STUDY OF FRACTURE INSTABILITY IN BRITTLE MATERIALS BY STRAIN ENERGY DENSITY

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The study of fracture instability, achieved with the analysis of the distribution of the strain energy density (dW/dV) on the cracked brittle body, is presented. This study is an extension of the strain energy density theory, based on the hypotheses on the role of stationary values of (dW/dV). A parameter S_{LG} is introduced which can be used as a measure of system fracture instability; S_{LG} depends upon the combination of loading type, material properties and geometry configuration. The theoretical predictions derived from the application of the present analysis corroborate with experimental results.

INTRODUCTION

The problem of energetic stability of crack propagation has been approached by Gurney, Mai and Atkins (1,2,3) with the method of global energy balance. The study of fracture stability or instability of a cracked body with elastic behavior is achieved when the toughness equations is known. The governing equations of fracture stability depend upon the constraints on the cracked body and geometry stability factor (gsf) of the testpiece. These constraints lead to controlled crack propagation under displacement-controlled or load-controlled conditions (2,3).

In the present work the problem of fracture instability of the cracked body is approached from a different viewpoint, based on the strain energy density (SED) theory (4,5). It constitutes an extension of strain energy density theory based on the hypotheses on the meaning of stationary values of strain energy density. From these hypotheses there may result: the crack path, the crack

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path stability and the stability of cracking for testpieces with the toughness equations unknown.

CRACK PATH PREDICTION

Consider a plane body subjected to a system of external forces with a preexisting crack whose the tip is point 0 (Figure 1). Once the stresses σ_{ij} and strains ε_{ij} are known referred to the coordinate system xy, the volume energy density can be computed from

$$\frac{dW}{dV} = \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij} = \frac{1}{2} \sigma_{ij}\varepsilon_{ij}, \tag{1}$$

if linear elastic deformation is assumed. If r is the distance of a point from the crack tip in the neighbourhood of 0, then: $dW/dV = S/r$ where S is the strain energy density factor.

The maximum of relative minima of strain energy density which appears at point L on the circumference of a circle of radius r_c and centered at 0 is denotes by $[(dW/dV)^{max}_{min}]_L$. According to strain energy density criterion, the initiation of fracture from the point 0 takes place along the direction OL, when the $[(dW/dV)^{max}_{min}]_L$ reaches a critical value $(dW/dV)_c$. The quantity $(dW/dV)_c = S_c/r_c$ is derived from the true stress-true strain diagram of the material in uniaxial tension and is a material parameter. The material parameter S_c is related to the fracture toughness value K_{Ic} by

$$S_c = \frac{(1+\nu)(1-2\nu)}{2\pi E} K_{Ic}^2 \tag{2}$$

For fast crack propagation the body does not have sufficient time to redistribute the stresses during the fracture process. In such a case, the crack path in a cracked body, beyond the point L can be predicted as follows:
Hypothesis (A) : For fast fracture, the crack path can be determined from the conditions before fracture initiation takes place.
Hypothesis (B) : The fracture trajectory can be obtained from the curve which begins from point L, shows the maximum gradient of (dW/dV) and passes from point G where the global minimum value (dW/dV) appears.
Assuming that the analytical expressions of the curve fracture trajectory, referred to coordinate system xy, is (x(s),y(s)) then, according the hypothesis (B) we have the differential equations

$$\frac{dx}{ds} = -\frac{\partial U/\partial x}{|\nabla U|}, \qquad \frac{dy}{ds} = -\frac{\partial U/\partial y}{|\nabla U|} \tag{3}$$

where $U(x,y)=(dW/dV)>0$ is the function of the strain energy density. From the solution the equations (3) the eventual segment LG of the crack path may be determined. If we have but the contours of strain energy density which can result from the use of a computer program of finite elements, then the crack path coincides with the "gorge" which begins at point L.

