THE APPROXIMATION OF A COHESIVE CRACK BY EFFECTIVE ELASTIC CRACKS

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The concept of equivalent or effective elastic crack is introduced as an approximation of more detailed fracture models like the cohesive crack models. Some specially interesting equivalences are defined and discussed on general grounds: load-displacement equivalence, load-CMOD equivalence, and J-CTOD equivalence.

In the second half of the paper these equivalences are explored for large sizes with the help of the asymptotic method, already developed by the authors.

INTRODUCTION

Substitution of an actual fracture process —a plastic or a cohesive zone surrounding the crack tip— by an effective or equivalent crack was probably the first approximation to non-linear fracture problems. The *equivalent* linear elastic problem has to be solved in conjunction with an associate R-curve or, otherwise stated, a crack growth rule has to be independently hypothesized as a relationship between the effective crack growth resistance and the effective crack extension. This is not the only price one has to pay for simplifying the cohesive crack; it was proven that R-curves are geometry and size dependent —i.e., are not a material property— and, consequently, the equivalence becomes severely restricted.

In spite of these shortcomings there is some evidence [1] showing that for usual geometries and available sizes, differences between R-curves can happen to be well inside the experimental scatter and, in this respect, they may be considered a material property for practical purposes, so that exploring the implication of different definitions of equivalences may be rewarding from a scientific and practical point of view. The first steps towards a systematic analysis of equivalences are presented in this paper.

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THE EQUIVALENT ELASTIC CRACK

In this work, the detailed fracture process is approximated by a cohesive crack as defined in [2]. Monotonic mode I fracture is considered.

The concept of *equivalent crack* emerges from this particular example: Two geometrically identical cracked samples, as shown in Fig. 1, are loaded under displacement control u. One sample is made with a cohesive material, as defined above, and the other is made with a linear elastic material. The measured responses of the two samples —i.e., the loads P and P_{eq}— to every displacement u will be different, but we can make both responses to match each other by choosing a suitable crack length in the elastic sample. In doing so the P-u curves of both specimens are the same, but, in general, the equivalence ends here; stress or displacement fields, or relevant parameters like *CMOD* or *CTOD*, are not the same. Moreover, the price paid for the equivalence is that the linear elastic material has not a constant crack growth resistance. Instead, a R-value changing with crack length is needed in order to keep the P-u equivalence. As we shall see later, the dependence of the crack growth resistance —or R-curve, as is usually known— is not a material property, but depends on the geometry and specimen size.

At first sight, the advantages of defining this equivalence are not obvious since there are not simple rules for the generation of the R-curves for every geometry and size. However, in some circumstances this can be done as we shall see later.

P-Y Equivalences

This kind of equivalence is shown in Fig. 1. The actual sample is sketched on the left, its cohesive zone has grown monotonically up to c, and the corresponding load is $P(c)$. The equivalent sample, made with an elastic non cohesive material, is sketched on the right and it is loaded with *same P* value (hence the P equivalence labeling). Notice that the crack length is not a_0 but $a = a_0 + \Delta a^{P\text{-}Y}$, where P stands for the imposed load and Y for the magnitude related with the second degree of freedom. One should realize that the stress and displacement fields of the right hand sample are known when the load and the crack length are known, and since load is fixed only one degree of freedom remains.

P-u Equivalence. When the load-point-displacement u is chosen as a second variable, one arrives at the load-displacement equivalence. If P and u are measured in the actual sample, the equivalent elastic crack length can be computed from the equation:

$$C_{eq}(a_0+\Delta a^{P\text{-}u}) = \frac{u}{P} \tag{1}$$

where C_{eq} is an expression for the compliance of the non cohesive sample. Notice that the equivalent crack growth resistance $R_{eq} = K^2_{eq}(a_0+\Delta a^{P\text{-}u})/E$ corresponding to each pair (P,u) needs not be constant. In fact it is different, giving rise to $R^{P\text{-}u}$-$\Delta a^{P\text{-}u}$ curves. Because the compliance and the RH member of (1), needed for $\Delta a^{P\text{-}u}$ evaluation, are geometry-dependent , so will be the R-curves.

