NUMERICAL PROCEDURE FOR DETERMINING WEIBULL PARAMETERS BASED ON THE LOCAL APPROACH

F. Minami,[1] A. Brückner-Foit, B. Trolldenier[2]

The scatter of toughness observed in the ductile-brittle transition region of ferritic steels can be described either in terms of the Weakest Link model or in terms of the Local Approach. It is demonstrated how the measured toughness values can be used to determine the parameters of the statistical distribution of the Weibull stress. A modified Weibull stress is introduced which takes the effect of a multiaxial stress state into account. The influence of the change of the stress state with the specimen size is discussed in terms of the parameters of the Weibull stress.

INTRODUCTION

The scatter of fracture toughness results in the ductile-brittle transition region of ferritic steels can be described by the Weakest Link model (1)-(3) or by the Local Approach (4), (5). In this theory, a characteristic stress σ_w called the Weibull stress is introduced whose statistical distribution $F_{\sigma_w}(\sigma_w)$ is a Weibull distribution with two parameters, the shape parameter m and the scale parameter σ_u. Both parameters depend only on the material.

Recently, an estimation procedure for the Weibull parameters of $F_{\sigma_w}(\sigma_w)$ has been proposed (6), (7). It is shown in this paper how the results of this estimation procedure depend on the details of the evaluation procedure of the Weibull stress. The example used are fracture toughness results obtained with CT-specimens of a reactor pressure vessel steel.

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1 Department of Welding and Production Engineering, Faculty of Engineering, Osaka University, 2-1 Yamada-Oka, Suita, Osaka 565, Japan

2 Institute for Reliability and Failure Analysis, University of Karlsruhe, P.O. Box 3640, D-7500 Karlsruhe, West Germany

LOCAL APPROACH FOR BRITTLE FRACTURE

The failure probability of cracked or notched specimens whose failure is triggered by the weakest link or by the worst flaw is given by:

$$P_f = 1 - \exp\left(- \frac{M_0}{V_0} \cdot \int_{V_{pl}} \int_{a_c(\vec{r})}^{\infty} f_a(a)\, da\, dV_{pl} \right) , \tag{1}$$

where V_{pl} is the volume of the yielded region containing the weak spots of size a, and M_0 denotes the average number of weak spots in the unit volume V_0. The critical flaw size a_c can be calculated from:

$$a_c(\vec{r}) = \frac{E \cdot G_c \cdot \pi}{2(1 - \nu^2) \cdot \sigma_{eff}(\vec{r})^2} \tag{2}$$

if the weak spots are described as penny-shaped cracks. In Eq.(2), G_c denotes the resistance of the matrix material against unstable crack propagation and $\sigma_{eff}(\vec{r})$ is the effective applied stress acting on the weak spot at location $\vec{r}$. This stress is normally assumed to be the maximum principle stress.

In the Local Approach, $f_a(a)$ is assumed to be of the form a^{-r} for large values of a. Hence Eqs.(1) and (2) yield:

$$\frac{M_0}{V_0} \cdot \int_{V_{pl}} \int_{a_c(\vec{r})}^{\infty} f_a(a)\, da\, dV_{pl} = \frac{1}{V_0} \cdot \int_{V_{pl}} \frac{\sigma_{eff}(\vec{r})^m}{\sigma_u^{\ m}}\, dV_{pl} \ . \tag{3}$$

The Weibull stress σ_w is introduced to rewrite the right hand side of Eq.(3):

$$\sigma_w = \left(\frac{1}{V_0} \cdot \int_{V_{pl}} \sigma_{eff}(\vec{r})^m\, dV_{pl} \right)^{1/m} \tag{4}$$

and the failure probability Eq.(1) at a given load level L is determined by the corresponding value of the Weibull stress:

$$P_f(L) = 1 - \exp\left(- \frac{\sigma_w(L)^m}{\sigma_u^{\ m}} \right) . \tag{5}$$

ESTIMATION OF WEIBULL PARAMETERS

Each load level L can be characterized by the corresponding value of the applied load or of the displacement $V(L)$ for notched specimens or by the value $J(L)$ of the applied J-integral for cracked specimens. The relation $\sigma_w(L) - V(L)$ or $\sigma_w(L) - J(L)$ can be used to calculate the statistical distribution of the Weibull stress and hence the parameters m and σ_u.

