TO APPROACHES OF ELASTIC-PLASTIC FRACTURE MECHANICS

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In elastic-plastic fracture mechanics two approaches, namely the COD and J-integral are employed. They both have certain priorities and drawbacks. Critical crack size is in engineering practice calculated from the design curves. Several design curves were suggested. Accuracy of approach must be considered complexly, including the methods for determination of δ_c and J_{IC} characteristics.

In last twenty years the fracture mechanics has become the means for designing of demanding structures. At the same time considerable attention is paid to improvement of methods for testing the fracture mechanics characteristics and precission of calculation of the critical crack size, mainly by use of design curves. The contribution is devoted to several discussed questions of the elastic-plastic fracture mechanics.

FRACTURE MECHANICS APPROACHES

In the field of validity of the linear-elastic fracture mechanics - LELM, the behaviour of a crack in the body is described by the stress intensity factor: $K_I = S_n Y \sqrt{\pi a}$. The limit state of brittle failure is characterized by plane-strain fracture toughness K_{IC}. The K_I data for idealized shapes of cracks and basic modes of loading are given in the manuals and codes. Temperature dependence of fracture toughness K_{IC} must be determined experimentally, e.g. according

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to (1).

Over the limits of LELM the elastic-plastic fracture mechanics - EPLM is valid. Two approaches: the COD and J-integral are employed. The approach of equivalent energy is a modification of J_{IC} aaproach. The J_{IC} approach is more perfect than the COD approach since it better expresses the plastic strengthening of material and at the boundary of K_{IC} - J_{IC} approaches they both seem to be equivalent. An advantage of the COD approach consists in the fact that the δ_c value can be directly measured on the tested model whereas the J_{IC} value for the model can be only calculated. Conformity of δ_{CE} with K_{IC} can be attained by a suitably defined δ_c and an appropriate design curve.

The two-criteria K_r - S_r approach is used at the boundary of fracture mechanics approaches and the plastic failure of materials. It processes the previous fracture mechanics characteristics K_{IC}, J_{IC} as well as R_e, R_m, thus it is a calculation method.

CHARACTERISTICS OF THE ELASTIC-PLASTIC FRACTURE MECHANICS

The fracture toughness J_{IC} and the critical crack tip opening displacement δ_c belong to the "calculation" characteristics of materials and therefore they must not depend on the size of the experimental body. At the test by the COD approach the values of $\delta_c = \delta_u$; δ_i; δ_m, similarly as J_{IC}; J_i; J_m can be determined. The characteristics of material resistance against the initiation of a brittle crack are only δ_c and J_{IC} which correspond to formation of a instantaneous failure and meet the limiting criteria of validity (2, 3).

Fracture toughness J_{IC} is determined from the force F - force displacement f diagram according to the relationship:

$$J_{IC} = J_{CE} + J_{CP} = \frac{K_C^2}{E'} + \frac{X.A_{CP}}{B.b} \qquad ....(1)$$

$$K_C = \frac{F_C.Y}{B.\sqrt{W}} \quad ; \qquad E' = \frac{E}{1-\nu^2} \quad ; \qquad b = W - a$$

for SEB specimen $X_1 = 2.0$; for CT specimen $X_2 = f(\frac{a}{b}) =$

= 2.2 - 2.3.
According to Landes and Herrera(4) for the CT specimen is valid:

$$J_{CP} = \frac{n}{n+1} \frac{X.F_C.f_{CP}}{B.b} \qquad ....(2)$$

where n is the hardening coefficient; $n \doteq 6$.
It can be proved that for the equivalent energy approach the equation (1) is valid

$$K_{CE} = K_{CJ} = \sqrt{K_C^{\,2} + \frac{X.E'.A_{CP}}{B.b}} \qquad ....(3)$$

The critical crack tip opening displacement δ_c is determined from the force F - notch edge opening displacement V according to the relationships:

$$\delta_c = \delta_{CE} + \delta_{CP} = \frac{K_C^{\,2}}{c.E'.R_e} + V_{CP}\left[1 + \frac{1}{r}\frac{a+z}{W-a}\right]^{-1} \qquad ....(4)$$

according to BS 5762 (2)..... c = 2 ; $1/r = 2.5$ ....(4a)

according to VÚZ draft (5) ..... c = 1 ;

$$1/r = 3 \frac{W}{a} \frac{R_e}{S_n} \qquad ....(4b)$$

where S_n is the nominal stress in the cross section below the notch.