CMOD Equivalence. When the *CMOD* (Crack Mouth Opening Displacement) is chosen instead of the displacement associated to the load, one has a *P-CMOD* equivalence. The equivalent elastic crack can be computed from an equation similar to (1),

$$C^{CMOD}_{eq}(a_0+\Delta a^{P\text{-}CMOD}) = \frac{CMOD}{P} \tag{2}$$

where C^{CMOD}_{eq} is the corresponding compliance associated to *CMOD* for a non cohesive sample.The same comments as before, regarding the specificity of R-curves, can be done. Also it can not be stated, at first sight, that equations (1) and (2) are equivalent and, hence, there is no reason to equate $\Delta a^{P\text{-}u}$ and $\Delta a^{P\text{-}CMOD}$.

X-Y Equivalences

The above reasoning can be generalized to a couple of variables *(X-Y),* where load needs not to be one of them. Now, the actual specimen and equivalent (or *virtual*) specimen are not bearing the same load, in general, and the equivalent load $P^{X\text{-}Y}$ and effective crack extension $\Delta a^{X\text{-}Y}$ corresponding to the virtual specimen can be computed by equating *X* and *Y* in both specimens:

$$X_{eq}[P^{X\text{-}Y}, \Delta a^{X\text{-}Y}] = X \tag{3}$$

$$Y_{eq}[P^{X\text{-}Y}, \Delta a^{X\text{-}Y}] = Y \tag{4}$$

J-CTOD Equivalence.The couple *(J-CTOD)* is an example of the generalized *(X-Y)* equivalence. The variable *CTOD* is the Crack Tip Opening Displacement .The variable *J* needs some remarks: When the cohesive sample is considered, *J* is the J-integral taken over a path always surrounding the cohesive zone. Under such circumstances it was shown [3] that:

$$J = W_F(CTOD) \tag{5}$$

where $W_F(w)$, is the specific work supply, or work done against the cohesive stress to open a unit area of cohesive crack up to *w* [2]. For the non-cohesive sample, *J* is equal to K_I^2/E.

According to this, the expression of Eq. (3) will be, using (5) as the expression of the RH member:

$$\frac{1}{E}\left[K_{Ieq}(P^{J\text{-}CTOD}, \Delta a^{J\text{-}CTOD})\right]^2 = W_F(CTOD) \tag{6}$$

The expression for the Eq. (4) may be obtained in the form [4]:

$$\frac{8}{\sqrt{2\pi}\,E} K_{Ieq}(P^{J\text{-}CTOD}, \Delta a^{J\text{-}CTOD})\,(\Delta a^{J\text{-}CTOD})^{1/2}\, L\left(\frac{\Delta a^{J\text{-}CTOD}}{D}\right) = CTOD \tag{7}$$

where the function $L(\Delta a/D)$ must be obtained for a particular geometry from linear elastic analysis, and depends implicitly on the initial crack length, but verifies that it tends to 1 when the size grows to infinite: $L(0) = 1$ for any geometry.

By substitution of K_{Ieq} from one equation into the other, one gets $\Delta a^{J\text{-}CTOD}$ as a function of *CTOD*. From this result it is also possible to obtain $P^{J\text{-}CTOD}$ as a function of *CTOD*.

This particular equivalence was considered previously by the authors in [1], under the name "R-CTOD approximation". It has the advantage that, given the softening function, the governing *R-CTOD* curve is immediately found and is size and geometry independent. The *R-*Δa curves are not. Since this is an approximation, there is no reason to expect that both loads —P and P_{eq}— coincide. They really do not. However, it was found, for notched beams, that the maximum load can be accurately predicted using the equivalent *J-CTOD* model. As an example, for ordinary concrete notched beams, the error was found to be less than 5% for beam depths larger than 8 cm [1].

<u>A Simple Analytically Computable Example</u>

To illustrate the differences between the different equivalences, $\Delta a^{X\text{-}Y}$ values at peak load are shown in Fig. 2 versus the specimen size for a very simple geometry. The specimen chosen is a central cracked panel subjected to uniform loading. The softening relation is supposed of Dugdale type (rectangular softening), for simplicity, with a tensile strength of σ_R and a critical crack opening w_C. The specimen size is characterized by A_0 (half initial crack length). The characteristic length is $l_{ch} = E{\cdot}w_C/\sigma_R$. Figure 2 shows the evolution of the equivalent crack extension at peak load versus the inverse of the specimen size for three equivalences; *P-CMOD*, *J-CTOD* and *P-CTOD*, where the *CMOD* is understood as the opening of the crack at its centre (this last equivalence, for brevity, has not been discussed in the paper).