For this purpose, a finite element analysis has to be performed which determines the value of the stress tensor at the nodal points. Eq.(4) can be rewritten in the following form:

$$\sigma_w{}^m = \sum_k \frac{1}{V_0} \int_{V_k} \sigma_{eff}{}^m \, dV_k \ , \tag{6}$$

where V_k denotes the volume of an element. In (7) the integral over each element was approximated by the mean value of the effective stress σ_{eff} which was assumed to be equal to the stress normal to the crack plane. A more refined integration procedure can be introduced which relies on the interpolation procedure given by the shape functions of the finite elements and on a Gaussian integration algorithm. Moreover, the stress normal to the crack plane is replaced by the maximum principal stress.

A second alternative is to take the random orientation of the planes of the weak spots into account and to use the same model as for multiaxial loading of ceramic components (see e.g. (8) and references therein). The following expression is obtained for the effective stress σ_{eff}:

$$\sigma_{eff} = \left(\frac{1}{2\pi} \int_0^{\pi} \int_0^{\pi} \left(\sigma_n^2 + \frac{4}{(1-\nu)^2} \tau^2 \right)^{m/2} \sin \theta \, d\theta \, d\phi \right)^{1/2} \ , \tag{7}$$

where the coplanar energy release rate was used as a fracture criterion for the weak spots, and the angles ϕ, θ define the orientation of the plane of the weak spot relative to the pincipal stress axes. The stress σ_n is the resulting stress normal to the flaw plane and τ the resolved shear stress.

The estimation procedure for the Weibull parameters m, σ_u is summarized in the flow chart shown in Figure 1. The loading variable and the corresponding Weibull stress with an assumed starting value for m are calculated at each load step L. The Weibull stress at which fracture of a given specimen occured can be determined from this relation between the loading parameter and the Weibull stress. Hence, a sample of Weibull stresses σ_w at fracture can be generated for a given test series, where a certain number of nominally identical cracked or notched specimens was loaded until they failed by cleavage fracture. The parameters of the failure probability Eq.(5) can be estimated by the maximum likelihood method. Convergence of the estimation procedure is obtained if the maximum likelihood estimate $\hat{m}$ for the shape parameter m is approximately equal to the starting value used for the calculation of the Weibull stress.

Side-grooved CT-specimens of three different thicknesses ($B = 25mm$, $B = 12mm$, $B = 6.5mm$) were fractured $T = -55°C$. The material used was a reactor pressure vessel steel with German designation 20 MnMoNi 55. The value of the J-integral at cleavage fracture, J_{cl}, was determined according to ASTM E 813. The stress tensor ahead of the crack tip was evaluated in an elasto-plastic finite element analysis at IWM Freiburg, West Germany. The Weibull parameters for the Local Approach were calculated using the estimation procedure explained above. Table 1 shows the Weibull parameters obtained with the effective stress σ_{eff} in Eq.(6) equal to the maximum principal stress σ_1 and with σ_{eff} given by Eq.(8). Figure 2 contains the statistical distributions $F_{\sigma_w}(\sigma_w)$ of the Weibull stress where

$$F_{\sigma_w}(\sigma_w)(\sigma_w(L)) = P_f(L) \tag{8}$$

and P_f is given in Eq.(5).

DISCUSSION

The differences between the values of the Weibull parameters obtained for both definitions of the effective stress σ_{eff} are negligible. This means that only the maximum principal stress is important for cleavage fracture.

Whereas the statistical distributions of the Weibull stress agree quite well for both $B = 25mm$ and $B = 12mm$ there seems to be a shift to higher values of the Weibull stress for the very small specimens with $B = 6.5mm$. However, the sample of small specimens which failed by cleavage fracture without prior stable crack extension is too small and no conclusions about tendencies of the Weibull parameters can be drawn at present. Additional experiments with cracked and notched specimens are in preparation.

