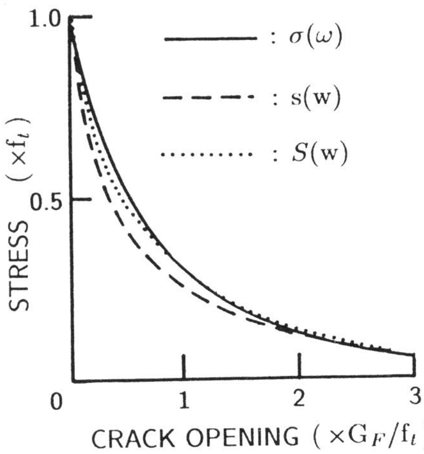

Fig. 2 Estimated Tension Softening Curves for FE-analysis

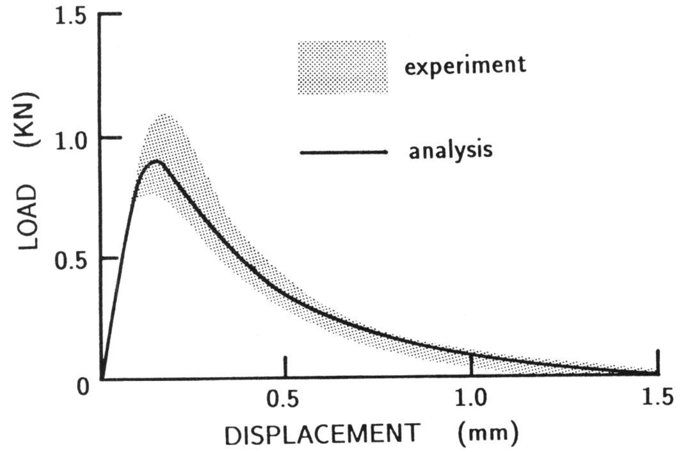

Fig. 3 Load–Displacement Curves of Plain Concrete

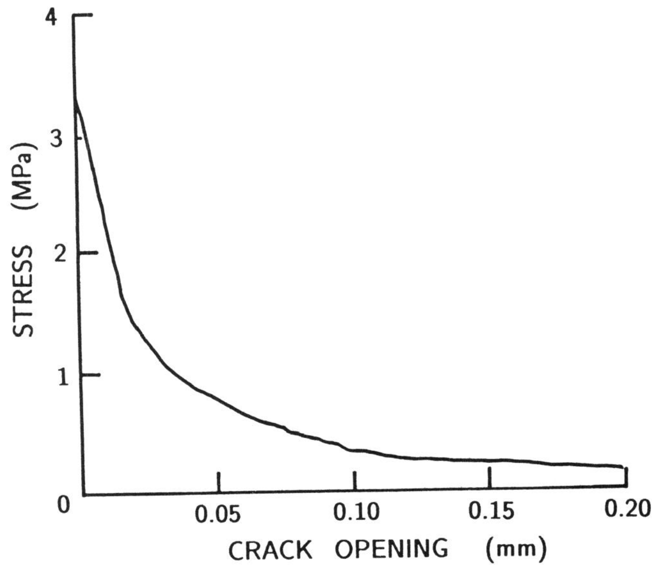

Fig. 4 Experimental Example of Tension Softening Curve