Between δ_c and J_{IC} the following relationships are valid:

$$J_{CE} = G = \delta_{CE}.R_e; \qquad J_{IC} = m.\delta_c.R_e \;;$$

$$J_{CP} = m'.\ \delta_{CP}.\ R_e \qquad ....(5)$$

On comparison of the elastic components J_{CE} and δ_{CE} in equations (1) and (4) the c = 1. On comparison of the plastic components J_{CP} and δ_{CP} in (1) (4) (5) the rotation factor for the CT specimen ($f_{CP} = V_{CP}$; z = = 0) can be calculated:

$$\frac{1}{r} = \frac{b}{a}\left(\frac{n+1}{n}\frac{m'.B.b.R_e}{X_2.F_C} - 1\right) = \frac{b}{a}\left(C_2 \frac{R_e}{S_n} - 1\right)$$

i.e. the rotation factor depends on the relative stress R_e/S_n. It was also proved, (6) that at the chenge of the crack length a/W = (0.3 - 0.7) it is suitable to use the correction member W/a for the rotation factor in the sense of equation (4b).

Determination of δ_c according to BS 5762 is incorrect theoretically and gives conservative data.

CALCULATION OF CRITICAL CRACK LENGTH ACCORDING TO DESIGN CURVES

Calculation of critical crack length for the elastic-plastic condition of a failure is a very complex matter. Therefore, the equivalent length of a central crack passing through the plate thickness $\bar{a}_c$ is determined according to design curves in engineering practice. For other crack shapes and other stress fields the $\bar{a}_c$ datum is recalculated according to the shape and correction factors. For the COD approach the limit curves are expressed in the following way:

$$\bar{a} = \frac{E.\delta_c}{\pi.R_e} \; \frac{1}{c.\varphi} \; ; \qquad ....(6)$$

$$\varphi_1 = \left(\frac{S}{R_e}\right)^2 \quad ... \quad \left(\frac{S}{R_e}\right) \leq 0.5 \qquad ....(6a)$$

$$\varphi_2 = \left(\frac{e}{e_e} - 0.25\right) \; ... \; \left(\frac{e}{e_e}\right) > 0.5 \qquad ....(6b)$$

where $e = S/E$; $e_e = R_e/E$

according to BWI (7) $c = 2$; $\bar{a} = \bar{a}_m$ acceptable length of equivalent crack ....(6c)

according to VÚZ (5) $c = 1$; $\bar{a} = \bar{a}_c$ critical length of equivalent crack ....(6d)

According to corrected WES 2802 draft (8)

$$a_c = \frac{E.\delta_{CB}}{R_e} \; \frac{1}{\varphi} \; ;, \qquad .... (7)$$

$$\varphi_1 = 2\left(\frac{e}{e_e}\right)^2 \quad \ldots\ldots \quad \left(\frac{S}{R_e}\right) \leq 1.0 \quad \ldots\ldots\ldots \quad (7a)$$

$$\varphi_2 = \left(3.5\ \frac{e}{e_e} - 1.5\right)\ldots\ldots \quad \left(\frac{e}{e_e}\right) > 1.0 \quad \ldots\ldots\ldots \quad (7b)$$

For the J_{IC} approach Landes and Begley (9) have suggested :

$$a_c = \frac{E.J_{IC}}{\pi.R_e^2}\ \frac{1}{\varphi} \quad ; \quad \ldots\ldots\ldots \quad (8)$$

$$\varphi_1 = \left(\frac{S}{R_e}\right)^2 \quad \ldots\ldots \quad \left(\frac{S}{R_e}\right) \leq 1.0 \quad \ldots\ldots\ldots \quad (8a)$$

$$\varphi_2 = \left(2\ \frac{e}{e_e} - 1\right) \quad \ldots\ldots \quad \left(\frac{e}{e_e}\right) > 1.0 \quad \ldots\ldots\ldots \quad (8b)$$

