The Structural Integrity of High-Duty Materials

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Abstract

The paper seeks to provide a general framework for treatment of structural integrity issues and directs attention to the questions that have to be answered if a new material is to be used with confidence in engineering applications. Some exemplary figures are given for high-duty metallic, non-metallic, and composite systems, but the main thrust of the paper is to highlight the general importance of initial defect population and damage accumulation.

Introduction

The prime objective of design engineering is to generate components or structures which satisfy functional needs. This must be done in a manner which achieves technical efficiency, given the financial restraints associated with any particular type of engineering application. Increasingly, concern is being expressed with respect to STRUCTURAL INTEGRITY. First, this involves a consideration of the complete loss of function of the system and of any consequences that this may have with respect to human safety or economic disaster, such as may be associated with the system's "safe shutdown" which implies subsequent inability to operate. Second, it requires that attention be paid to the failure rate of components, whose inability to continue to act would not cause complete loss of function of the system as a whole, but would require unplanned "outages", whilst repairs or replacements were being effected.

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The initial result of this is to produce irritating, perhaps expensive, strictly, non-critical, economic penalties, but it is important to note that too great an occurrence of such non-critical outages could lead to loss of public confidence in the overall design concept or the engineering system as such, so that economic disaster might ensue, because the system was rejected as being too unreliable. The full structural integrity concept should therefore involve not only the failure rate of each type of component, but also the consequences of such failure, so that appropriate judgments can be made. The question of public acceptability is increasingly important, and it should be recognised that the majority of the public has experienced little technical training.

It is within this general framework that the present paper addresses itself to "The Structural Integrity of High-Duty Materials". As described above, Structural Integrity is concerned with:

a) complete loss of function of an engineering system
b) failure-rate of non-critical components
c) consequences of such loss of function or failure.

Before turning to specific details, it is appropriate to give consideration to the terms FUNCTION and DUTY.

Function

An engineer's initial design is set by the FUNCTION of a system; by its basic *raison d'être*. A bridge must cross a river and bear traffic; it must perhaps also permit shipping to ply up and down the river. There are various engineering responses to this functional need, but the basic function of the system is the same for all designs; the total span is fixed and traffic and meteorological conditions are common.

Some factors inherent to function are as follows:

a) **Dimensions** A structural component may be fixed in one dimension only, as for the length of a strut or tie-bar; it may be specified in two dimensions, as for an aircraft fuselage panel, rivetted to ribs and stringers; possibly, maxima may be set in three dimensions if there are space limitations for accommodation of a component.

These factors are of importance in materials selection because different figures of merit pertain to the different dimensional constraints. In a minimum weight design to minimise elastic deflection under a given load in a strut or tie-bar, it is necessary to maximise specific stiffness (E/ρ) where E is Young's modulus and ρ is the specific gravity; to maximise the buckling load of a strut, (E/ρ^2) should be maximised; to maximise the buckling load of a panel, (E/ρ^3) should be maximised. The incentive to use materials which are of lower density therefore varies, depending on the application. A convenient "map" to assist initial selection of materials on these criteria has been described by Ashby.(1)

b) **Shape** As for a bridge, there may be engineering responses to a functional requirement which differ quite dramatically in appearance, but, in other cases, shapes are virtually fixed, because they have evolved as the most efficient means of serving a functional purpose. Examples may be found in systems involving fluid flow: nozzle guide vanes and blading in gas turbines or steam turbines; ships propellers. In conventional designs, using conventional materials, shape **and** size can be dictated by the required combinations of flow-rate and compression ratio. Effective utilisation of materials with high (E/ρ) or composite materials may require radical changes in design.

c) **Refractoriness** The maintenance of integrity at high temperatures is a basic requirement, which has led to a great deal of interest recently being shown in ceramics and ceramic-matrix composites (CMCs) for high-temperature applications. There is considerable experience of a number of monolithic oxide systems and carbon as refractories, e.g. in metal-processing, or as nozzles on rocket motors, but in such applications, either the loads are compressive, or the duration of loading is for a few minutes only, and the item is expendable. The ability of a material to withstand a combination of temperature, environment and load is a feature of its structural integrity.

d) **Elastic load-bearing and elastic displacements** The function of an engineering system often requires that loads of a given magnitude can be sustained without significant plasticity (i.e. local yielding is permitted, but general yielding is not) and a design code may therefore limit stresses to a fraction (perhaps two-thirds) of the yield stress, ignoring local stress-concentrations. It is important to note, however, that many systems would suffer loss of function if elastic displacements under load were excessive (parts which fitted, or gave clearance, in the unstressed state might not fit or might jam when stressed) and designs can be displacement-limited rather than load-limited. The big incentive to use materials of high specific stiffness as well as high specific strength is to minimise elastic displacements at high stress in a minimum weight design. A contributory factor is thermal expansion coefficient (α) in any application in which temperature variations are encountered: if α is large, the associated dimensional changes may not be tolerable: in constrained systems, large thermal stresses may be generated.

