FRACTURE OF RODS IN GENERALIZED THERMOELASTICITY

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If in the linear generalized thermoelasticity a prescribed heat source is replaced by an exponential function that depends on stresses and temperature,and characterizes absorption of radiation, a nonlinear theory of irradiated thermoelastic body is obtained. A linearized version of such theory is proposed, and one-dimensional harmonic solution is discussed. A failure criterion depending on the duration of electromagnetic pulses and their intensities is postulated.

INTRODUCTION

It is well known that a linear theory of thermoelasticity can be used to describe adequately a nonlinear fracture process in which rapid growth of temperature takes place, cf. Nicolis and Prigogine (1).

An analysis of fracture of a laser rod within linear dynamic theory of thermal stresses was presented before by Lysikov (2), who assumed that laser irradiation of the rod can be modelled by suitable heat source depending nonlinearly on both the temperature and stress fields. Such a formulation of the problem is similar to that of a combustion process as it is presented by Volpert and Khudiaev (3). In this article Lysikov's results are extended to include effects of the relaxation times of generalized thermoelasticity and of thermoelastic coupling on the fracture of a laser rod, cf.monographic article of Ignaczak (4).

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First a linear version of the problem is formulated, and a plane harmonic solution is studied. Four eigenfrequencies are obtained: two of them represent the acoustic modes while remaining two correspond to the entropy modes. A failure of the laser rod is to occur for one of the two entropy modes, with positive imaginary part of frequency.

BASIC EQUATIONS

For a one-dimensional motion the dimensionless system of field equations is taken in the form

$$\begin{cases} u'_{xx} - (\theta + t_1\,\dot{\theta})'_x = \ddot{u} & (1) \\ \theta'_{xx} + r = \dot{\theta} + t_0\,\ddot{\theta} + \varepsilon\,\dot{u}'_x & (2) \\ r = r_0 \exp\left(- \dfrac{M + m\,\sigma}{T} + M\right) & (3) \\ \sigma = u'_x - \theta - t_1\,\dot{\theta} & (4) \end{cases}$$

where u,T,r and σ represent respectively the displacement, temperature, heat source and stress fields, whereas $\theta = T - 1$. Moreover ε is the classical thermoelastic coupling constant, M denotes a width of the energy gap in a stressless state, and m accounts for the influence of the stress on the gap. Coefficients $t_1, t_0, t_1 \geq t_0 > 0$ are the relaxation times, as described in reference (4), and r_0 is a constant.

LINEARIZATION

A linearization of the exponential heat source leads to

$$r = r_0 \left| 1 - m\,u'_x + (m+M)\,\theta + m\,t_1\,\dot{\theta} \right| \qquad (5)$$

If we put $\theta = \psi - 1/(m+M)$, then from (1)-(4) we obtain

$$\begin{cases} u'_{xx} - t_1\,\dot{\psi}'_x - \psi'_x = \ddot{u} & (6) \\ \psi'_{xx} + r_0(m+M)\psi - \varepsilon\,\dot{u}'_x - r_0 m\,u'_x = t_0\ddot{\psi} + (1 - m\,r_0 t_1)\dot{\psi} & (7) \end{cases}$$

These are the displacement-temperature field equations of the theory under consideration.

DISPERSION RELATION

Eqs (6) - (7) yield the dispersion relation

$$(\omega^2 - k^2)\left| t_0\,\omega^2 - k^2 + (m+M)r_0 + i(1 - m\,r_0 t_1)\,\omega \right| = k^2(t_1\,\omega + i)\,(\varepsilon\omega + i\,m\,r_0) \qquad (8)$$

The limiting cases of eq.(8) are those given in reference (2) ($\varepsilon=0$) and in reference (5) ($r_o=0$).

If we put $\varepsilon=0$, m=0 in (8) we obtain

$$\omega^o_{1,2} = \pm k \,, \qquad \omega^o_{3,4} = i\ \eta_{3,4} \tag{9}$$

where

$$\eta_{3,4} = \frac{1-mr_o t_1}{2\,t_o} \left| -1 \pm \sqrt{1+4\ t_o \frac{(m+M)r_o - k^2}{(1-mr_o t_1)^2}} \right| \tag{10}$$

Clearly, the frequencies $\omega^o_{1,2}$ correspond to acoustic modes while $\omega^o_{3,4}$ to entropy modes.

For $\varepsilon \neq 0$, $m \neq 0$ the approximate solutions of eq.(8) take the form

$$\omega_{1,2} = \omega^o_{1,2} + \Delta\omega_{1,2} \,, \quad \omega_{3,4} = \omega^o_{3,4} + \Delta\omega_{3,4} \tag{11}$$

where corrections $\Delta\omega_i$, i=1,2,3,4 are to be found by approximative methods.

FAILURE CRITERION

A failure of the material is to occur for the modes with positive imaginary part of frequency. For zero thermoelastic coupling this occurs for the mode corresponding to ω^o_3. For a small positive values of coupling parameters the absorption energy increases for a mode corresponding to ω_3 and its time growth is determined by the characteristic time $\tau = i/\omega_3$. For k=0 we obtain

$$\tau = (1 - m\ r_o t_1)/|(m+M)r_o| \tag{12}$$

Therefore in this case a failure is not to occur if the time length of impuls is shorter than τ.

REFERENCES

(1) Nicolis, G. and Prigogine, I., "Self-Organization in Nonequilibrium Systems", John Wiley,New York, 1977.

(2) Lysikov, Yu.I., Zh.Prikl.Mekh. i Tekhn.Fiz., n°4(146), pp.56-59, (1984).

(3) Volpert, A.I. and Khudiaev, S.I.,"Analysis in classes of discontinuous functions and equations of mathematical physics", in Russian, Moscow, 1975.

(4) Ignaczak, J., "Generalized thermoelasticity and its applications", in: R.B.Hetnarski (ed.), Thermal Stresses III,Elsevier Sci.Publ.,B.V.,pp.281-354, 1989

(5) Tao, D. and Prevost,J.H.,J.Therm.Stresses,7,79 (1984)