Turner suggested already the second modification (10) of equation (8) in the form:

$$\varphi_1 = \left(\frac{e}{e_e}\right)^2 \left(1 + 0.5\left(\frac{e}{e_e}\right)\right)^2 \ldots\ldots \left(\frac{e}{e_e}\right) \leq 1.2 \quad \ldots\ldots \quad (9a)$$

$$\varphi_2 = 2.5\left(\frac{e}{e_e} - 0.2\right) \quad \ldots \quad \left(\frac{e}{e_e}\right) > 1.2 \quad \ldots\ldots \quad (9b)$$

For the two-criteria approach several equations are used. The basic equation (11) is :

$$K_r = S_r \left[\frac{8}{\pi^2}\ \ln \sec\left(\frac{\pi}{2}\ S_r\right)\right]^{-0.5} \quad \ldots\ldots\ldots \quad (10)$$

The failure assessment diagram according to (12)

$$K_r = (1 - 0.14.L_r^2)\ (0.3 + 0.7\ \exp\ (-0.65\ L_r^6)) \quad \ldots\ldots\ldots \quad (11)$$

For pressure vessel with a crack on the external surface with dimensions c/1 the S_r is calculated

$$S_r = \frac{2.p.m}{h(R_e + R_m)}\ \frac{h/c - 1/m}{h/c - 1} \quad ;$$

$$m = \sqrt{1 + 0.263 \frac{l^2}{r.h}} \qquad \ldots\ldots\ (12)$$

The calculated dependences of the equivalent crack length $\bar{a}_c$ on the relative stress are shown in Figs. 1, 3 and 4. The points for the conditions of vessel failure are plotted in these diagrams. Evaluation of conditions for failure of the first vessel by use of the two-criteria approach is shown in Fig. 2.

CONCLUSIONS

Approval of the approaches of the elastic-plastic fracture mechanics is by no means a simple matter. The δ_c and J_{IC} characteristics have a great scatter. Also in model tests the cracks occur in the materials with heterogenous properties, in an inhomogeneous field of residual stresses etc.. On the basis of our experience and the mentioned examples we recommend:

1. For determination of the elastic-plastic characteristics of fracture toughness the equation (1) should be used for J_{IC} and for δ_c the equations (4, 4b) should be used. The BS 5762 approach is incorrect and yields conservative data.

2. The critical crack size $\bar{a}_c$ and not the admissible size $\bar{a}_m$ should be determined. The measure of safety a_c/a_m depends on many factors as the probability of J_{IC}, S_T but also significance of a structure: nuclear power equipment, bridges, cranes etc.

3. For calculation of the equivalent crack size $\bar{a}_c$ by the J_{IC} approach the equations of Landes (8) should be used, by the δ_c approach the equations (6a, b, d) are advised.

4. Behind the limits of validity of the J_{IC} approach the failure assessment diagram $K_r - L_r$ according to equation (11) should be used.

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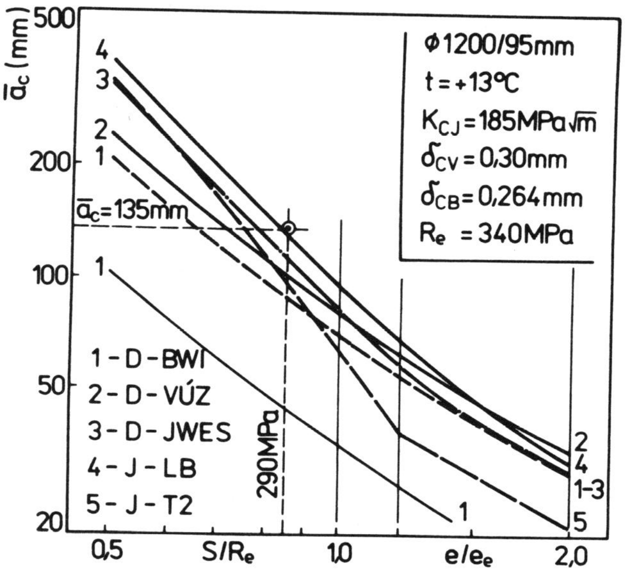