(e) **Weight and Cost** In recent years, technical judgments on the selection and viability of materials have often been made initially on the basis of a minimum weight design. This is clearly of great significance in transportation systems, where reduction of weight can give advantages of higher performance, reduced fuel costs, or higher payload. It is a relatively straightforward step to cost alternative engineering responses to a functional requirement, using different materials, to form a judgment on the most appropriate solution to the particular need. Factors to be taken into account should normally include the costs of any re-tooling, re-design of detail (particularly joints) and re-training of staff, in addition to the basic price of material. These costs can then be discounted over the period of the estimated production run to decide on whether or not the change is profitable.

What is often not assessed properly in such cost-benefit analysis is the structural integrity issue: will loss of function occur, will the failure-rate be excessive, will the consequences be such as to be significantly detrimental to the economic assessment?

Duty

Some aspects of duty have been treated in the section on Function, but it is convenient to categorise duty primarily in terms of loading sequences and/or temperature sequences. The loads may be "dead" loads, as produced by the "dead-weight" of a structure or by a uniform centripetal force (as in an engine running at constant angular velocity). They may be "live" loads, i.e. - variable loading, arising from start-up/shut-down or take-off /landing sequences, from periodic power adjustments or in-flight manoeuvring, from traffic loading, from wind gusts, wave surges etc. An important type of loading not always taken into account is that experienced in construction or erection, when out-of-balance forces may occur. Out-of-balance forces may also be engendered during service, e.g. by wear of bearings. Conventional engineering design limits the stresses generated by loads to a fraction of the yield stress, but, increasingly, the design engineer is being asked to consider "what ....if ....?" questions, to cater for the possibility of a system being subjected to gross overloads and to ensure that, if it is, a "safe shutdown" situation is obtained.

Similar considerations apply with respect to thermal effects although there are major differences in that the duty imposes displacements (via thermal expansions and contractions) rather than loads. If a component's "gauge length" contains a soft region (as may be present in a welded joint), plastic strain can become localised, even though no stress - concentrator is present. The material's response to the imposed duty is highly sensitive to temperature and, at high temperatures, to strain-rate. Ashby's deformation maps provide a clear illustration of material response in such conditions.(2)

A further feature of thermal excursions is that internal, or "residual", stresses may be created as a result of thermally-induced displacements which differ spatially. Examples are found in fusion-welded joints, where the contraction of a molten weld pool on solidification sets up tensile stresses, balanced by compression on either side; or in metal-matrix-composite (MMC) systems, in which the matrix usually possesses a higher value of thermal expansion coefficient than that of the ceramic reinforcement so that the reinforcement is in compression and the matrix is in tension, after cooling from the processing temperature. Some relaxation of these stresses by plastic flow may occur. It is important to note that the magnitude of these stresses varies with temperature, so that they are of less significance for high temperature applications, and of more significance for low temperature applications.

The initial design places limits on the stresses generated by applied loads or displacements, and restricts these to a fraction of the yield stress. The use of the term "high-duty" therefore requires careful consideration. One connotation is that there is a relatively high probability of occurrence of accidental overload and that loss of function in such an instance would have severe consequences with respect to safety or loss of investment. Another connotation is that the duty is composed of a stress or strain spectrum of such severity that there is a significant probability of component failure, or even loss of function of the system, in a period less than the design lifetime. Under the normal static design load, component failure or loss of function could be anticipated only where there were contributory effects from the chemical environment or, at high temperatures, from thermally-activated deformation and fracture processes. For such effects or for the case of varying stresses or strains, curves should be available to give failure life as a function of stress (or strain) or stress-range (or strain-range). In such cases, "high-duty" refers to the situation where the lifetime expected from the anticipated duty does not exceed the required lifetime by a large margin.

To summarise, "high-duty" refers to those situations in which a probability of component failure or loss-of-function of the system must be recognised and taken into account in design. One example may be drawn from the nuclear industry, in which it might be accepted that component failure could occur once every ten thousand reactor-years, but that loss of function of the system must occur only once in ten million reactor years.(3) These figures are based on the perceived dangers of radio-active releases. It might be noted, however, that the 40 years anticipated life of the proposed European Fast Reactor (EFR) involves only approx. 750 controlled reactor trips (on-off cycles).(4) In contrast, a turbine disc in a civil aero-engine may experience between 10^4 and 10^5 on-off cycles in its anticipated life, with stresses at the bore approaching the material's yield stress. It is to attempt to improve on the performance of metallic alloys in applications such as aero-engines that intense effort is being put into the development of MMCs, CMCs, glass/ceramics and intermetallics, and it is pertinent therefore to examine their properties alongside those of high-strength metallic alloys in a general assessment of the Structural Integrity of High-Duty Materials.

