A CONSTITUTIVE MODEL FOR CRACK CYCLIC BEHAVIOUR OF PLAIN CONCRETE

D.A. Hordijk*, H.W. Reinhardt**

A new constitutive model for the cyclic behaviour in the softening zone of concrete is proposed. The model exists of a number of continuous functions and is based on experimental results obtained from deformation-controlled uniaxial tensile tests performed in the Stevin Laboratory of the Delft University of Technology. The model is complete, in the sense that every loading path can be predicted, i.e. unloading may take place at any point of the softening curve, till any lower stress-level and also within another loading cycle. A flow chart of the model is given which makes that it can easily be implemented as a mathematical subroutine in numerical progammes.

INTRODUCTION

Nowadays, the fact that there is a post-peak behaviour for plain concrete under tensile loading is well accepted among researchers in the field of fracture mechanics of concrete. Therefore, most of the existing numerical programmes have an option for the softening branch which becomes active after the tensile strength is reached. Initially, linear softening relations were applied. Later on, supported by the fact that analyses showed a proper input for the shape of the softening diagram to be important, also expressions that approach the real material behaviour more appropriately, were implemented.

Compared to the stress-crack opening relation for concrete, its behaviour under crack closure and crack re-opening has been studied scarcely. A very limited number of models have been proposed so far. For a review see Hordijk and Reinhardt (1). Yet, most of the FE-codes, if they take care of unloading beyond peak load in tension, apply a simple linear unloading and reloading relation. Experimental results show that such a relation deviates

* TNO-Institute for Building Materials and Structures, Lange Kleiweg 5, NL-2288 GH Rijswijk, The Netherlands

** Institute for Engineering Materials, Stuttgart University, Pfaffenwaldring 4, D-7000 Stuttgart 80, West-Germany

strongly from the real post-peak cyclic material behaviour. For experimental results see, for instance, Reinhardt (2).

In the Stevin Laboratory of the Delft University of Technology, the fatigue behaviour of plain concrete is studied by means of nonlinear fracture mechanics (3). For that study, the post-peak cyclic behaviour of concrete served as input. Therefore, a new constitutive model was proposed that describes the material behaviour appropriately. An essential of the proposed model is that the damaging effect due to a loading cycle is incorporated.

Due to the fact that the unloading and reloading curves of a loop follow different paths, reversals in deformation direction that occur within a loop should also be modelled properly. To the authors' opinion, propositions for this behaviour has not been given so far. The model that will be presented here is complete. This means that every possible loading path is determined by the model.

CONTINUOUS-FUNCTION-MODEL

The model gives a description for the stress-crack opening relation, while crack opening is defined according to the Fictitious-Crack-Model by Hillerborg et al. (4). It consists basically of continuous functions and is therefore called "Continuous-Function-Model" (CFM).

Loops starting from and returning at the envelope curve

The three basic equations are, respectively, empirical expressions for the unloading curve (I), the gap in the envelope curve (II) and the reloading curve (III) (see Fig. 1). It has been chosen to use only characteristic points in the σ-w relation (f_t, w_c, w_{eu}, σ_{eu}, σ_L) as variables in the expressions. The parameter w_c is the crack opening where stress can no longer be transferred and is defined as $5.14G_F/f_t$. The expressions that were chosen are based on a close inspection of the experimental results. The softening relation is described by the following expression:

$$\frac{\sigma}{f_t} = \{1+(c_1\frac{w}{w_c})^3\}\exp(-c_2\frac{w}{w_c}) - \frac{w}{w_c}(1+c_1^3)\exp(-c_2) \quad (1)$$

with $c_1=3$, $c_2=6.93$ and $w_c=5.14G_F/f_t$.

Starting from point (w_{eu}, σ_{eu}) at the envelope curve, the unloading curve (I) is determined by:

$$\frac{\sigma}{f_t} = \frac{\sigma_{eu}}{f_t} + \{\frac{1}{3(w_{eu}/w_c)+0.4}\}[0.014\{\ln(\frac{w}{w_{eu}})\}^5 - 0.57\sqrt{(1-\frac{w}{w_{eu}})}] \quad (2)$$

When reloading starts from a lower stress level σ_L, the gap in the envelope curve (II) is known by an expression for w_{inc}:

$$\frac{w_{inc}}{w_c} = 0.1\frac{w_{eu}}{w_c}\{\ln(1+3\frac{\sigma_{eu}-\sigma_L}{f_t})\} \quad (3)$$