Figure 1 Equivalent crack length - relative stress dependence for MnV steel at +13 °C

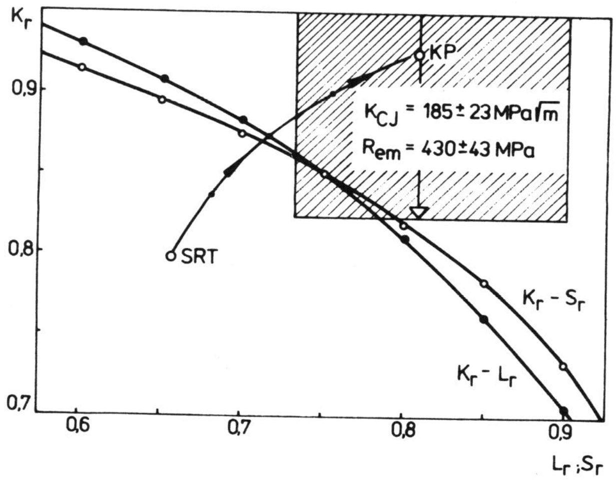

Figure 2 Assessment of MnV steel pressure vessel failure with K_r-S_r and K_r-L_r approaches

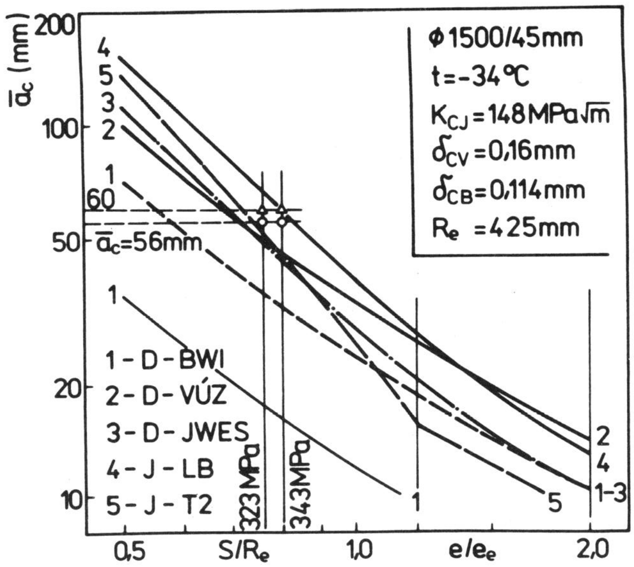

Figure 3 Equivalent crack length - relative stress dependence for SA weld metal in Fe 510 E steel at -34 °C

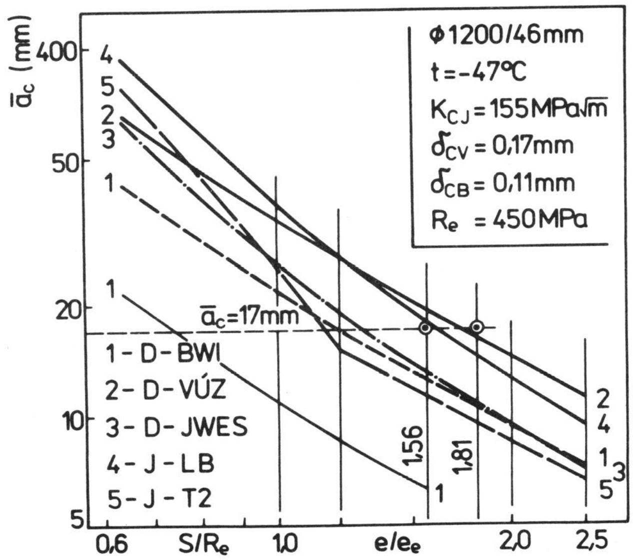

Figure 4 Equivalent crack length - relative strain dependence for MA weld metal in Fe 510 E steel at -47 °C