BEHAVIOUR OF CRACKS UNDER THREE-DIMENSIONAL MODES

J. Royer and V.A. Le*

This work deals with brittle fracturing under tridimensional modes, and more precisely with the study of mode III incidence on critical loading sustained by a crack. Original specimen geometry and a corresponding loading set-up allowed a mixed mode to be applied on a part through crack in the cross-section of a round bar by using a classical testing machine. The geometric parameters of the device made it possible to adjust the ratio of the different modes. Stress intensity factors on the crack tip were calculated using a new formulation based on boundary integral equations developed especially for this application. The recording of the applied load versus displacement provided the critical load at the onset of crack instability in perspex samples.

INTRODUCTION

In the field of linear elastic fracture mechanics, many studies, e.g. (1-3), have been performed for plane mixed modes, and different specimen geometries have been elaborated. Difficulties become greater with mode III since the problem is three-dimensional. The different criteria proposed for this case (4) usually relate to a generalization of plane models, unfortunately with only rare attempts at validation. Admittedly, it is true that validation of these critera is difficult for both experimental and numerical approaches.

In experimental work, a specimen geometry was studied with an end-shape permitting any type of loading. After the middle cross-section of this specimen was precracked by classical three-point bending, an original loading device made it possible to apply a three-dimensional mode to the crack. The study was limited to quasi-static loading using

* Ecole Nationale Supérieure de Mécanique, Laboratoire Mécanique des Structures, 1 Rue de la Noë - 44072 Nantes Cédex, France

perspex samples. The ratio between modes III and I remained steady at all times, and its value could be adjusted from the geometric parameters of the device which was brought into play using a standard testing machine.

For numerical study, the stress intensity factors (SIF) were derived from a new approach for finite three-dimensional cracked solids based on boundary integral equations. The linear elastic solution provided the three components of the displacement jump vector across the crack and the different SIF K_I values were calculated from these jumps.

EXPERIMENTAL DEVICE

Experimentation was performed on circular perspex samples (25 mm diameter) with ends designed to apply to any type of loading, including torque. A sharp notch normal to the specimen axis was machined in the middle cross-section, and a crack was then started using a classical three-point bending set-up. Crack-front geometry could be described by an ellipse with 5 and 7 mm semi-axes.

Cracked specimens were then subjected to mixed loading by using the special device scheme illustrated in Figure 1 in which two identical specimens were connected to the loading apparatus. The boundary conditions at the ends were equivalent to simple supports with respect to bending and to fixed ends with respect to torsion. As shown in Figure 1, the F action line applied on the lever could be shifted away from distance d of the specimen axis. The two-specimen set and its equipment was equivalent to a beam with a 2ℓ span (ℓ = 125 mm). Two moments were thus applied on each cross-section including cracks : a torsion moment (Mx = Fd/2) and a bending moment (Mz = $F\ell/4$). In these conditions, the cracks of the two loaded specimens were subjected to a mixed mode : mode I was governed by the bending moment and modes II and III by the torsion moment. At any time during loading, these modes remained proportional to F, and the ratio $\delta = K_{III}/K_I$ kept a steady value depending only on the ratio d/ℓ.

STRESS INTENSITY FACTORS

Although there were difficulties in establishing interpolations between values of different studies (5-7), significant discrepancies exist among seldom published values and concerning the only mode I. So a new specific method for tridimensional calculus of cracked elastic solids with finite dimensions was developed by V.A. Le (8).

The displacement field was defined by using Kupradze potential of the second kind. The resulting stress vector is written as the sum of a surface integral over the crack and a line integral along its edge. The kernel of the surface integral depends explicitly on local metrics related to a parametrization of the crack surface, and is moreover shown to be an invariant with respect to the choice of this parametrization. The line integral allows surface as well as kinked crack problems to be dealt with, for which it has been shown that there is a set of line integrals having a sum equal to zero.

Considering the circular lateral free surface of specimen as another crack leads to the problem of two crossed cracks in an infinite medium. The first is the actual crack in the cross-section ; the second (circular-shaped) is long enough to simulate a cylinder with free ends. This approach allows the so-called auxiliary problem to be solved.