The Technical Basis of Structural Integrity

To carry out a structural integrity assessment, it is necessary to understand and characterise:

a) the stresses (or strains) deriving from the duty,
b) the initial defect population and its distribution,
c) the mechanisms of damage accumulation,
d) the final failure condition.

Features of the stresses have been described in general terms above and it is, nowadays, possible to deduce rather accurate values of maximum stress, σ, (or strain, ε) equivalent stress, $\bar{\sigma}$, $(\bar{\varepsilon})$, or stress range $\Delta\sigma$ $(\Delta\varepsilon)$. Techniques employ analytical or finite element calculations, photo-elastic analysis, strain-gauges, or "hot-spot" experiments on models (examining infra-red output). Some dispute still exists on the best method for analysing load spectra, but the degree of confidence in the values obtained for stresses derived from applied loads or strains in the elastic range is high. Effects of local plasticity are not generally treated so well and rather little attention is paid to the accurate evaluation of residual stresses.

Given a knowledge of the stresses (or strains) deriving from the duty, the assessment recognises that the lifetime is controlled by the accumulation of damage from a state characterised by the initial defect population to one which causes failure. Assurance of structural integrity then relates also to the sensitivity of initial non-destructive inspection (NDI) and to the feasibility and the practicability of the timescale of periodic, in-service NDI. The following section is therefore devoted to a description of the generation, characterisation and control of the initial defect population.

Defects : Characterisation and Control

High duty, metallic alloys for aerospace, defence, or mechanical engineering applications are usually made from rather clean, relatively expensive, starting materials and are treated with a degree of care throughout their processing. Production routes may involve casting and mechanical working, or powder processing, followed by hipping, forging and extrusion. Surface quality is usually high and final components are often surface-ground. The main type of defect is likely to be a non-metallic inclusion, although porosity may also be a concern, particularly in powder products. Particulate MMCs are produced initially by co-spray or powder routes, followed by hipping or extrusion. For Al/SiC systems, the nature and size of the SiC particles is significant, because large, angular particles tend to crack more easily. Attention needs to be paid to the presence of any pores in the material, and to the degree of homogeneity: areas of low volume fraction of particulate may be soft; clumping of particulate may be equivalent to large defects. For reinforcement of MMCs by whiskers or fibres, a possible route is squeeze-infiltration by molten metal. Here, there is a question of whether the wettability is sufficient to give bonding. The existence of pores and the fragility of fibres during extrusion raise points of concern. For monofilament reinforcement, a casting route may be able to be used for some systems, but, for a Ti alloy matrix, it seems that diffusion bonding or superplastic forming of thin foils must be used. Questions arise concerning the degradation of fibre or the interface during processing and the ability to produce a high volume fraction of monofilament in a cross-ply lay-up. For CMCs, note should be taken of general fabrication problems for ceramics, in that sintering tends to result in pores, or brittle, glassy phases at grain-boundaries. For SiC/continuous fibre SiC(e.g. Nikalon) a vapour infiltration process has been proposed, but again the ability of this process to produce a pore-free product is in question, and it is not clear that a suitable consolidation process exists.

In addition to the defect population inherent to the material, defects may be introduced in the assembly of components. Often the quality assurance (QA) procedures do not treat assembly in a closely specified manner, but attention should be paid to mechanical effects (e.g. scoring), to thermal effects (strain-induced cracking) and to environmental effects (possible chemical attack from cutting lubricant). Similar defects could be induced during the disassembly and re-assemby required for periodic inspections.

The control and characterisation of defects can be treated in three ways: process control, proof testing, and complete NDI. Process control would make sure that defects greater than a given size did not exist in the starting material by understanding and controlling each stage of the process route. This is not easy to effect, due to the large number of processes and stages in each process route. It must be combined with meticulous assembly routines and QA to ensure that defects of concern are not introduced during manufacture and assembly.

Proof testing subjects a component to stresses higher than the duty stress, prior to the component entering service. Any over-large defects then produce failure in the proof test and (at the cost of a rejected component) this ensures that components which enter service do not contain defects greater than a certain maximum size. There are two points to note. First, in metallic alloys, where plastic flow occurs at stress concentrations, beneficial compressive stresses are present during subsequent service at lower stress. This effect may not apply to brittle ceramics, in which overstress may produce local microcracks, and hence increase "damage". Secondly, the overstress is often only some 20% greater than the duty stress, so that the "window" between the size of an eliminated defect and that of a defect of concern is not large. No protection is given for accidental overloads greater than the overstress load in the proof test.

