AN INFINITE PLATE WITH LINES OF DISCONTINUITIES ALONG ARCS OF A PLATE UNDER UNIFORM HEAT FLOW

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The plane problems of heat conductivity and thermoelasticity for a plate containing crack or rigid fibers along arcs of a circle under uniform heat flow at infinity is theoretically studied. Using the complex variable technique the temperature and stress problem are reduced to well-know boundary value problems. The thermal stress intensity factor is defined and results are given for specific cases.

INTRODUCTION

Within the framework of linear thermoelasticity a great variety of crack and inclusion problems have been analyzed by many authors. The Griffith crack problem was studied by Florence and Goodier (1), while in the same problem Sih (2) derived the thermal stress intensity factors and the local thermal stress field in the crack tip. A great number of papers have been published dealing with the problem of the penny-shaped crack (Martin- Moran (3), Barber and Comninou (4), Olesiak and Sneddon (5)) and speciale interest has been noted regarding the crack problems in anisotropic plates (Atkinson and Clements (6), Sturla and Barber (7)). Solutions for thermoelastic inclusion problems have also been studied (Secine (8), Kattis (9)).

In this work, the temperature and thermoelastic problem in an infinite elastic and isotropic plate with cracks or inclusions along arcs of circumference in analyzed. Using the complex variable method the temperature

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and thermoelastic problems are reduced to well-known boundary value problems. The thermal stress intensity factor is defined and results are given for two special cases.

FORMULATION AND SOLUTION OF THE PROBLEMS

We consider an infinite isotropic plate on the z - plane containing cracks or rigid inclusions which lie along n arcs $\Gamma_s = \alpha_s b_s$ $(s=1,2,\dots,n)$ of a circumference of radius α (Fig.1). The union of these arcs will be denoted by Γ such that $\Gamma = \Gamma_1 + \Gamma_2 + \dots + \Gamma_n$ and the positive direction of Γ_s is from a_s to b_s, where α_s, b_s represent the ends of the arc Γ_s. He suppose that constant heat flow $q = |q|\exp(i\gamma)$ is applied at infinity and the cracks or inclusions are thermally insulated. The following boundary conditions are valid on the edges on the cracks or inclusions

$$q_r^{\pm}(\sigma) = 0 , \quad \sigma \in \Gamma , \tag{1}$$

$$(\sigma_{rr} + i\,\sigma_{r\theta})^{\pm}(\sigma) = 0 \quad \text{or} \quad (u+iv)^{\pm}(\sigma) = 0, \quad \sigma \in \Gamma, \tag{2}$$

where the plus (+) and minus (-) superscripts denote the boundary values of the functions at point σ of Γ from the left and right, respectively, as one moves around Γ in the positive direction.

The Temperature Problem

According to (9) the polar components of the heat flow q_r, q_θ and the temperature T can be expressed in terms of two sectional holomorphic function F(z), P(z) in the form

$$-\frac{q_r}{k}\sqrt{\frac{z}{\bar{z}}} = \frac{z}{\bar{z}}F'(z) - \frac{\alpha^2}{\bar{z}^2}P'\left(\frac{\alpha^2}{\bar{z}^2}\right), \tag{3}$$

$$i\frac{q_\theta}{k}\sqrt{\frac{z}{\bar{z}}} = \frac{z}{\bar{z}}F'(z) + \frac{\alpha^2}{\bar{z}^2}P'\left(\frac{\alpha^2}{\bar{z}^2}\right), \tag{4}$$

$$T + iV = F(z) + P\left(\frac{\alpha^2}{\bar{z}}\right), \tag{5}$$

where k is the thermal conductivity of plate and V is an auxilliary harmonic function. For infinite isotropic regions containing thermally isotropic holes and lying under constant heat flow q at infinity, the functions F(z), P(z) have the expansions

$$F'(z) = \frac{-\bar{q}}{2k} + 0\left(\frac{1}{z^2}\right), \quad P'(z) = 0\left(\frac{1}{z^2}\right) \tag{6}$$

$$P'(z) = \frac{\alpha^2 q}{2kz^2} + 0(1) \quad , \quad F'(z) = 0(1), \tag{7}$$

at $t=\infty$ and $z=0$, respectively. Using the equations (1) and (3) we arrive at the following boundary value problems to determine the sectional functions $F(z)$, $P(z)$ with line of discontinuity Γ

