THE DISTRIBUTION OF STRESS INTENSITY FACTORS FOR A LARGE CRACK DISTRIBUTED BY A SMALL CRACK

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The influence of a small, statistically distributed crack on the stress intensity-factors $K_{I,II}$ of a macrocrack is investigated. The K-values for the two interacting cracks are calculated approximatively following the concept by Gross (2). Using the Monte-Carlo-method frequency distributions of the K_I, K_{II} were calculated. These can be used for safety statements of cast steel or resin structures.

INTRODUCTION

The main emphasis of this paper is laid on some practical conclusions and not on theoretical aspects of fracture mechanics. In the case of cast- steel and -resin structures flaws arrise during the production process. Specifically in the region of notches with high stress concentrations these flaws may grow during cycling loading. Numerous cases are reported elsewhere on the failure of cast structures by a sudden instable crack after a certain number of load cycles. From the technical point of view it is now very important to give a proper estimation for the safety of such a structure. Specifically the evaluation of the stress intensity factors is accompanied by a lot of uncertainities as the position and the dimension of the flaw but also the influence of neighboring microdefects. This study describes a procedure and demonstrates some results of the influence of a statistically distributed adjacent microcrack in the sense of a frequency distribution of the K_I, K_{II} values of a

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flaw simulated by a line crack.

SOLUTION METHOD

After substitution of the real flaw system by two cracks, a"long" crack interacting with a "short" crack (microcrack), see Figure. 1, the first problem is the calculation of the stress-intensity factors K_I, K_{II} - specially for the long crack. In the literature of the last 20 years numerous papers were published reporting stress-intensity-factors. In this contribution only papers are cited both treating two arbitrary cracks and presenting algorithms for calculating the K_I, K_{II} - values.

A standard method is based on the introduction of two complex stress functions with the Kolosov-formulas. The papers mentioned below follow this concept with different ways of numerical realisation. Isida published 1970 a general solution method in (1). An "explicit" algorithm was communicated by Gross 1982 in (2) approximating the kernel of the integral equation by Chebyshev polynominals. Observing some geometrically defined restrictions the equations published by Gross are used in this paper because of the easy applicability. A lot of papers were presented also by Y.Z. Chen solving 1984 the problem under discussion in (3) with testcases for colinear cracks. W.Günther published 1985 in (4) a similar solution like that solution of Gross with the same restrictions with respect to the asymptotic solution but only for two cracks. Rubinstein recently showed 1986 in (5) a very general solution for interacting microdefects - an inclusion, a hole or a microcrack - using the same approach as Gross and Günther. Nemat-Nasser et al presented 1987 an effective calculation procedure based on an earlier paper for the difficult case when a large crack interacts with a system of small cracks which are situated very close to its tip in (6) resp. in (7). The authors of this paper might have used this concept because of its generality if they had been aware of its availability in 1984 at the beginning of this project.

An other concept than that mentioned above using Green's function for a force dipole was applied by M.Kachanov and colleagues in a set of papers, e.g. in (8) solving specially the influence of a series of colinear cracks on a macrocrack. Following a similar procedure Rose presented 1986 in (9) very simple equations for K_I, K_{II} of a semiinfinite crack disturbed by a small crack for $\sqrt{x_1^2 + y_1^2}/a \geq 5/3$, see Figure 1, with an error smaller then 5 % . Not directly applicable for this project but generally efficient is the boundary integral method, see e.g. Kuhn (10) or Ang (11).

As mentioned above the method of Gross (2) was followed in this paper because of its ease for programming, its accuracy and its clear definition of invalidity - the crack tips must not be situated in the intersection area of the two circles in Figure 1. The distribution of K_I/K_0, K_{II}/K_0 for the two crack tips L, R of the long crack under an angle γ_2 with crack length A = 2 mm influenced by a short crack under an angle γ_1 with a crack length a = 1 mm in a sheet under uniaxial tension σ in y-direction are shown in Figure 2 as a sample.

Now the long crack is assumed to be a "deterministic" crack with a given length A in the x-direction, $\gamma_2 = 0$. The coordinates x_1, y_1, the length of the small crack a and the angle are assumed to be statistically distributed. The general concept of probabilistic fracture mechanics as outlined by Harris and Lim in (12) could be followed. In this paper a Monte-Carlo-simulation is applied using the following proposals by Schueller, (13) for the different distributions: a, x_1, y_1 possess an exponential probability density of the type

$$f(\xi) = \alpha e^{-\alpha\xi} \quad (0 \leq \xi \leq \infty, \alpha = \text{const} \geq 0). \tag{1}$$

The probability distribution is $F(\xi) = 1 - e^{-\alpha\xi}$ with a mean value $\bar{a}$, $\bar{x}$, $\bar{y}$ and a standard deviation $\frac{1}{\alpha}$. The angle γ_1 is assumed to be uniformely distributed.

The Monte-Carlo-method starts with a random number U, $0 \leq U \leq 1$, which is assumed as the probability of an event, e.g. the crack length a. This leads to the following equations

$$U = 1 - e^{-a/\bar{a}}, \quad a = -\bar{a} \ln (1 - U), \tag{2}$$

$$x_1 = x_0 - \bar{x} \ln (1 - u), \quad y = \pm \bar{y} \ln (1 - U). \tag{3}$$

x_0 is a defined minimum distance of the mid point of the small crack. For γ follows $\gamma = \pi . U$. Different combinations for $A/\bar{a}$, $\bar{x}$, $\bar{y}$ are studied by taking 2000 different random numbers U per case. The frequency distributions of K_I, K_{II} of the long and the short crack are shown for a specific set of $A/\bar{a}$, $\bar{x}$, $\bar{y}$ in

Figure 3 as a demonstrative sample.