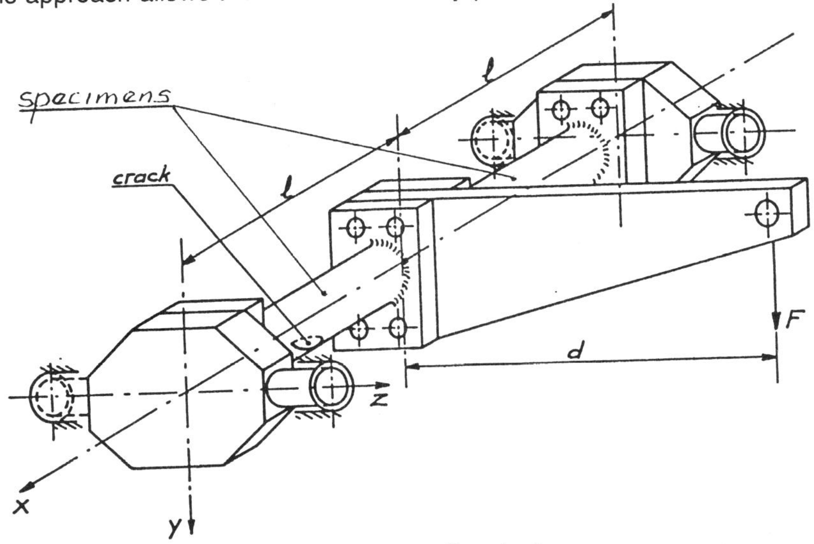

Fig. 1. Scheme of the loading device

Let us denote :

. S_{cr}: a semi-elliptic crack with a Cartesian parametrization : $F_{cr} : \Delta_{cr} \ni (y_1, y_2) \rightarrow y \in S_{cr}$

.$S_{cy\ell}$:a circular crack with a parametrization : $F_{cy\ell} : \Delta_{cy\ell} = [0,2\pi]x[-h,h] \ni (\theta, z) \rightarrow y \in S_{cy\ell}$

. L : the intersection line between the two crack surfaces : $= \bar{S}_{cr} \cap S_{cy\ell}$

. $S_{cr} \ni y \rightarrow \varphi_{cr}(y) \in R^3$, $S_{cy\ell} \ni y \rightarrow \varphi_{cy\ell}(y) \in R^3$: are respectively the unknown densities on S_{cr} and/$S_{cy\ell}$

. $\phi_{cr} = \varphi_{cr} \circ F_{cr}$, $\phi_{cy\ell} \circ F_{cy\ell}$

$S_{cy\ell}$ is delineated by the upper and lower crack fronts at the ends of the bar and by a vanishing contour surrounding L, with clockwise orientation. Thus, the equation system of the problem can be written :

$$\forall y_0 \in S_{cr},\ t(y_0, n_{y0}) = vp\int_{\Delta cr} Noy_{\Delta cr}(\phi^{cr})(y_0,y)\,dudv + \int_{\Delta cy\ell} Noy_{\Delta cy\ell}(\phi^{cy\ell})(y_0,y)dudv$$

$$\forall y_0 \in S_{cy\ell},\ t(y_0, n_{y0}) = \int_{\Delta cr} Noy_{\Delta cr}(\phi^{cr})(y_0,y)\,dudv + vp\int_{\Delta cy\ell} Noy_{\Delta cy}(\phi^{cy\ell})(y_0,y)dudv$$

$$\forall \tilde{y} \in \partial S^{cr}\backslash L,\quad \lim_{Scr \ni y \to \tilde{y}} \varphi^{cr}(y) = 0$$

$$\forall \tilde{y} \in \partial S^{cy\ell}\backslash L,\quad \lim_{Scy\ell \ni y \to \tilde{y}} \varphi^{cy\ell}(y) = 0$$

$$\forall \tilde{y} \in L,\quad \lim_{Scr \ni y \to \tilde{y}} \varphi^{cr}(y) + \lim_{Scy\ell\ sup \ni y \to \tilde{y}} \varphi^{cy\ell}(y) - \lim_{Scy\ell\ inf \ni y \to \tilde{y}} \varphi^{cy\ell}(y) = 0$$

where the kernels in the first two equations denoted as $Noy_{\Delta cr}(\phi^{cr})(y_0,y)$ and $Noy_{\Delta cy\ell}(\phi^{cy\ell})(y_0,y)$ are given by :

