

STRESS ANALYSIS BY F.E.M. IN CRACK TIP AND FREE EDGE  
REGIONS IN COMPOSITE MATERIAL STRUCTURAL ELEMENTS

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In structures made from composites, geometrically simple, an analysis of stress distribution is shown in the free edge region, introducing then a crack in the delamination mode. The analysis is performed by the finite element method using singular elements. The behaviour of stresses is given for cross-ply and angle-ply laminates with and without crack in the delamination mode, in a thin layer introduced in the model between the orthotropic laminas.

INTRODUCTION

Classical solutions for the stresses and the strains around a crack tip in an isotropic medium are known (1). Singular behaviour is indicated for stress distributions along a radial direction from the tip. The finite element method is usually based upon assumptions for displacements and stresses defined in terms of polynomial functions over elements of finite size; it is impossible to obtain exact representation of singular behaviour in a singular region. To overcome this problem many authors chose a finite element mesh with a substantial refinement around the tip (2), some other authors introduce special elements in the crack tip zone (3). An interesting approach, also adopted in this paper, is represented by the use of standard parabolic isoparametric elements, with mid-side node appropriately placed (4) (5) (6).

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FREE EDGE STRESS FIELD

The importance of a three-dimensional approach to stress distribution problems in composite laminates has been focused by Pipes and Pagano: a system of second order coupled partial differential equations describes the problem (7). A review on the past works on free edge stress problem can be found in the paper of Salomon (8). Finite elements (9) (10) (11) and finite differences (12) (13) (14) are applied to this problem. Attention has been paid also to some other methods: Reissner's principle was used by Pagano (15), Galerkin's method by Wang and Dickson (16). Delamination phenomena can be the cause of collapse in composite laminated structures: a large number of data in literature has well focused the rule of delamination (7) (9) (10). A laminate of four laminas, as shown in Figure 1 - (A), is considered in this work for the sake of focusing the full three-dimensional stress distribution in multi-layered plates. A numerical approach by three-dimensional finite elements is offered, by interposing between orthotropic layers a thin homogeneous and isotropic layer which represents the matrix; the model presents 4100 hexahedral linear elements (Figure 1 and Figure 3). The orthotropic material behaviour is described by

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} \\ & G_{22} & G_{23} & G_{24} & G_{25} & G_{26} \\ & & G_{33} & G_{34} & G_{35} & G_{36} \\ & & & G_{44} & G_{45} & G_{46} \\ & & & & G_{55} & G_{56} \\ & & & & & G_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{12} \\ \epsilon_{23} \\ \epsilon_{31} \end{bmatrix} \quad (1)$$

where (units in MPa):

$$\begin{aligned} G_{11} &= 1.338 \cdot 10^5 & G_{12} &= 0.356 \cdot 10^4 & G_{13} &= 0.356 \cdot 10^4 \\ G_{22} &= 0.954 \cdot 10^4 & G_{23} &= 0.398 \cdot 10^4 & G_{33} &= 0.102 \cdot 10^5 \\ G_{44} &= 0.460 \cdot 10^4 & G_{55} &= 0.274 \cdot 10^4 & G_{66} &= 0.460 \cdot 10^4 \end{aligned} \quad (2)$$

the rest of  $G_{ij}$  is zero. The matrix is considered a material homogeneous isotropic. The stress values are given near the free edge, in the external interlaminar matrix layer, considering the stacking sequences  $+\phi, -\phi$ s for various values of  $\phi$ , for laminates upon uniform axial extension (see Figure 1 - (A)). The interlaminar stress distributions referred in Figure 2 point out the free edge action on interlaminar stress component values, which present, for some authors, a singular behaviour in moving from the centre of the laminate to the free edge.

STRESSES TRENDS AT DELAMINATING CRACK TIP ZONE

In angle-ply and cross-ply laminates, a crack is opened between laminae, in the free edge region, where the most important values for interlaminar stresses, in the undamaged laminates, are observed (Figure 2): in the interlaminar external matrix for  $-45,+45$ s and  $90,0$ s stacking sequences, that is in the interlaminar matrix between the orthotropic layers having  $+45, -45$  and  $0, 90$  degrees stacking sequences respectively, as shown in Figure 1. The analysis is conducted by singular finite elements where the quarter point technique, described in (4) (5) and (6), is applied. These elements, shown in Figure 3, are posed in the interlamina in the form of rings of elements around the crack tip, as shown in Figure 1, detail (C): the tip is posed along the x direction, the same of the load. The stress distributions are given approaching to the crack tip, along the  $\bar{Y}/C$  direction, in Figure 4 and in Figure 5. The stress distribution obtained shows, in the  $-45,+45$ s case, a singular behaviour for the normal components, particularly for the  $\sigma_{zz}$  component, the most exalted also in the  $90,0$ s case. The normal interlaminar component  $\sigma_{zz}$  is the most exalted stress in the cases of stacking sequences considered. The other components, having a singular distribution approaching the crack tip in the  $90,0$ s case, are the  $\sigma_{yz}$ ,  $\sigma_{xx}$  and  $\sigma_{yy}$  components.

CONCLUDING REMARKS

The significant concentration for interlaminar stresses, due to the presence of the free edge, computed in undamaged laminate, induce to expect collapse by delamination. In a crack tip region, near the free edge, these interlaminar components are exalted when a delaminating crack is introduced. The delaminating component  $\sigma_{zz}$  is very significant and exalted near the crack tip. This fact suggests that delamination probably continues across the laminate inducing the separation of the laminae (opening mode I), but in an orthogonal direction to the load. The interlaminar stress component  $\sigma_{zz}$  is, moreover, the most dangerous for the bonding of laminae: in this interlaminar thin zone, the matrix is forced to stand an important normal stress, stronger than the shear interlaminar stresses, less dangerous for the bonding. As concluding remarks the results of this report can be useful for the understanding of the behaviour of detected defects in bonding of laminates.