$$[F'(\sigma)]^+ - [P'(\sigma)]^- = [F(\sigma)]^- - [P'(\sigma)]^+ = 0, \quad \sigma \in \Gamma. \tag{8}$$

when the behaviour of the function at $z=\infty$ and $z=0$ is expressed by (6) and (7) the solution have the form

$$F'(z)+P'(z) = \beta_0, \quad F'(z)-P'(z)=D(z)X(z), \tag{9}$$

where

$$X(z) = \prod_{s=1}^{n} \{(z-a_s)(z-b_s)\}^{-1/2}, \quad D(z) = \sum_{s=-1}^{n} d_s z^s. \tag{10}$$

The unknown coefficients β_0, d_s $(s=-1,0,\ldots,n)$ are determined, taking into account the equations (6) and (7) and the condition of the univalence of the temperature T. In the case where there is one crack or inclusion ($n=1$, $a_1=\alpha\exp(-i\varphi)$ and $b_1=\alpha\exp(i\varphi)$) the solution is

$$F(z)+P(z) = -\frac{q}{2k}[z \exp(-2i\gamma) + \frac{\alpha^2}{z}], \tag{11}$$

$$F(z)-P(z) = -\frac{q}{2kX(z)}[\exp(-2i\gamma) + \frac{\alpha}{z}] \tag{12}$$

The Thermoelastic Problem

Following Muskhelishvili [10] the polar components of the stresses and displacements can be expressed by equations

$$\sigma_{rr}+i\sigma_{r\theta} = W(z) + \bar{W}(\bar{z}), \tag{13}$$

$$2(\sigma_{rr}+i\sigma_{r\theta}) = W(z) - \frac{\alpha^2}{\rho^2}\Omega(\frac{\alpha^2}{z}) + (1-\frac{\alpha^2}{\rho^2})[\bar{W}(\bar{z}) - \bar{z}\bar{W}'(\bar{z})], \tag{14}$$

$$4\mu(\frac{\partial u}{\partial\theta} + i\frac{\partial v}{\partial\theta}) = iz\{\varkappa W(z) + \beta\Psi(z) + \frac{\alpha^2}{\rho^2}\Omega(\frac{\alpha^2}{z}) - $$

$$-(1-\frac{\alpha^2}{\rho^2})[\bar{W}(\bar{z}) - \bar{z}\bar{W}'(\bar{z})]\}, \tag{15}$$

$$\Psi(z) = F(z) + \bar{P}(\frac{\alpha^2}{z}), \tag{16}$$

where $\varkappa=3-4\nu$ and $\beta=2a_t E$ for plane deformation, $k=(3-\nu)/(1+\nu)$ and $\beta=2a_t E$ for generalized plane stress, a_t is the temperature coefficient of linear expansion, E is Young's modulus, μ is shear modulus, and ν is Poisson's ratio. For infinite regions bounded internally by simple-closed contours C_k, $k=1,2,\ldots,l$, the functions $W(z)$, $\Omega(z)$ have the expansions

$$W(z) = -\frac{\beta G}{1+\varkappa}\frac{1}{z} + 0\left(\frac{1}{z^2}\right), \quad \Omega(z) = -W(0) + 0\left(\frac{1}{z^2}\right), \qquad (17)$$

$$W(z) = W(0)+0(z) \quad, \quad \Omega(z) = -\frac{\beta G}{1+\varkappa}\frac{1}{z} + 0(1), \qquad (18)$$

at $z=\infty$ and $z=0$, respectively, where

$$G = \frac{1}{2\pi i}\oint_C \Psi(\sigma)d\sigma \quad (C=C_1+C_2+\ldots+C_l), \qquad (19)$$

when the temperature function $\Psi(z)$ has been determined, the functions $W(z)$, $\Omega(z)$ are derived following well-known procedures (Muskhelishvili (10)). Thus, for the case of an insulated crack with the temperature functions (11), (12) the complex functions have the form

$$W(z)+\Omega(z)=\beta_0-\frac{\beta G}{1+\varkappa}, \quad W(z)-\Omega(z)=\left(\frac{\gamma_{-1}}{z}+\gamma_0+\gamma_1 z\right)X(z), \qquad (20)$$

