THE STRUCTURAL LIFE PREDICTION UNDER CREEP OR HIGH-CYCLE FATIGUE CONDITIONS

Y.Samarin*, V.Astafjev**, V.Maclakov*, V.Semenychev*

A concept of structural design and structural operation under technical state conditions for creep or high-cycle fatigue is suggested. The general aspects of this concept are
(i) the description of deformation process;
(ii) the analysis of failure associated with creep rupture, high-cycle fatigue, crack initiation and propagation;
(iii) the establishment of appropriate design criteria;
(iiii) the control and identification of structural responces.

INTRODUCTION

Components of the structures in high temperatures have a finite lifetime caused by time dependent deformation of material (creep). Failure could also occur under cyclic loading (fatigue). Improved inspection methods reveal the presence of cracks in components. The crack may result from manufacturing or may be formed under service conditions. Cracks could be initiated by thermal or mechanical loading or nucleated from coalescing microvoids. Design methods must be able to predict the lifetime under these circumstances. Hence, understanding of the growth of defects and deterioration of material has to be improved. The development of creep mechanics has been given by Rabotnov (1). The later developments are found in the textbook byBoyle and Spence (2). The theory of deformation and fracture under cyclic loading has been described by Troshchenko (3).

* Kujbyshev Politechnical Institute, Kujbyshev, 443010, USSR.
** Kujbyshev State University, Kujbyshev, 443011, USSR.

CONSTUTIVE EQUATIONS AND DAMAGE LAWS

Constitutive equations. The description of first and second stage of deformation under creep conditions is based on the theory of incomplete recovery, proposed by Samarin (4)

$$p(t) = u(t) + v(t) + w(t)$$

$$u = \sum u_k,\ \dot{u}_k = d_k((1-f)a_k\sigma^s - u_k) \quad \ldots\ldots\ldots\ldots(1)$$

$$v = \sum v_k,\ \dot{v}_k = d_k(fa_k\sigma^s - v_k)H(fa_k\sigma^s - v_k)$$

$$\dot{w} = B\sigma^n$$

where p, u, v, w are creep, recoverable viscoelastic, unrecoverable viscoplastic and viscous strain (Fig. 1), H is the Heavyside function, the dot means time derivative. Analogous constutive equations based on tensorial state variables follow from Astafjev (5).

The constutive equations (1) can also describe the first and second stages of deformation under high-cycle fatigue, where $p = \Delta\varepsilon$ is the width of inelastic hysteresis loop and $\sigma = \sigma_a$ is the amplitude of stress under symmetric cycle (Figure 2).

The equations (1) with stochastic coefficients are offered to describe the behavior of a set of identical members (σ and p is the generalized force and displacement in this case). For individual member a model with measured responces at the beginning of the creep curve is built by Eremin et al (6) (Figure 3).

Creep damage laws. As proposed in ref. (5) and by Radchenko et al (7) the constitutive equations (1) are valid for description of the third stage of creep deformation by replacing the stress σ on the "effective stress" $\sigma/(1-\omega)$. The creep damage parameter ω changes in accordance with the evolution equation of Kachanov-Rabotnov (1)

$$\dot{\omega} = A(\sigma/(1-\omega))^m,\ \omega(0) = 0 \quad \ldots\ldots\ldots\ldots(2)$$

High-cycle fatigue damage laws. For high-cycle fatigue conditions the constitutive equations (1) can describe the third stage of cyclic deformation by replacing the amplitude of stress σ_a on the "effective amplitude" $\sigma_a(1+q)$. The high-cycle fatigue parameter q characterises the material softening and is described by the following evolution equation

$$q = L\sigma^r p \qquad (3)$$

Material parameters determination. The experimental determination of material parameters d_k, a_k, f, s, B, n, A and m (or L and r) are based on a series of creep (or fatigue) and lifetime curves. The parameters determination is reduced to the solution of a set of linear equations. The use of regularisation methods, special measure regime etc. provide the calculational and statistical stability of this approach.

FRACTURE CRITERION OF ELEMENTARY VOLUME

Energetic fracture criterion. The rupture of structures is analysed using the thermodynamic of irreversible process relations. The energetic fracture criterion proposed in ref. (5) is formulated as "the elementary volume δV of material is fractured and elementary crack δS is initiated when the internal energy density U of this volume has reached the critical value U_*".

Creep conditions. Taking into account the internal energy density dependence on stress, temperature and internal state variables as proposed in ref. (5) this criterion may be written as

$$U(\sigma, T, \omega_*) = U_* \qquad (4)$$

The simpest approximation of condition (4) given in ref. (5) is

$$\sigma/(1 - \omega_*) = \sigma_* \qquad (5)$$

where ω_* is the value of damage parameter at fracture moment and σ_* is the ultimate stress.