<u>ASYMPTOTIC ANALYSIS OF THE EQUIVALENT CRACK</u>

Let us explore, now, the *equivalences* for large specimen sizes. The authors developed a method particularly appropriate for analyzing cohesive crack models when the specimen size is large [5, 6] and some results will be briefly summarized here.

It is assumed that the size of the cohesive zone, c, remains bounded as the specimen size (characterized by D) grows and that the stress and displacement fields can be developed in series of c/D. The zeroth order asymptotic approach is obtained when terms of the order of c/D are neglected. When the linear terms are also included —neglecting terms of the order of $(c/D)^2$— the first order approach is under consideration.

For zero order approach, the stress and displacement fields *far from the cohesive crack tip* are the same as the corresponding fields of an elastic crack of length a_0. This is a well known far field property. A not so obvious result obtained using the asymptotic analysis is that the cohesive zone length c may be expressed as:

$$c = \frac{\pi E}{32}\,\frac{\mathrm{CTOD}^2}{W_F(\mathrm{CTOD})} <\zeta^{1/2}>^{-2} \tag{8}$$

where all terms have been previously defined, except $\zeta = x/c$, (x is the coordinate of a point inside the cohesive zone, measured from the initial crack tip) and < > means a weighted average value on the (0,1) interval. The weight function is the solution of the asymptotic problem as formulated in [5], and depends on the softening function. More details, not necessary for our purpose, can be found in [5, 6].

When the first order approach is considered a new far field property —not so trivial— is deduced:

For a cohesive material and a general geometry under mode I loading, every far field may be approximated, up to order c/D, by the corresponding elastic field of a crack of length $a_0 + \Delta a^{FF}_{\infty}$ *where* Δa^{FF}_{∞} *is an effective (or equivalent) crack increment given by*

$$\Delta a^{FF}_{\infty} = c <\zeta> \tag{9}$$

where $<\zeta>$ has the same meaning as before.

Far Field (FF) Equivalences

Let us consider the equivalences based on variables associated with fields far away from the cohesive zone (*FF*, far fields). Among the three equivalences mentioned before, the equivalences *P-u* and *P-CMOD* belong to this class when the specimen size is very large. In fact loads are applied on the boundaries and *u* is the associated displacement, the same happens with the *CMOD*.

The far fields of both specimens —the cohesive one and the elastic equivalent— coincide for very large sizes, according the results just mentioned. In consequence, when the values of *P* and *u* are imposed, also the values of *CMOD* will coincide and the same will happen for other far field variables. This result has an important implication on the equivalent elastic cracks i.e., all equivalent elastic cracks based on far-field variables should coincide for very large sizes. For the equivalences considered here we have:

$$\Delta a^{FF}_{\infty} = \Delta a^{P\text{-}u}_{\infty} = \Delta a^{P\text{-}CMOD}_{\infty} \tag{10}$$

where subscript ∞ means infinite (very large) size.

It is possible to derive a lower bound for Δa^{FF}_{∞} as a function of *CMOD*. From the results (8) and (9) and the Bunyakovsky-Schwarz inequality the lower bound is found as:

$$\Delta a^{FF}_{\infty} = c <\zeta> = \frac{\pi E}{32} \frac{CTOD^2}{W_F(CTOD)} \frac{<\zeta>}{<\zeta^{1/2}>^2} > \frac{\pi E}{32} \frac{CTOD^2}{W_F(CTOD)} \tag{11}$$