The third method is to use complete NDI, followed by the rejection of components found to contain defects of concern. For structural integrity purposes, it is necessary to carry out NDI on the finished component, but, for economy (to avoid costs of machining/assembly on defective material) it is likely that preliminary NDI would be carried out on the bulk material. The main questions concern the sensitivity and reliability of NDI techniques to detect sizes of concern with high probability. Some figures are discussed in the following section.

Defect Sizes of Concern

In this discussion, the expression $K = \sigma_{app} (\pi a)^{1/2}$ is used to estimate critical values of defect size, a_{crit}, for simplicity.

A system that has been studied in great detail is that of a PWR pressure vessel. For operation on the "upper shelf", where the fracture toughness, K_{IC}, is greater than 200 MPa$\sqrt{m}$, the critical defect size for a uniform hoop stress in the pressure vessel shell of 300MPa (two-thirds of the yield stress) is approx 150mm. More refined calculations which include effects of stress concentrations give a value of approx 65mm.(3) To establish the structural integrity of the PWR pressure vessel reliance is placed on high-quality steelmaking and manufacturing techniques combined with multiple 100% ultrasonic scans to ensure the absence of any defect of size greater than 25mm. No defects of this size are expected, but reliance is placed on NDI to detect any rogue occurrences.(5)

A completely contrasting example is given by the example of two grades of maraging steel for intended use in rocket motor-cases, where the function is to contain the pressure due to the explosion of the fuel system, such that the rocket is propelled, rather than the case bursting. Grade G150 has a yield stress of 2.3 GPa (a hoop stress of 1.54 GPa) and a fracture toughness of 33MPa$\sqrt{m}$ which implies a value of a_{crit} of 0.15mm. Grade G125 has a yield stress of 1.9 GPa (a hoop stress of 1.27 GPa) and a fracture toughness of 70MPa$\sqrt{m}$. This implies a value for a_{crit} of 0.97mm. The former value is not detectable with any high degree of reliability and assembly or service abuse could easily produce scratches of this size. The main conclusion is that use of this grade is not able to be justified in terms of structural integrity.

Without going into details of the calculations at this stage, it is instructive to consider two examples for which the critical crack sizes are quite large, but for which sub-critical crack growth might be anticipated in service, so that restrictions are placed on the initial defect population to ensure that the design lifetime can be achieved. One example concerns powder formed nickel-base alloys for gas-turbine discs. For an applied stress level of 1GPa, the maximum initial defect size which would be consistent with a life of ten thousand cycles is 0.1mm. This is below the highly reproducible NDI limit, but process control is available via the mesh size of the sieve used to grade the powder. If the applied stress were increased to 1.3GPa, however, the defect size would decrease to 40μm. This is not only virtually impossible to detect by NDI, but is technically more difficult to control by sieving and is less attractive economically, because the yield of powder is smaller.(6,7)

The second example is that of a forged steel crankshaft in a car, which must typically have a life of greater than a hundred million cycles (100,000 miles at 60 m.p.h. and 4000 r.p.m. equates to 4 x 10^8 cycles). Here, integrity must be insured by a combination of factors: limits on the permissible stress-amplitude (via the S-N curve), consistency of material to reduce the size of initial defects (process control), attention to machining processes and surface condition, prevention of abuse during assembly (QA procedures), and employment of surface treatments to produce hard layers and compressive stresses.

It is important that, when fatigue data are collected, they are obtained on representative test volumes with representative surface conditions. The Rolls-Royce philosophy with respect to turbine discs, for example, requires that spin-rig testing be carried out on actual discs. This is an important feature of the structural integrity case, but does not obviate the need for small-specimen testing and understanding of the fatigue process. It would be highly uneconomic to carry out spin-rig testing on material which was unlikely to achieve the desired performance.

In contrast to these figures for high-duty metallic alloys, consider calculations for an Al/SiC MMC, containing, say 15-20% SiC by volume. The value of plane strain toughness, for this type of material, is unlikely to be more than about 24 MPa√m (although values of 70 MPa√m may be obtained in thin sheet) and the fatigue-crack growth-rate, with values in MPa and m, is given approximately by(8) :

$$da/dN = 2 \times 10^{-12} \Delta K^4 \qquad \ldots\ldots 1).$$

The 0.2% proof stress for such a composite is approx. 400 MPa and typical potential applications are in airframe components, which are often designed to a fairly high fraction of the yield stress. If the design stress is taken as 300 MPa (0.75 proof stress), the critical crack size in plane strain is only 2mm, although it could be as high as 20mm in thin sheet. For parts of airframe structures it is unlikely that NDI is able to detect defects, even of 2mm, with a high degree of reliability and the conclusion must be that particulate MMCs of this type are not, in the present state of development, suitable for use under tensile stresses of normal design magnitude. They are, of course, highly attractive for use as compression members, such as struts, where their high values of specific stiffness show to advantage. If the critical crack length is to be increased to a tolerably inspectable value (probably by visual means) of, say, 20 mm, the maximum value of tensile stress that could be countenanced is slightly less than 100MPa (0.25 yield stress).