High-cycle fatigue conditions. In the textbook by Fedorov (8) it was shown that for high-cycle fatigue conditions the change of heat flux per unit cycle ΔQ is proportional to the work of stresses ΔA. The approximation of this relationship may be written in the form

$$\Delta Q = (1 - k(\sigma_a - \sigma_{-1})^r)\,\Delta A \qquad (6)$$

where σ_{-1} is the shakedown limit stress. The law of energy conservation $\Delta U = \Delta A - \Delta Q$ allows us to write the energetic fracture criterion $U = U_*$ as

$$k_f k\int_0^{N_*}(\sigma_a - \sigma_{-1})^r \sigma_a \Delta\varepsilon dN = U_* \qquad (7)$$

where $\Delta A = k_f \sigma_a \Delta \varepsilon$ is the area of inelastic hysteresis loop, k_f is the shape factor of the loop.

Length of initiated crack. The energetic fracture criterion enable us to estimate the characteristic size d of fractured volume δV and the length of initiated crack. As shown in ref. (5) the energetic balance of volume-surface transition lead to the following

$$d \sim 2\gamma / U_* \quad \text{.................(8)}$$

where γ is the effective surface energy density. Thus, the energetic fracture criterion $U = U_*$ of elementary volume δV means that the volume δV of characteristic size d is fractured and the initial crack δS of characteristic length d is coinceived.

CREEP CRACK GROWTH

Creep crack growth equation. The growth of initiated (or technological) crack in structures is analysed also under energetic fracture criterion. Thus, during the crack growth process internal energy density U in volume δV near the crack tip has a critical value U_*. Using the approximation (5) equation for crack length l(t) proposed by Astafjev (9) can be written as

$$\sigma(l(t) + d,\ t)/(1 - \omega(l(t) + d,\ t)) = \sigma_* \quad \text{..(9)}$$

Hutchinson (10), Rice and Rosengren (11) stress field asympthotics and evolution equation (2) lead (9) to the following creep crack growth equation

$$\int_0^z \left(\frac{c(x)}{z-x+1}\right)^{\alpha} \tau'(x)dx = 1 - c^{\beta}(z) - \tau_o \frac{c^{\alpha}(0)}{(1+z)^{\alpha}} \quad \text{.....(10)}$$

where $z = (l(t) - l_o)/d$, $\tau = t/t_*$, $c(z) = C^*(l(t))/C^*_{cr}$ are dimentionless crack length, time and anvariant C^*-integral, $\alpha = m/(n+1) < 1$, $\beta = (m+1)/(n+1) \leq 1$, $t_* = 1/A(m+1)\sigma_*^m$, $C^*_{cr} = BI_n \sigma_*^{n+1} d$.

Crack start conditions. Equation (10) gives us the expression for dimentionless time τ_o and dimentionless rate $(\tau'(0))^{-1}$ of crack start

$$\tau_o = (1 - c^{\beta}(0))/c^{\alpha}(0) \quad \text{......................(11)}$$

$$\tau'(0) = \alpha\tau_o - \beta(c(0))^{\beta-\alpha-1}c'(0) \quad \text{............(12)}$$

Relationship (11) shaws that C^*_{cr} is analogous to K_{IC}

of linear fracture mechanics. For $C^*(l_o) = C^*_{cr}$ crack starts immediately. When $C^*(l_o) < C^*_{cr}$ ($c(0) > 1$) and $\tau_o > 0$ there exist conditions for which $\tau'(0) = 0$. This means that the crack after time τ_o begins to propagate dinamically (unstable crack growth). If $\tau'(0) > 0$ then after time τ_o the stable crack propagation with initial crack growth rate $(\tau'(0))^{-1}$ begins.

Stable crack growth conditions. For $z \gg 1$ equation (10) has the following solution

$$c^{\alpha}(z)\,\tau'(z) = \frac{\sin \pi\alpha}{\pi}\left(\frac{1-c^{\beta}(0)}{z^{1-\alpha}} - \beta\int_0^z \frac{c^{\beta-1}(x)\,c'(x)\,dx}{(z-x)^{1-\alpha}}\right) \quad \ldots (13)$$

When $c(z) \ll 1$ this expression may be written in form

$$\dot{l}(t) = A(m+1)\frac{\pi}{\sin \pi\alpha}\left(\frac{C^*(l(t))}{BI_n}\right)^{\alpha}(l(t) - l_o)^{1-\alpha} \quad \ldots (14)$$

Relation (14) coinsides with the analogous of Kubo et al (12) obtained in assumption $\omega(l(t) + d, t) = 1$.