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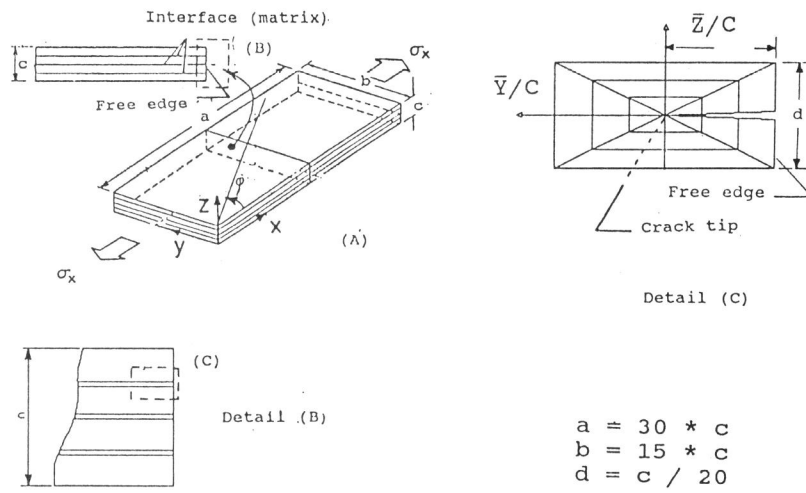


Figure 1 Geometries investigated

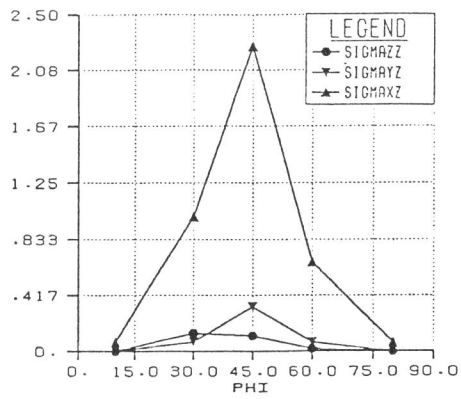


Figure 2 Stress distribution in  $+\phi, -\phi)_S$ .  $SIGMAIJ = \sigma_{ij} / \sigma_{xx\infty}$

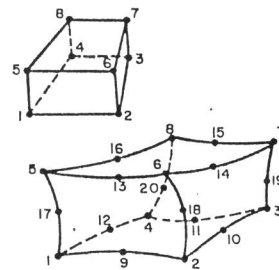


Figure 3 Elements used

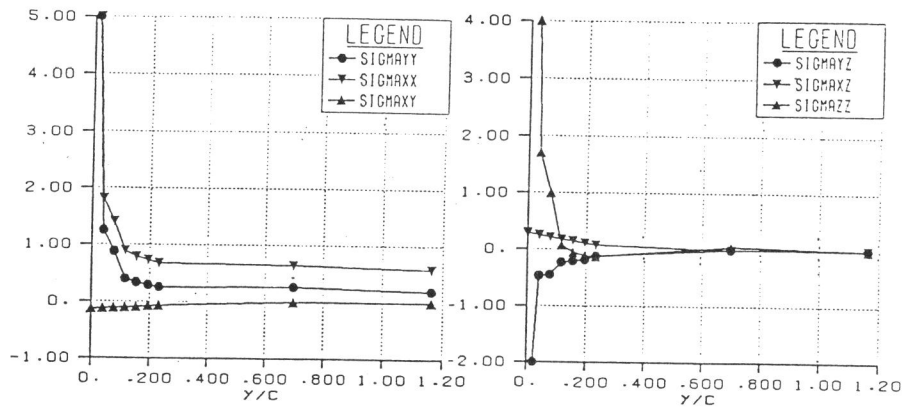


Figure 4 Stress distribution for  $90,0)_S$  laminate.  
 $SIGMA_{IJ} = \sigma_{ij}/\sigma_{xx\infty}$

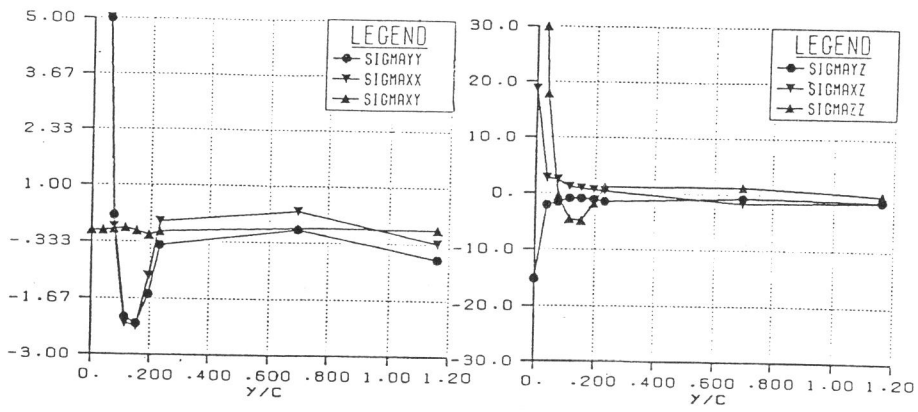


Figure 5 Stress distribution for  $-45,+45)_S$  laminate  
 $SIGMA_{IJ} = \sigma_{ij}/\sigma_{xx\infty}$