CRACK PATH STABILITY

The stability of the crack path when crack propagation can results from analysis of the distribution of the strain energy density on the cracked body according to the following hypothesis.
Hypothesis (C) : The stability of the crack path is determined when the equation (3) has a unique solution.
In the case the contours of strain energy density exist, then the stability of the crack path can be deduced from the sharpness with which the curve of the "gorge" is drawn. Using a computer program with finite elements we take the contours of strain energy density on the idealization of DCB-type specimen with varying dimensions. The stability of crack path for DCB-type specimen was studied in (3). This reference, presents experimental results where some testpieces of DCB-type showed instability of crack path. For similar geometry specimens, the map of contours of strain energy density shows many global minima of strain energy density out of the axis of the initial crack. According to the hypothesis (C) there is a great probability that the initial crack, when propagated, may deviate from its initial direction.

STABILITY OF CRACKING

The fast propagation of the crack from L to G (Figure 1) does not leave sufficient time margins for redistribution of the stress field. For the point G where the stationary $[(dW/dV)_{min}]_G$ value of strain energy density appears the following hypothesis may be made:
Hypothesis (D) : When the tip of the propagated crack draws near to point G, the growth rate of the crack decreases sharply. Thus, controlled-loading may stop crack growth.

Consider the length $l = \int_L^G ds$ of the curve which is the fracture trajectory between the local, L, and global, G, minima of (dW/dV). When the initial crack starts growing the difference between the values of (dW/dV) at points L and G is

$$\Delta(dW/dV) = [(dW/dV)_{min}^{max}]_L - [(dW/dV)_{min}]_G \tag{4}$$

If we assume that the crack is propagated with average velocity υ then the necessary time for the crack tip to cover curve LG is $\Delta t=l/\upsilon$. The velocity of crack growth depends on the value of $\Delta(dW/dV)$.

To confine the failure, due to crack propagation, to a local scale two conditions must be met: point G inside the body with length l as small as possible and Δt, i.e. the time needed for the crack tip to make from L to G, as long as possible. This time increase may be effected only if the mean velocity of crack growth is decreased, depending on $\Delta(dW/dV)$. Consequently the quantity

$$S_{LG} = l\Delta(dW/dV) \quad , \qquad (N/m) \qquad (5)$$

may be used as a measure of mechanical system instability; this quantity involves a combination of material properties, loading types and structure geometry.

APPLICATION

The composite beams of aluminum and epoxy layers are considered. They have a span S_1=20 cm, a width W=5 cm and a thickness B=1 cm (Figure 2). The widths of aluminum and epoxy layers are denoted by c and d respectively. A crack of length a=1 cm is located in epoxy and at a distance L_1 from the mid span, where the beam is loaded by force P. A series of different cases, corresponding to various values of the widths c and d and of the location of the crack, were considered. The distance L_1 was taken equal to 0,1,2,3,4,5,6,7 and 8 cm, while the following combinations of the widths c,d were examined: (c,d) (0.0,5.0), (0.5,4.5), (1.0,4.0), (1.5,3.5), (2.0,3.0), (2.5,2.5) and (3.0,2.0). The modulus of elasticity and Poisson's ratio for the aluminum and epoxy layers take the values E_{AL} = 71 GN/m^2, ν_{AL}=0.34 and E_{EP}=3.4 GN/m^2, ν_{EP}=0.35, respectively. The fracture toughness of epoxy is K_{Ic}=610 KN m$^{-3/2}$.

The stress analysis of the composite beam was perfomed by a finite element computer program which uses twelve-node isoparametric elements. A special circular core element surrounds the crack tip and reproduces the singular nature of stresses there. The program directly calculates the stress intensity factors K_I and K_{II}, to a high degree of precision. Results were drawn on the plotter as contours lines of the (dW/dV) on the specimens idealization.

RESULTS AND CONCLUDING REMARKS

Experiments were pefrormed in aluminum-epoxy composite beams (for c=0.0 cm and c=1.5 cm), subjected to three point bending with crack in epoxy at various distances from the mid span. The specimens were tested in a loading machine on wich load increase was controlled (dP>0).

It was observed that the crack path was unstable when the crack was near the support point and stable when nearing the midpoint of the beam. These experimental results are in close agreement with theoretical predictions based on hypothesis (C). The stable crack paths on fractured specimens were in very good agreement with the theoretically predicted LG curves deduced through application of hypotheses (A) and (B). In some specimens cracking was arrested in the neighbourhood of point G. This was achieved by appropriate manipulations on the loading machine after crack propagation had started. These results justify hypothesis (D) on the role of point G, where the global minimum of (dW/dV) appears. Theory in (4) for corresponding cases in a soft testing machine predicts unstable crack propagation and, thus, global failure.