CONCLUSION

For all the sets of mean values A/a, x, y investigated a maximum increase of the stress intensity factor K_I for the long crack of about 15 % for 2000 variations by one dataset occured. Only in one of 100 analysed cases the K_I value is 5 % higher than the mean value. Here the short crack is nearly as long as the long crack. Unfortunately the method of Gross calculates too small values for K_I, K_{II} for this situation. About 5 % of the investigated cases lead to a "pathologic" crack configuration with the crack tips in the "forbidden" area. These cracks were skipped in the analysis - but now an improved procedure is under elaboration allowing also these cases.

SYMBOLS USED

σ = constant stress in y-direction (N/mm^2)

$K_{I,II}$ = stress intensity factors ($N/mm^{3/2}$)

K_0 = reference stress intensity factor $\sigma\sqrt{\pi a}$ ($N/mm^{3/2}$)

a, A = crack length of the short and the long crack (mm)

x_1, y_1 = midpoint coordinates of the short crack (mm)

x_0 = minimum distance of the mid point of the small crack (mm), in this case a.

U = assumed random number

a, x, y= mean values of a, x_1, y_1

γ = inclination of the crack with respect to the x axis

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ILLUSTRATIONS

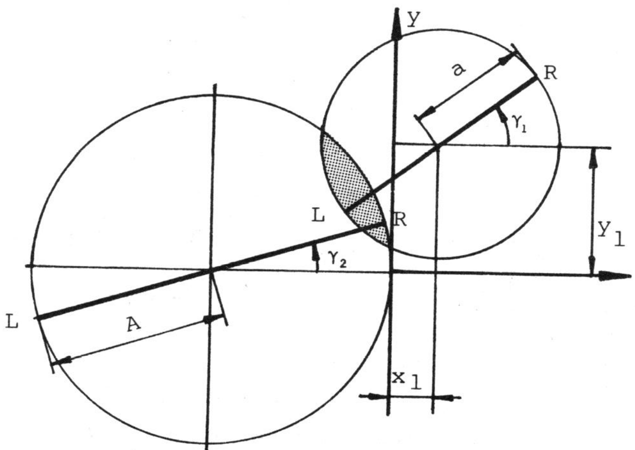

Figure 1 Crack Geometry

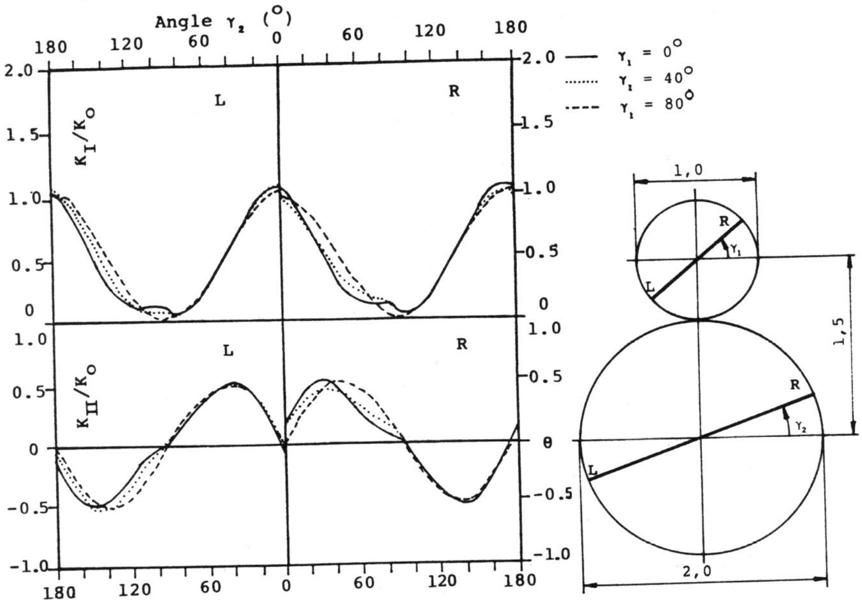

Figure 2 Stress intensity factor for the long crack

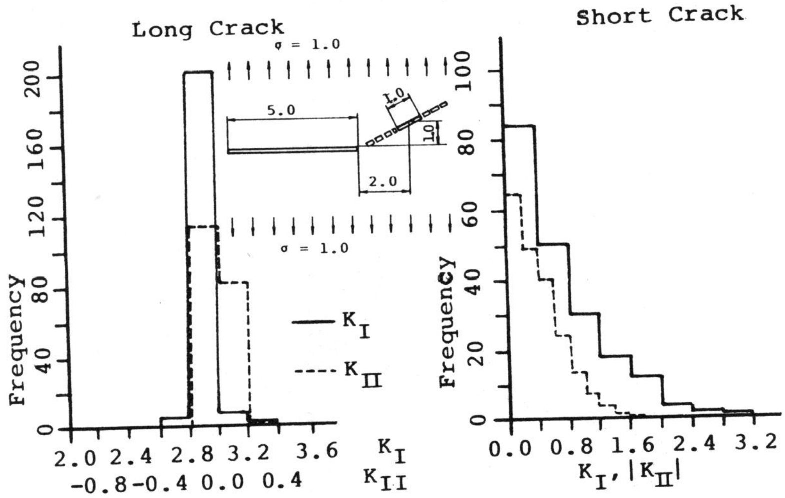

Figure 3 Frequency distribution of K_I, K_{II} for the long and the short crack