Critical crack length. The growth of the crack length leads to the growth of $c(z)$ and there is a critical crack length l_{cr} when $\tau'(z_{cr}) = 0$. It may be evaluated from (10) as follows

$$C^*(l_{cr}) = C^*_{cr}(1 - (1-\alpha)(1 - c^{\beta}(0)))^{1/\beta} \quad \ldots\ldots\ldots (15)$$

Contrary to the linear fracture mechanics condition $K_I(l_{cr}) = K_{IC}$ the expression (15) shaws that l_{cr} depends on both C^*_{cr} and applied load and also on initial crack length l_o. The qualitative behaviour of dependece $l(C^*)$ is illustrated in figure 4.

Crack growth under variable loading. Under two-step loading the creep crack growth equation (10) predicts- (i) a sudden crack growth up on Δ and the increase of crack rate just after the low-to-high stress change; (ii) partial crack arrest on $\Delta\tau$ and decrease of crack rate after the high-to-low stress changes (Figure 5); (iii) a sudden crack growth up on Δ and partial crack arrest on $\Delta\tau$ after short overloading (Figure 6). This conclusion corresponds to the experimental data of Taira et al (13) for 316 SS at 650°C (Figure 7). The experimrntal data from ref. (13) and theoretical relationship between $l(t)$ and C^* from equation (14) are plotted in figure 8.

CONCLUSIONS

The use of offered equations for the whole deformation process permit to prognose the behaviour of individual member with sufficient precision that is to solve the question of its deformational and strength properties under creep or high-cycle fatigue conditions.

REFERENCES

(1) Rabotnov, Yu.N., "Creep Problems in Structural Members", North-Holland, Amsterdam, 1969.

(2) Boyle, J.T. and Spence, J., "Stress Analysis for Creep", Butterworths, London, 1983.

(3) Troshchenko, V.T., "Fatigue and Nonelasticity of Metals", Naukova Dumka, Kiev, 1971 (in Russian).

(4) Samarin, Yu.P., "Constitutive Equations of Materials with Complex Rheology", Kujbyshev State University, Kujbyshev, 1979 (in Russian).

(5) Astafjev, V.I., Prikl. Mech. i Techn. Phis., No.6, 1987, pp. 156-162 (in Russian).

(6) Eremin, Yu.A., Radchenko, V.P. and Samarin, Yu.P., Mashinovedenie, No.1, 1984, pp.67-72 (in Russian).

(7) Radchenko, V.P., Samarin, Yu.P. and Chrenov, S.M., Dokl. Akad. Nauk SSSR, Vol.288, No.3, pp. 571-574 (in Russian).

(8) Fedorov, V.V., "Damage and Fracture Kinetic of Solids", FAN, Tashkent, 1985 (in Russian).

(9) Astafjev, V.I., Mech. Tverdogo Tela, No.1, 1986, pp. 127-134 (in Russian).

(10) Hutchinson, J.W., J. Mech. Phys. Solids, Vol.16, No.1, 1968, pp. 13-31.

(11) Rice, J.R. and Rosengren, G.F., J. Mech. Phys. Solids, Vol.16, No.1, 1968, pp. 1-12.

(12) Kubo, S., Ohjii, K. and Ogura, K., Eng. Fract. Mech., Vol.11, 1979, pp. 315-329.

(13) Taira, S., Ohtani, R. and Kitamura, T., Trans. ASME, J. Eng. Mater. Techn., Vol.101, 1979, pp. 154-168.