Variation of the instability parameter S_{LG} with the ratio of beam reinforcement for different location of the crack on the beam is shown in figure 2. It may be observed that, as beam reinforcement increases with augmented aluminum contribution, beam stability also increases. Furthermore, for unchanged aluminum contribution the structure becomes progressively unstable as the crack nears the mid-point of the beam.

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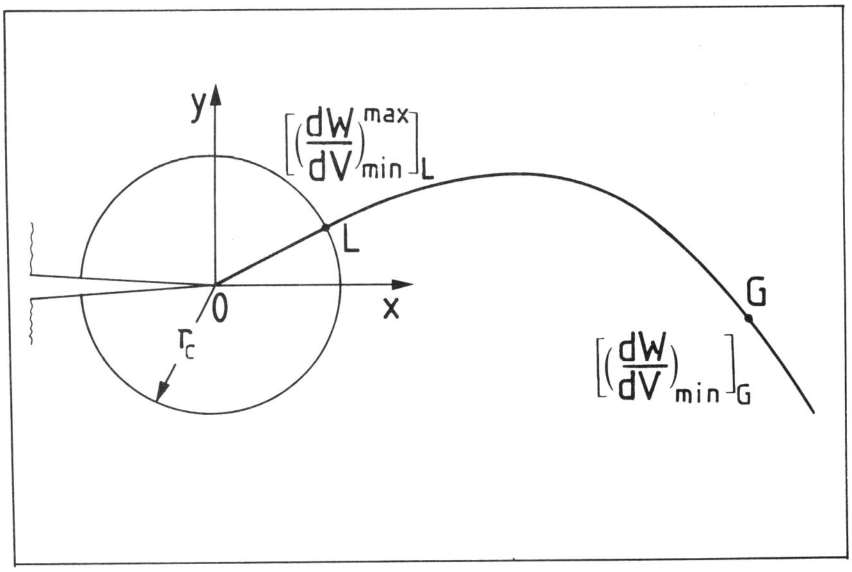

Figure 1 Fracture trajectory OLG passing from the stationary values of (dW/dV)

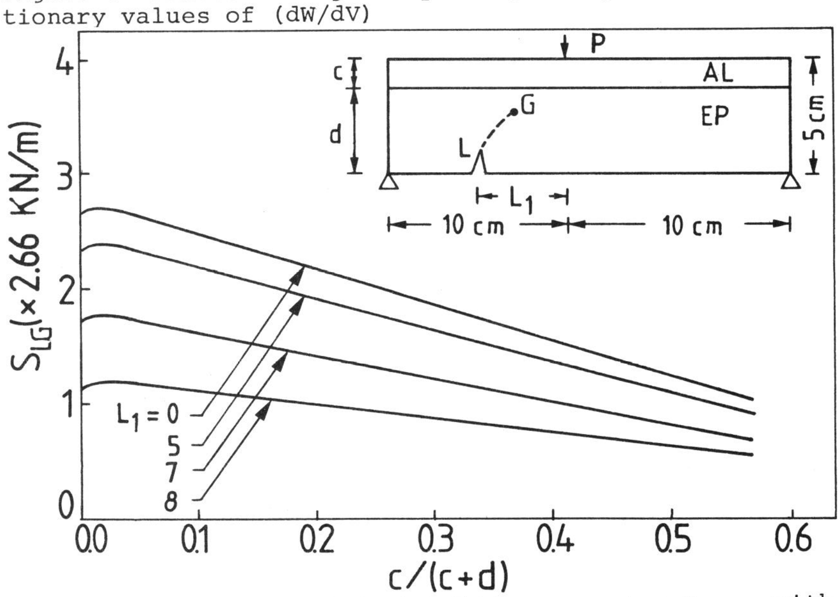

Figure 2 Variation of instability parameter S_{LG} with ratio c/(c+d) for different L_1