In summary, in load-controlled situations, structural integrity, in terms of loss-of-function, could be established for thick sections under monotonic load only if tensile stresses were less than 100 MPa : for thin sections, it can be established for the conventional design stress of 300 MPa. There is an additional ameliorating factor for components such as thin fuselage, in that the ribs and stringers provide stiffening, which changes the stressing from that of pure load-control by introducing a degree of displacement-control.

Aircraft fuselage is, however, subjected to fatigue loading during take-off/landing cycles. For a civil aircraft, the projected design life might be of the order of 20,000 cycles (4 cycles/day x 250 days/year x 20 years or equivalent figures). If the NDI limit is taken as 2mm and the critical defect size is 20mm, the life of 20,000 cycles can be tolerated only if the cyclic tensile stress range is limited to 115 MPa (by integrating equation 1) with appropriate limits and assuming off-on loading, i.e. a stress ratio R = o). For thick sections, the margin between the NDI limit and the critical defect size is vanishingly small, so that a limiting cyclic tensile stress range cannot be based on crack growth rates and can only be based on non-exceedence of the fatigue threshold. A conservative value of the threshold (to cater for lack of closure) might be taken as $2\text{MPa}\sqrt{\text{m}}$; for an NDI limit of 2mm, the limiting tensile stress range is 25MPa,. These figures should be viewed in the light of the fact that the MMCs have been developed primarily for high specific stiffness, rather than fracture toughness or fatigue properties: the SiC particles, in fact, have a deleterious effect similar to that of iron - and silicon-containing inclusions in conventional aluminium alloys.

These considerations clearly place defined limits on the fields of application of particulate Al-based MMCs and limits of this sort may be even more closely defined for ceramic-based systems, in which smaller defects can lead to catastrophic failure, and for which a distribution of inherent material defects must be anticipated, leading to a distribution of failure strength. A probabilistic approach to structural integrity is then required.

The Probabilistic Approach to Reliability and Strength

If R denotes reliability and F denotes unreliability, as fractions,(9) then:

$$R + F = 1 \qquad \ldots\ldots 2).$$

In general R (or F) is a function of some other variable, e.g. time, t (at constant stress, σ) or stress, σ, with t fixed. It is also possible to consider varying time combined with varying stress. If the variable is time:

$$R(t) + F(t) = 1 \qquad \ldots\ldots 3).$$

where R(t) vs t is the survival curve and F(t) vs t is the mortality curve (e.g. the cumulative fractional probability of death up to, or at, a given age). The curve of F(t) vs t is the cumulative distribution function (CDF) and it is possible to define an associated probability density function (PDF), $f(t)$ which is the differential (here, with respect to time) of the CDF

$$f(t) \;=\; (d/dt)\,F(t) \qquad \ldots\ldots\; 4).$$

With respect to a mortality curve, $f(t)$ would represent the fractional probability of death at a given age. In a similar manner, there is a cumulative fractional probability of failure $F(\sigma)$ up to, and including a given stress, σ, and an associated PDF, $f(\sigma)$ given by

$$f(\sigma) \;=\; (d/d\sigma)\,F(\sigma) \qquad \ldots\ldots\; 5).$$

The PDF, $f(\sigma)$ then represents the probability of failing at a given stress, σ. A commonly used expression for $F(\sigma)$ is the Weibull distribution, which in its three-parameter form is given, for a test volume V, by

$$F(\sigma)_v \;=\; 1 - \exp\{\, -V(<\sigma-\sigma_o>/\sigma_s)^m\} \qquad \ldots\ldots\; 6).$$

for $\sigma > \sigma_o$, where σ_o is a datum or "cut-off": for $\sigma = \sigma_o$, the exponential term is $\exp(o) = 1$ and $f(\sigma) = 0$.