The coordinates of the returning point at the envelope curve (w_{er},σ_{er}) can now be found with

$$w_{er} = w_{eu} + w_{inc} \quad \ldots\ldots(4)$$

and eq. 1. The reloading curve (III), starting from the point at the lower stress level (w_L,σ_L) up to point (w_{er},σ_{er}) at the envelope curve, is determined by:

$$\frac{\sigma}{\sigma_L} = 1+\left[\left(\frac{1}{c_3}\frac{(w-w_L)}{(w_{er}-w_L)}\right)^{0.2c_3} + \left(1-\left(1-\frac{(w-w_L)}{(w_{er}-w_L)}\right)^2\right)^{c_4}\right]\left(\frac{c_3}{c_3+1}\right)\left(\frac{\sigma_{er}}{\sigma_L} - 1\right) \quad \ldots(5)$$

with for the coefficients c_3 and c_4:

$$c_3 = 3\left(3\frac{f_t-\sigma_L}{f_t}\right)^{\left(-1-0.5\frac{w_{eu}}{w_c}\right)}\left\{1-\left(\frac{w_{eu}}{w_c}\right)^{\left(\frac{0.71f_t}{f_t-\sigma_L}\right)}\right\} \quad \ldots\ldots(5a)$$

$$c_4 = \left[2\left(3\frac{f_t-\sigma_L}{f_t}\right)^{-3} + 0.5\right]^{-1} \quad \ldots\ldots(5b)$$

<u>Inner loops</u>

The above expressions describe the behaviour of a loop starting from and returning to the envelope curve. Here also a proposal will be given for the behaviour when the deformation direction reverses within such a loop. For the description of these so-called "inner" loops, a counter i is used. This counter is taken zero for the envelope curve and increases with 1 each time there is a reversal in crack opening direction before the crack opening at the previous reversal is reached (see Fig. 2). The essential for inner loops in this model is that they return to the same point as where they started from. This means that yet a damaging effect due to cycling within a loop starting from and returning at the envelope curve, is not taken into account. As far as notation is concerned, A_i is used for a point where the direction changes from an opening crack into a closing crack and L_i is used for the opposite reversal in crack opening direction. The meaning of the other applied variables can be obtained from Fig. 2. The unloading curves are determined by

$$\Delta\sigma_{A_i} = \sigma_{A_i} - \sigma_{u_1}(w_{A_i}) - \sum_{n=3}^{i}\left(\frac{w_{A_i} - w_{L_{n-2}}}{w_{A_{n-1}} - w_{L_{n-2}}}\right)^2 \Delta\sigma_{A_{n-1}} \quad \ldots\ldots(6)$$

$$\sigma_{u_i}(w) = \sigma_{u_1}(w) + \sum_{n=2}^{i}\left(\frac{w - w_{L_{n-1}}}{w_{A_n} - w_{L_{n-1}}}\right)^2 \Delta\sigma_{A_n} \quad \ldots\ldots(7)$$

and the reloading curves can be found with

$$\Delta\sigma_{L_i} = \sigma_{r_1}(w_{L_i}) - \sigma_{L_i} - \sum_{n=3}^{i}\left(\frac{w_{L_i} - w_{A_{n-1}}}{w_{L_{n-1}} - w_{A_{n-1}}}\right)^8 \Delta\sigma_{L_{n-1}} \quad \ldots\ldots(8)$$

$$\sigma_{r_i}(w) = \sigma_{r_1}(w) - \sum_{n=2}^{i} \left(\frac{w - w_{A_n}}{w_{L_n} - w_{A_n}}\right)^8 \Delta\sigma_{L_n} \qquad (9)$$

After completion of an inner loop, the counter i decreases with 1. Special attention has to be given to stress crack opening reversals at crack openings corresponding to the gap in the envelope curve. For that case, point A_1 (see inset of Fig. 2) must be determined by means of an iterative procedure.

MODEL PREDICTIONS AND CONCLUDING REMARKS

Comparisons between experimental results and model predictions can be found in (3). There the predictions have also been compared with those by the Focal-Point-Model by Yankelevsky and Reinhardt (5). Here only the result of a prediction for an arbitrary chosen loading path is shown in Fig. 3. Finally, a flow chart of the complete model is given in Fig. 4.

It can be concluded that the model gives an appropriate and complete description for the crack cyclic behaviour of plain concrete. It contributes to improve the description of concrete in FE-codes and other numerical programmes. It has recently been implemented in the DIANA finite element programme of TNO-IBBC (6). The results of some preliminary analyses of concrete beams under four-point bending and fatigue loading, are promising.