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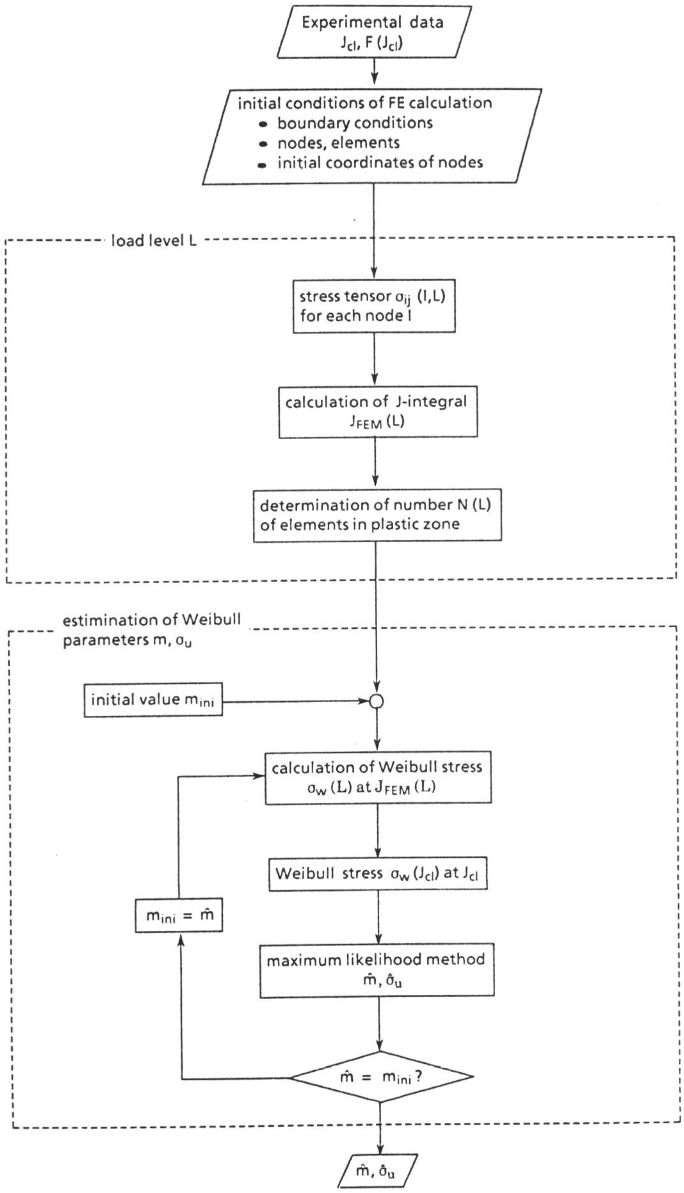

Figure 1: Flow chart for the estimation procedure of the parameters of the statistical distribution of the Weibull stress.

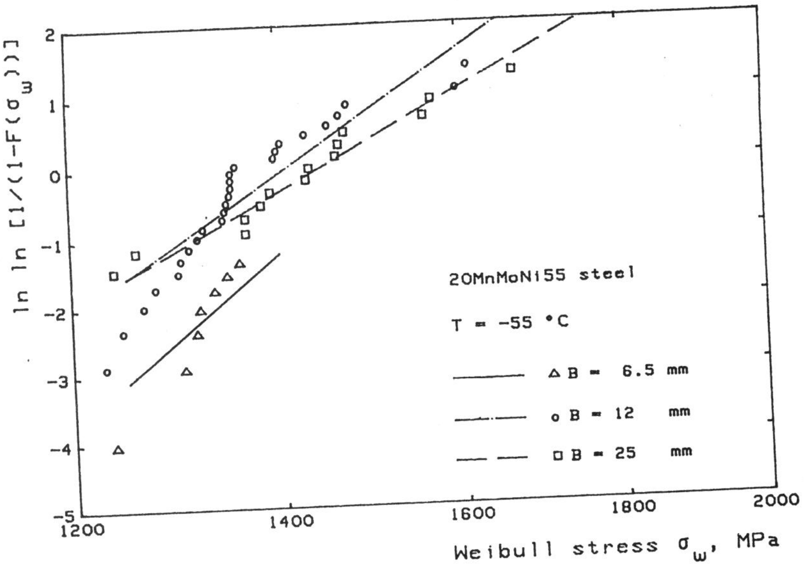

Figure 2: Statistical distributions of the Weibull stress for CT-specimens of three diffenrent thicknesses; σ_{eff} as in Eq.(7).

specimen thick-ness B	σ_{eff}	Weibullparameter m	Weibullparameter σ_u
25 mm	σ_1	10.9	1390 MPa
25 mm	Eq.(7)	10.5	1449 MPa
12 mm	σ_1	13.0	1370 MPa
12 mm	Eq.(7)	12.8	1411 MPa
6.5 mm	σ_1	15.2	1476 MPa
6.5 mm	Eq.(7)	16.4	1510 MPa

Table 1: Results of the estimation procedure for the parameters m , σ_u.