The term σ_s is a characteristic stress or "scale parameter", corresponding to the stress at which $F(\sigma) = 0.63$. The exponent m is the "Weibull modulus" and high values of m (steep slopes to $F(\sigma)$) are associated with spike-like distributions for $f(\sigma)$, i.e. consistent material. Values of m and σ_s are determined from a plot of

$$\ln\ln\{1/(1 - F(\sigma))\} = m\ln(\sigma - \sigma_o) \;-\; m\ln\sigma_s \;+\ln V \qquad \ldots\ldots\; 7).$$

The plot on "Weibull paper" (lnln vs ln) is then a straight line of slope m and when the lnln term is zero, i.e. $\ln\{1/(1-F(\sigma))\} = 1$, $F(\sigma) = 0.63$, $m\ln(\sigma-\sigma_o) = m\ln\sigma_s$, i.e. $\sigma_s = (\sigma - \sigma_o)$. Estimation of σ_o essentially requires trial and error, together with a rigorous analysis of "best fit" for linearity. For values of m lying between 1 and 2, the plot of $f(\sigma)$ vs σ on a linear scale gives a very good estimate of σ_o by extrapolation, but for $m>2$, the low σ tail of $f(\sigma)$ has a gradient of zero rather than infinity. For $m \geq 4$, the Weibull PDF is extremely similar in form to a Gaussian, but (in three-parameter form) incorporates a genuine lower-bound cut-off. A very large number of tests is required to distinguish one distribution from the other.

It is accepted that $F(\sigma)$ reflects the distribution of defects in material, but that the relationship between this distribution and failure stress is too complicated to model analytically, because there are variations in defect length, number of given length, and orientation of defect , together perhaps with a variation in (thermally induced) local stress. The effect of test volume is well recognised: if the probability of failure for two test volumes V_1 and V_2 is the same, equation 6). gives $V_1(\sigma_1 - \sigma_o)^m = V_2(\sigma_2 - \sigma_o)^m$ where σ_1 and σ_2 are the failure stresses for volumes V_1 and V_2 respectively. A larger test volume is more likely to contain a large defect and hence to fail at a lower stress.

Much attention is paid in the ceramics literature to the attainment of high m values, since these are associated with consistent material (little variation in defect distribution from one test sample to the next). From a design point of view, however, what is important is the failure stress that can be guaranteed with a given degree of reliability. The characteristic scale factor, σ_s is of little value, since $F(\sigma)$is then 0.63. What is required is either the value at which $F(\sigma)$ assumes the appropriate value - perhaps 10^{-4} - or, if it is possible to derive, the best estimate of σ_o. The measurements must of course be made on samples with representative surface conditions and QA must ensure that surfaces are not abused in service, since the materials may be very sensitive to small defects.

Alternative graphs may plot survival probability, $R(\sigma)$ or P_s as a lnln term vs. ln $(\sigma-\sigma_o)$. If the graph has P_s increasing from the bottom to the top of the ordinate, it has a slope of -m and, for design, it might be appropriate to seek the value of $(\sigma-\sigma_o)$ for which $R(\sigma) = 0{\cdot}9999$, or a similar specified value. The $R(\sigma)$ graphs provide a good basis for illustrating the treatment of a combination of stress variables and time variables through the derivation of the so-called SPT (strength-probability-time) diagram.

The SPT Diagram

The CDF, $F(\sigma)$ represents the effect of a distribution of defects. If these defects increase in size with time, final failure will occur generally at lower stresses and the $F(\sigma)$ curve will be altered. Let the crack growth be expressed by:

$$da/dx = A\ \sigma^v a^w \qquad \ldots\ldots\ 8).$$

where dx is a period variable (cycles or time) and A, v and w are constants : on a fracture mechanics approach $v = 2w$. Separation of variables and integration between a_i $(x = o)$ and a_f $(x = x_f)$ gives, for $w \neq 1$:

$$a_i^{(1-w)} - a_f^{(1-w)} = (w-1)\,A\sigma^v x_f \qquad \ldots\ldots\ 8).$$

For large w and $a_f \gg a_i$ (or a_f approximately constant), this reduces to the form

$$\sigma^v x_f = \text{const.} \qquad \ldots\ldots\ 9).$$

If x represents time and the time to failure at stress σ, is t, the time to failure at stress σ_1 is t_1 the time to failure, t_2 at stress σ_2 is given by:

$$\sigma_1{}^v t_1 = \sigma_2{}^v t_2 \qquad \ldots\ldots\ 10).$$

An even more general approach to derive a similar results derives from the "damage accumulation" concept invoked by Kachanov to treat creep damage. Here, at $t = o$, the material is assumed to be continuous, with load-bearing cross-sectional area, A_o.

Hence, $$\sigma_{nom} = P/A_o \qquad \ldots\ldots 11).$$

where P is the load. At time t, the "continuity" of the cross-section has decreased, such that the effective load-bearing area is A_{eff}. Hence, at t,

$$\sigma_{true} = P/A_{eff} = \sigma_{nom}\ A_o/A_{eff} \qquad \ldots\ldots 12).$$