$\forall\ y, y_o \in S,\quad Noy_\Delta(\phi)(y_o,y) = (2(\phi_{,u},e_r,F_{,v})n_{yo} - (1-2\nu)((\phi_{,u},e_r,n_{yo})\,F_{,v} + (F_{,v}.n_{yo})\,\phi_{,u} \wedge \vec{e}_r) + 3\,(e_r\,\phi_{,u})\,((F_{,v},\,n_{yo},e_r)\,e_r + (n_{yo}.\,e_r)\,e_r \wedge F_{,v}) - 2\,(\phi_{,v},e_r,F_{,u})\,n_{yo} + (1-2\nu)\,((\phi_{,v},e_r,n_{yo})F_{,u} + (F_{,u}.n_{yo})\,\phi_{,v} \wedge \vec{e}_r) - 3\,(e_r.\phi_{,v})\,((F_{,u},n_{yo},e_r)\,e_r + (n_{yo}.e_r)e_r \wedge F_{,u})$

$S^{sup}_{cy\ell}$ and $S^{inf}_{cy\ell}$ designate respectively the upper and lower part of the cylinder separated by the cross-section, including S_{cr}.

The unknowns are the displacement jumps across the crack surfaces identical to the densities $\phi_{cy\ell}$ and ϕ_{cr}. Knowledge of these densities based on computation of the above equations provides the complete elastic solution to the problem. Nevertheless, the SIF only values were necessary, and these were derived directly from the ϕ_{cr} components.

That gave the values of SIF K_I and K_{III} at point A defined as the mid-point of the crack front on the symmetry axis :

- under pure bending, $K_I = 0{,}595\ \sigma_{max}\sqrt{\pi b}$
- under pure torsion, $K_{III} = 0{,}298\ \tau_{max}\sqrt{\pi b}$

Other comparisons of results thus derived were carried out on simpler problems for which the analytical solution was known : penny-shape crack under uniform pressure and torsion shear stresses, rectangular cracks under uniform pressure and uniform shear in the infinite medium. These led us to assume that opening mode values were more accurate than those in relation to the other modes.

EXPERIMENTATION

The results appear in Figure 2, which shows the values of maximum loads supported by the specimen versus ratio δ. For instance, if mode III is superimposed on mode I (both having the same value), the critical load supported by the crack is at least halved.

Crack propagation surface is also characteristic of the three-dimensional loading mode. The distortion of failed surface, which increases rapidly with mode III value, can be characterized by the spiral angle on the lateral surface of the specimen. As might be expected, there were many failure initiations along the fatigue crack front, which became more numerous when d values were higher. These conditions make the examination of crack surface rather delicate since the onset of multiple initiations is probably not simultaneous. In the case of extensive mode III, fragmentation can occur along the crack tip.

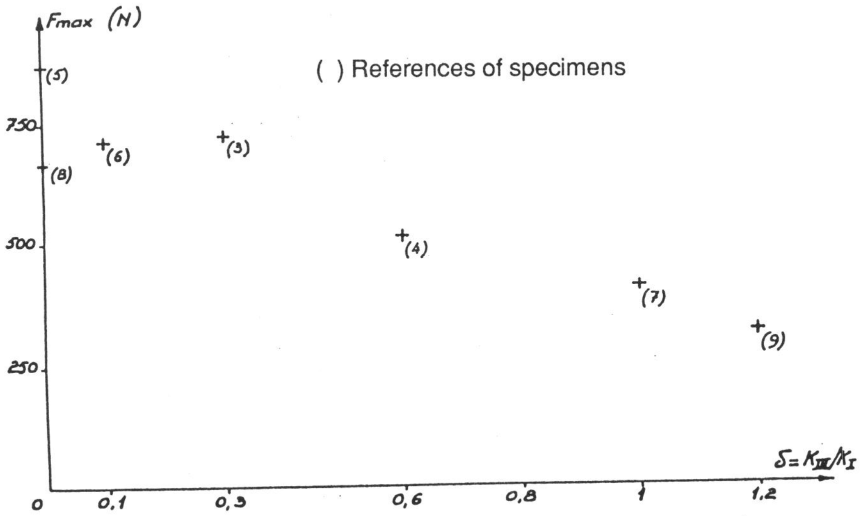

Fig. 2 - Maximum load versus ration δ

CONCLUSION

The results reported here demonstrate that under tridimensional modes failure of a cracked specimen machined in brittle material is possible and that these modes can be adjusted. Moreover, when SIF deduced from a numerical calculus based on the boundary integral equation method are known, it is possible to correlate the behavior of cracks in linear elastic fracture mechanics with that of the applied tridimensional mode. All the conditions are now satisfied to compare the results of the different criteria, although the accuracy of these results depends on solving some remaining problems. The purpose of our continuing experimentation will be to determine the point where the onset of instability occurs will steel specimens studied under fatigue conditions.

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