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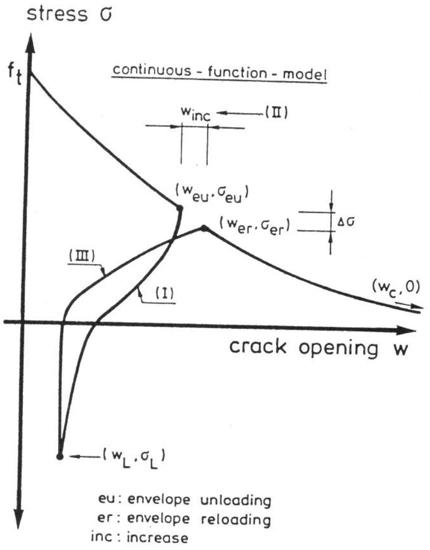

Figure 1 Set-up for the Continuous-Function-Model

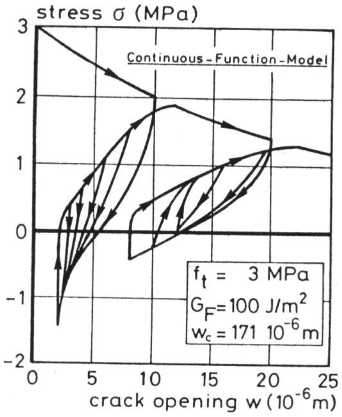

Figure 3 Model prediction for an arbitrary chosen loading path.

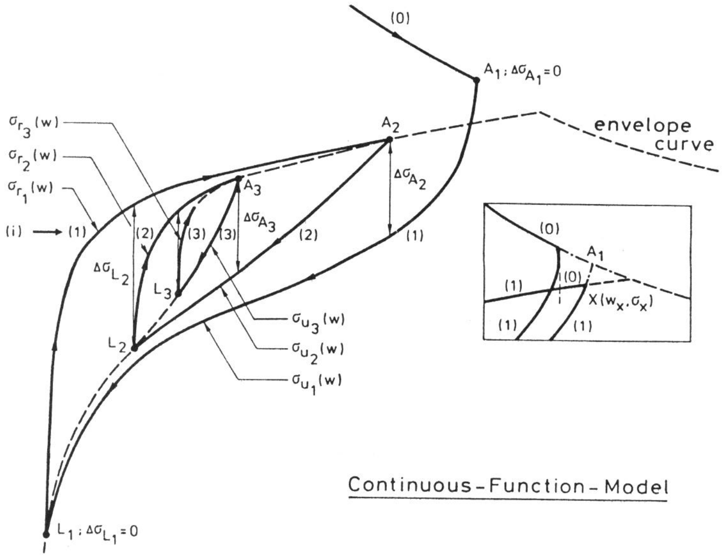

Figure 2 Procedure for inner loops.

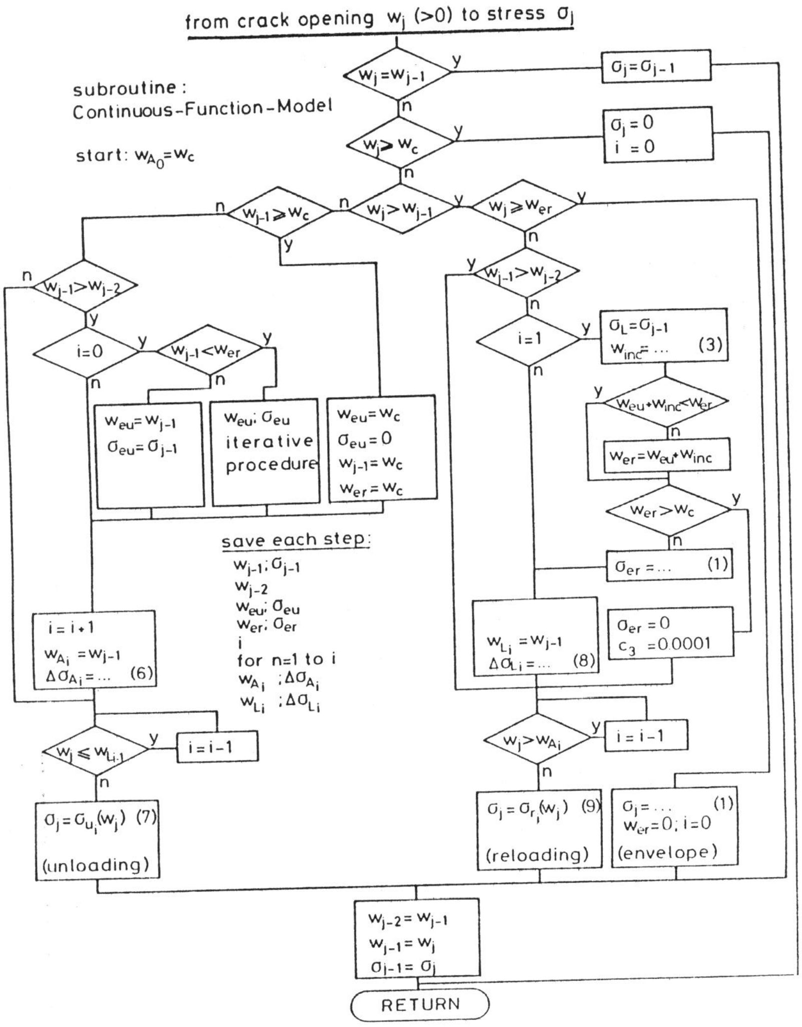

Figure 4 Flow chart for the Continuous-Function-Model (3).