THE EVOLUTION OF A SLENDER CRACK UNDER DIFFUSION MASS TRANSFER CONDITIONS

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Basic concepts and results of the theory of equilibrium cracks reveal elastic media, i.e. materials obeying Hooke's law up to fracture. So for an inelastic medium in the equlibrium equation, the transfer equations of corresponding defects should be added and the relation between the flows of defects and inelastic strain of the material should also be given. In the case of a slender plane crack, infinitely long in one direction, the inelastic deformation will change the crack profile h (x). As a result of stress relaxation due to inelastic processes, the functions ρ and h depend parametrically on time (1,2). The relation between these functions is governed by the choice of a special model of inelastic medium. In particular for an elastic medium this relation is presented in the well-known form:

$$\rho(x) = \frac{dh}{dx} \qquad (1)$$

Here crack evolution is studied in cases when stress relaxation is due to material self-diffusion, i.e. by a vacancies diffusion process. The investigation is based on crack equilibrium relation analysis as well as the relation between ρ and h governed by a diffusion mechanism of the crack recovery. The initial stage of a slender crack tip's diffusive blunting transforming the crack into a elongated cavity is considered.

Thus a slender equlibrium crack in a solid containing point defects (vacancies, impurity atoms etc.) is studied. It is supposed that the crack appeared and reached equilibrium size during a time period considerably less than the characteristic diffusion time. Elastic stresses and strains arise in crystal after the crack appearance, their gradients are the driving force of the point defects diffusion process (for the sake of definiteness we will speak about vacancies further). As a result the tip crack blunting can take place, since in the vicinity of the tip the most intensive diffusion flows occur.

The system of equations which describes self-consistently the crack evolution is formulated here. This system consists of the equation of equilibrium

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$$\int_{-L}^{L} \frac{\rho(\zeta)\,d\zeta}{\zeta - x} = \frac{2\pi(1-\nu)}{G}\left[P(x) - S(x)\right], \tag{2}$$

that of the crack evolution

$$\frac{\partial h}{\partial t} = -\omega\left[J_y(x,+0) - J_y(x,-0)\right], \tag{3}$$

and in the relationship for the chemical potential

$$\mu = \mu_0 + T\ln c + \omega K \varepsilon_{ii} \tag{4}$$

and for the vacancy flow

$$\bar{J} = -\frac{c\mathcal{D}_v}{T}\nabla\mu = -\mathcal{D}_v \nabla c - \frac{c\omega K \mathcal{D}_v}{T}\nabla\varepsilon_{ii} \tag{5}$$

Here G and ν are the shear modulus and the Poisson ratio, respectively; ρ is the density of the crack-forming dislocations, $S(x)$ is the cohesion force, $P(x)$ is the external load, ω is the volume per atom, c is the vacancy concentration, D_v is the diffusion coefficient, K is the all-directional compression, T is the temperature, ε_{ii} is the spur of the deformation tensor, the detailed discussion of these quantities being described in (1,2). The boundary conditions on the crack surface are also discussed; the general form of these is:

$$\sigma_y = \eta\,(\mu_1 - \mu_2) \tag{6}$$

where η is the permeability coefficient of the crack.

The main relationships are obtained by the solution of the equation system; these describe the evolution of the crack tip and its profile

$$h(x,t) = \frac{h_0}{d_0^{3/2}}\left(L_0 - x - \sqrt{\mathcal{D}^* t}\right)^{1/2}\left(L_0 - x + 0{,}5\sqrt{\mathcal{D}^* t}\right)$$

$$x < L_0 - \sqrt{\mathcal{D}^* t}, \quad L(t) = L_0 - \sqrt{\mathcal{D}^* t}, \quad \mathcal{D}^* = 2\mathcal{D}_v \frac{\omega^2 c K}{T}, \tag{7}$$

here L_0 is the initial length of the crack and d_0 the dimension of its tip.

The crack tip at first blunts quickly and then the rate of blunting decreases. Time of the crack tip blunting t_0, i.e. time of its transforming into the elongated cavity, may be estimated from the above expressions as the time of the crack tip shifting at the distance d_0. From that we obtain

$$t_0 \approx \frac{d_0^2}{\mathcal{D}^*} = d_0^2/2\mathcal{D}\,\frac{\omega K}{T} \qquad (8)$$

This value is considerably (by two to three orders) less than the time $d_0^2/\mathcal{D}_{0v}$ obtained from elementary considerations about the diffusion blunting of the cavity of size d_0. The presence in (8) of the factor $\omega K/T$ shows the governing role of vacancy diffusion under the influence of the field of elastic stresses generated by the crack, in comparison with the diffusion due to the concentration gradient. The contribution of a second flow term in the expression for the flow in the region of interest to us exceeds the contribution of the first term by a factor $\omega K/T$, therefore neglecting the term proportional to ∇c in the formula for the flow is justified. According to numerical estimations the value t_0 varies over a wide range from a thousandth of a second at temperatures near the melting point to 10^6 sec at $T \ll T_{pl}$

The diffusion mechanism of the crack blunting studied in the present work may be one of the basic reliable interpretations of such phenomena as brittle-ductile transition, super plasticity, cracking in solids etc.

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