"Continuity", ϕ, is defined from the expression

$$A_{eff} = \phi\ A_o \qquad \ldots\ldots 13).$$

and it is assumed that the rate of decrease of continuity is related to the instantaneous true stress:

$$d\phi/dt = -\ c\ \sigma_{true}{}^p \qquad \ldots\ldots 14).$$

where c and p are constants and the negative sign is present because ϕ decreases as t increases. From 12) and 13) it is possible to write:

$$d\phi/dt = -\ c\ (\sigma_{nom}/\phi)^p \qquad \ldots\ldots 15).$$

Hence $$\phi^p\ d\phi = -\ c\ \sigma_{nom}{}^p\ dt \qquad \ldots\ldots 16).$$

Integrate the left-hand side from $\phi = 1$ at $t = o$, to $\phi = o$ at $t = t_f$ and the right-hand side from $t = o$ to $t = t_f$:

$$\{\phi^{(1+p)}/_{(1+p)}\}_1^o = -\ c\ \sigma^p{}_{nom}\ t_f \qquad \ldots\ldots 17).$$

Or $$\sigma^p{}_{nom}\ t_f = \text{const.} \qquad \ldots\ldots 18).$$

as for equation 9). Equation 10) would then follow.

The application of equation 10) to deduce the SPT diagram proceeds as follows. It is conventional to break specimens at a constant strain-rate, with the stress increasing in linear fashion with time to a final value, σ_f, at fracture. Let this failure time be $t(\dot{\varepsilon})$. Had the stress σ_f been applied instantaneously, the time to fracture would be $t(\sigma)$. The relationship between $t(\dot{\varepsilon})$ and $t(\sigma)$ is obtained by letting $\sigma = lt$, where l is the linear slope of the curve of σ vs t. Then each increment of stress is σ dt and each contribution to damage is $\sigma^v dt$, or $(lt)^v dt$. Integration from $t = o$ to $t = t(\dot{\varepsilon})$ and substitution into 9) with $t(\dot{\varepsilon}) = x_f$ gives

$$l^v\{t\,(\dot{\varepsilon})\}^{\,1+v}/(1+v) = \sigma^v t(\dot{\varepsilon})/(1+v) = \text{const} \qquad \ldots\ldots 19).$$

Failure under an instantaneously applied stress, σ_f, gives $t(\sigma)$ from:

$$\sigma_f{}^v t(\sigma) = \text{const.} \qquad \ldots\ldots \quad 20).$$

Hence $$t(\dot{\varepsilon}) = (1 + v)\, t(\sigma) \qquad \ldots\ldots \quad 21).$$

If fracture tests are made under constant strain-rate loading, and a degree of sub-critical crack growth is involved, the distribution of failure stresses implies also a distribution of failure times and it is proposed(10) that the failure stress distribution be normalised to a standard failure time, $t(\sigma)$, under constant stress (of, say, 1s) by use of equations 21). and 10). Then, it is possible to estimate the shift in this normalised Weibull distribution for longer failure times, $t(\sigma)$. From equation 7) a given value of lnln $\{1/(1 - F(\sigma))\}$ is associated with mlnσ, taking $\sigma_o = o$ for simplicity. Writing equation 10) in ln form,

$$\ln \sigma_2 = \ln \sigma_1 = (1/v) \ \ln \ (t_1/t_2) \qquad \ldots\ldots \quad 22).$$

If there is a given probability of failure at a standard stress, σ_1, perhaps for a failure time t_1 of ls, an increase in failure time to 10s, with $v = 10$, gives

$$\ln \sigma_2 = \ln \sigma_1 - 0.23 \qquad \ldots\ldots \quad 23).$$

(i.e. $\sigma_2 = 0.79\, \sigma_1$) with a consequent effect on failure probability. The Weibull plot is shifted by a constant amount to produce a parallel line corresponding to the longer failure time. A set of such lines for different failure times is the Strength-Probability-Time (SPT) diagram.

There are several points to be made with respect to this analysis. The first is that although defect growth can be treated in terms of a stress intensity dependence, if $v = 2w$ in equation 8), this is strictly not necessary if other forms are plausible and for a large number of small, and multi-angled cracks, it is difficult to justify a deterministic stress intensity approach. Secondly, the forms of equations 9) and 10) and their relation to equation 7) need to be examined carefully, if a three-parameter Weibull is used. Threshold values pertaining to sub-critical crack growth mechanisms in long-term tests (which should presumably be included in 9) and 10)) may not be the same as those deduced statistically from the short-term fracture tests often used to derive the Weibull plots, so that application to produce SPT diagrams needs to be treated with caution. Of more importance is the fact that the standard derivation involves a normalisation procedure, but that stress changes in engineering components in service may involve a range of strain-rates. The reasoning leading to equation 21) assumes that the failure stress is the same for the two types of loading : in general, this will not be the case and careful iteration using the SPT diagram is required.