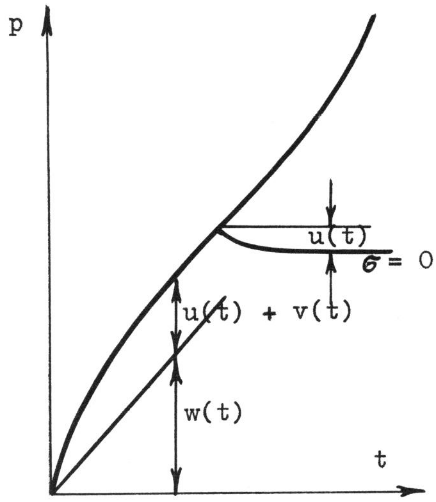

Figure 1 Creep curve with unloading

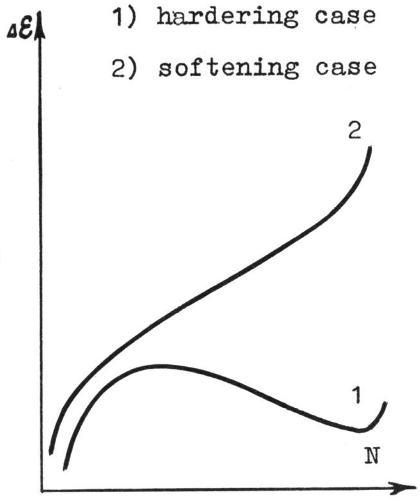

Figure 2 Fatigue curves for hardering and softening case

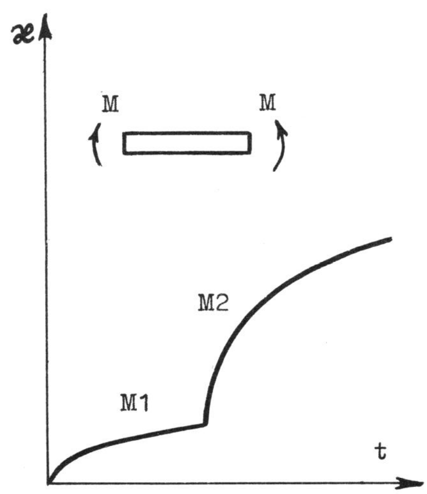

Figure 3 Curvature vs time dependence for bending

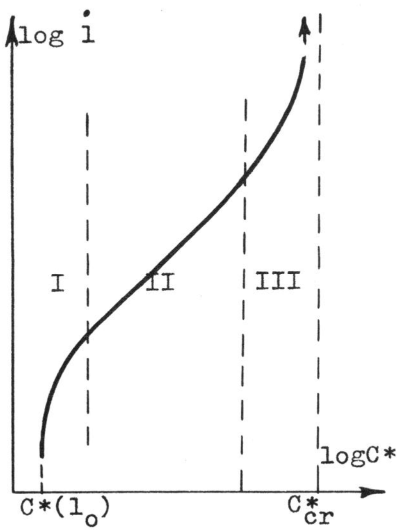

Figure 4 Dependence l vs C* according to equation (10)

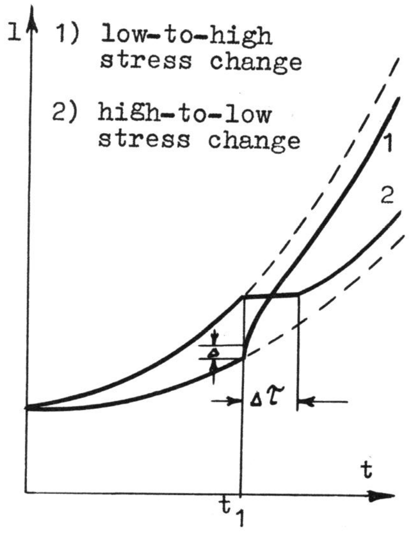

Figure 5 l(t) curves under two-step loading

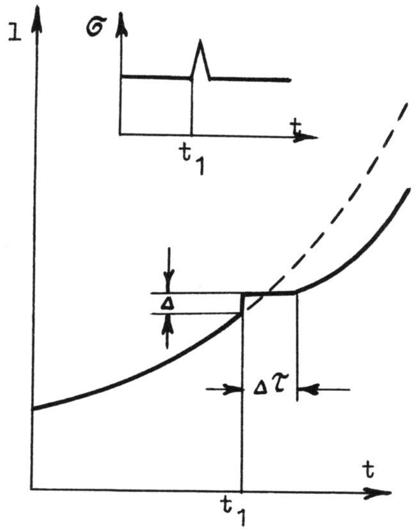

Figure 6 l(t) curve under short overloading

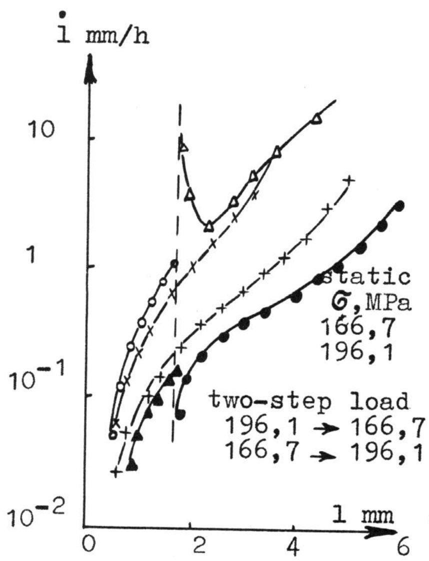

Figure 7 $\dot{l}$ vs l for 316 SS in two-step loading (13)

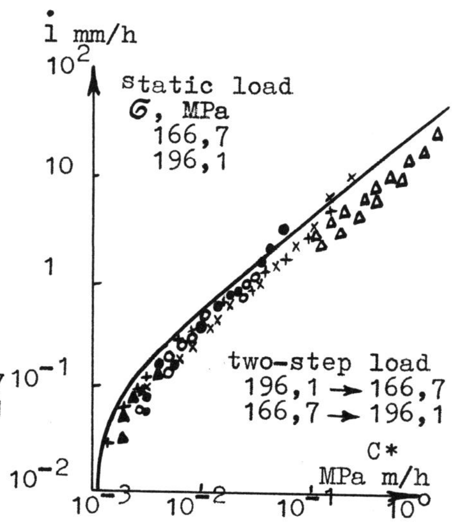

Figure 8 $\dot{l}$ vs C* for 316SS from ref.(13) and eq.(14)