where

$$G = -\frac{\alpha^2 q}{2k}\left[1-\cos\varphi+\frac{1}{2}\sin^2\varphi.\exp(-2\gamma i)\right], \quad \gamma_{-1} = -\frac{\alpha\beta G}{1+\varkappa},$$

$$\beta_0 = -f(\varphi)+ig(\varphi), \quad \gamma_1=f(\varphi)-ig(\varphi), \quad \gamma_0=-\frac{\beta G}{1+\varkappa}-\gamma_1\alpha\cos\varphi$$

$$f(\varphi) = -\frac{|q|\alpha\beta\cos\gamma\sin^2\varphi}{8k(1+\varkappa)}\,\frac{1+\cos^2\varphi/2}{1+\sin^2\varphi/2},$$

$$g(\varphi) = \frac{-\alpha\beta|q|\sin^4\varphi/2}{2k(1+\varkappa)}\sin\gamma$$

Introducing at the crack or inclusion tip a_1 the transformation $z=a_1-i\exp(-\varphi)t$ and defining the thermal complex by

$$K = K_I - iK_{II} = \lim_{t\to 0}[\sqrt{t}\,W(t)] \qquad (21)$$

the obtained stress intensity factors for the above case are shown in Figure 2 as a function of the crack half angle φ, when $\varkappa=2$ and $\gamma=0$ ($K_0 = q\alpha^{1.5}/[k(1+\varkappa)]$.

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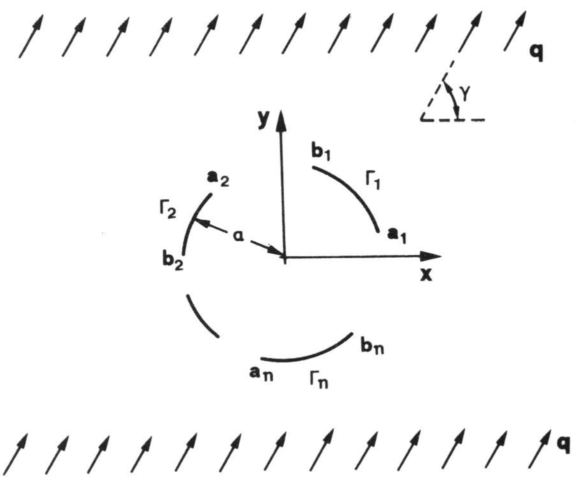

Figure 1 Geometry of the problem .

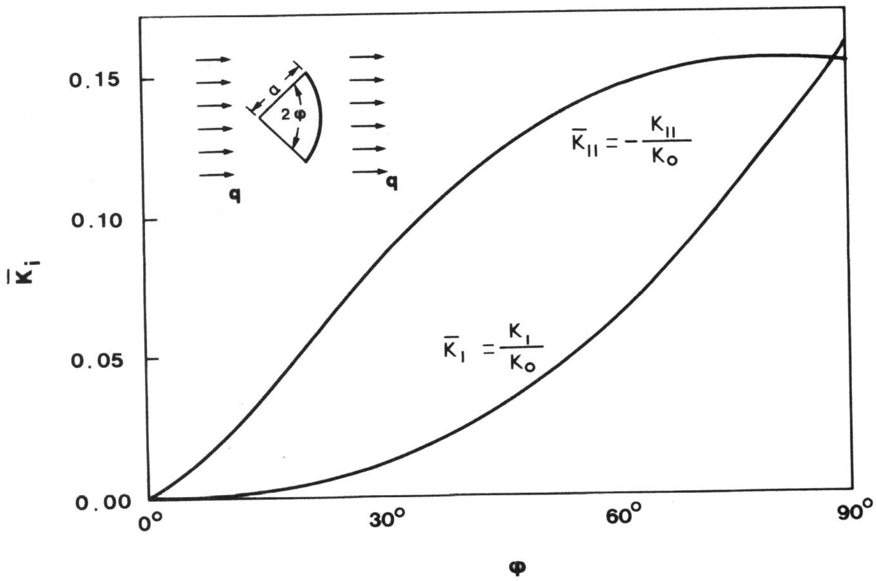

Figure 2 Thermal stress intensity factors as a function of crack half angle φ