In summary, techniques are available to treat the combination of stress variables and time variables on a statistical basis, but the main conclusion to be drawn is that the basic fracture tests have to be devised carefully, bearing in mind the possibility of sub-critical crack growth. Although high values of Weibull modulus give confidence in material consistency, data relating to failure probabilities of order 0.01% are sparse and yet these will certainly be needed to establish structural integrity, particularly in materials whose matrices are susceptible to environmentally - assisted sub-critical crack growth.

Concluding Remarks

The structural integrity of a material relates to its ability to withstand imposed duty without failure. For critical applications, a very low failure probability must be sustained, the precise value depending on the perception of risk and consequences. There are strong economic pressures which attempt to force designers to specify increasingly overous duty in their engineering systems, but the overall economics should reflect the structural integrity of the system and the cost of unplanned "outages"; both direct cost (the "tactical" assessment) and potential cost if customers lose confidence in the system (the "strategic" assessment). The present paper has concentrated on the techniques available to assess a material's resistance to failure emphasising the importance of the initial distribution of defects, their characterisation and control, defects which may be introduced during assembly procedures, and the growth of defects during service. For many systems, it is necessary to consider a distribution of defects and, here, it is necessary to invoke a statistical approach.

A point that has not been treated explicitly is that of accidental overload. It is possible to include such an event in the specification of duty, and, depending on application, assign a probability to its occurrence. It is necessary to take a comprehensive view of the ways in which overloads might be introduced, including out-of-balance loadings that might occur during erection or construction. It is also necessary to assess the consequences of what may be a complete failure of the system to decide whether noxious releases may be generated or whether fragmented parts can be contained. A ductile material may be able to withstand overloads in thick sections by its ability to deform plasticity, albeit perhaps impairing function by the permanent set induced: in thin pipe-walls, it may bulge unacceptably or burst with very large amounts of energy release, A brittle material is liable to fragment, unless the defect population has been designed to withstand the overload duty.

The perception of increased duty in future engineering systems has led to intense interest in ceramic and composite materials. At present, many of these are in early stages of development and it could prove to be unduly pessimistic to match existing property figures to perceived engineering duty. This is why the paper has concentrated primarily on assessment techniques. Nevertheless, certain guidelines emerge. If reliability has to be guaranteed at a particular level (say, 10^{-4}), this is the level at which the survival curves should be examined. High values of the Weibull modulus, although implying consistent material, are secondary to the evaluation of the combinations of stress and time associated with the appropriate survival probability. The overload assessment can be incorporated by relating frequency of occurrence to the survival curve to derive an overall probability of loss of integrity. Iteration with primary design and system economics is required, because reduction of stress requires increase in cross -section and hence increase in weight and material costs, with associated loss of performance. The catastrophic failure of monolithic ceramics under tensile loads necessitates the use of continuous-fibre reinforced materials for ceramic matrix composites and here the efficacy of the "pull out" mechanism in giving (post maximum load) toughening for load-controlled engineering systems needs to be examined closely.

There are many applications in which ceramic, MMC, CMC and GMC materials may be used to give weight-benefits, improved stiffness, or better temperature resistance where the loads are compressive or have only a small tensile component. Here, there are strong incentives to use the materials, to encourage their development and processing and to devise appropriate design methods. For duty which involves high tensile loads, it is important to proceed with caution, addressing at each stage the issues raised in this paper. The description "high duty" was chosen with care : only when a material exhibits structural integrity under high duty can it properly be referred to as a "high performance" material.

References

1. Ashby M.F. "Materials Selection in Conceptual Design" in "Materials 88 - Materials and Engineering Design". 9-13 May 1988, Institute of Metals, pp 2.1-2.13.

2. Frost H.J. and Ashby M.F. "Deformation - Mechanism Maps" Pergamon 1982.

3. "An Assessment of the Integrity of PWR Pressure Vessels" Second Report - Dr W. Marshall. June 1982 Addendum (Prof. Sir P.B. Hirsch) 1987 UKAEA.

4. First Consistent Design for the European Fast Reactor EFRUG 1989.

5. Whittle J. CEGB Proof of Evidence P13 on Non-Destructive Testing Sizewell "B' Power Station Public Inquiry Nov. 1982.

6. Wildgoose P., Turner N.G., Davies H.F., Helliwell B.J., Ubank R. and Harrison H. Powder Metallurgy <u>24</u> (1981) pp 75-86.

7. James M.N. and Knott J.F. Proc. AGARD Conf. No.393 "Damage Tolerance Concepts for Critical Engine Components" Oct 1985 pp 10.1-10.12.

8. King J.E. and Knott J.F. "Fatigue in Metallic Alloys Containing Non-Metallic Particles" in "Fatigue '90", Honolulu July 1990.

9. Bompas-Smith J.H. "Mechanical Survival" McGraw-Hill, New York 1973.

10. Davidge R.W. "Mechanical Behaviour of Ceramics" Cambridge University Press 1979.