Other Equivalences

When the variables chosen for the equivalence are related with the cohesive zone the far field property can not be exploited. This happens, for example, with the variable *CTOD* and with the equivalence *J-CTOD*. In principle, there is no reason to suspect that $\Delta a^{J\text{-}CTOD}$ and Δa^{FF} should coincide for very large sizes, and we will see that they do not. Indeed, the infinite size limit of the *J-CTOD* effective crack increase is obtained from (6) and (7) by putting $D \rightarrow \infty$ and taking into account that $L(0) = 1$. The result is:

$$\Delta a^{J\text{-}CTOD}_{\infty} = \frac{\pi E}{32} \frac{CTOD^2}{W_F(CTOD)} \tag{12}$$

which coincides with the lower bound found for the Far Field effective crack extension. This means that, although many equivalences merge for very large sizes, not all equivalences coincide. In particular we have found that

$$\Delta a_{\infty}^{FF} = \Delta a_{\infty}^{P\text{-}u} = \Delta a_{\infty}^{P\text{-}CMOD} > \Delta a_{\infty}^{J\text{-}CTOD} \tag{13}$$

A Further Look to the Example

The behaviour for very large sizes of the equivalent crack extensions for the example previously defined, is represented in Fig. 2 as the trend in the curves in a neighborhood of the origin. The ordinates at the origin are easily obtained for the Dugdale-type of softening because the peak load occurs when $CTOD = w_C$. Hence, from Eq. (12) it is easily found that

$$\Delta a_{\infty peak}^{J\text{-}CTOD} = \frac{\pi E}{32} \frac{w_C^2}{W_F(w_C)} = \frac{\pi}{32} l_{ch} \tag{14}$$

The two equivalences, $\Delta a^{P\text{-}CTOD}$ and $\Delta a^{J\text{-}CTOD}$, although different for small sizes, merge for very large sizes as shown in the figure.

For rectangular softening the relation $<\zeta>/<\zeta^{1/2}>^2$ is 4/3 [5] and then

$$\Delta a_{\infty}^{FF} = \Delta a_{\infty}^{P\text{-}CMOD} = \frac{\pi}{24} l_{ch} \tag{15}$$

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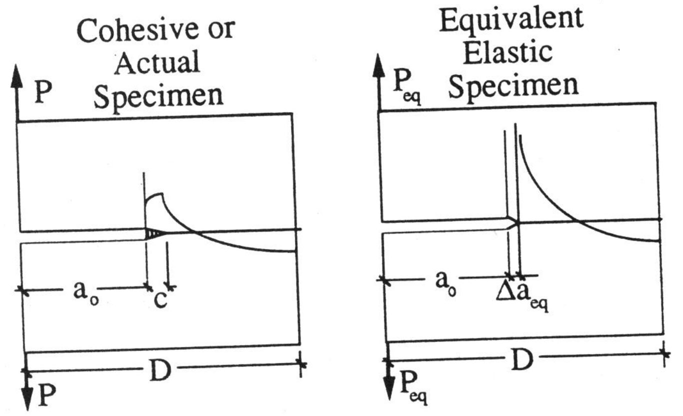

Fig 1. Definition of a general equivalence between the actual (or cohesive) specimen and a virtual elastic specimen.

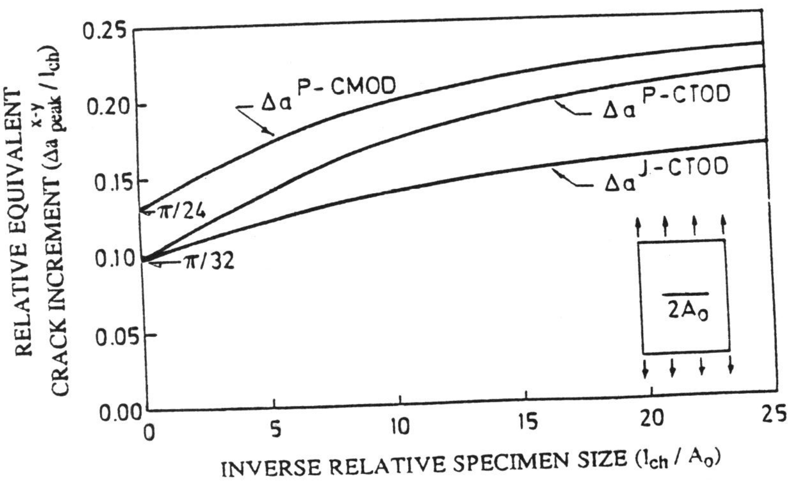

Fig. 2. Influence of specimen size on some equivalent crack extensions at peak load for Dugdale model (rectangular softening) and center cracked panel.