STUDY OF FRACTURE INSTABILITY BY THE LOCAL AND GLOBAL MINIMUM VALUES OF STRAIN ENERGY DENSITY

E.E. Gdoutos* and C.G. Thireos**

The strain energy density theory is used to study the fracture instability of a cracked body. The relative distance l between the local and global minimum values of the strain energy density function is introduced as a measure of fracture instability of the body. It is shown that the smaller the distance l, the more stable the system. Numerical results are obtained for a cracked plate with two symmetrical notches loaded by a uniform stress perpendicular to the crack axis.

INTRODUCTION

The problem of energetic stability of crack propagation has extensively been studied by Gurney and coworkers (1,2) within the framework of the global energy balance approach. Stable crack growth occurs when the rate of energy release is equal to the rate of the energy that is required for crack propagation. If more energy is released than that required for propagation crack growth becomes unstable.

* School of Engineering, Democritus University of Thrace, GR-671 00 Xanthi, Greece.

** Dept. of Theoretical and Applied Mechanics, The National Technical University of Athens, 5 Heroes of Polytechnion Avenue, GR-157 73 Athens, Greece.

Stability of cracking depends on the geometry of the testpiece, the material properties, the type and rate of loading and the environmental conditions. For elastic behavior stable cracking occurs when the rate of change with respect to crack length of the material resistance to crack growth is greater than a stability factor that depends only on the geometry of the testpiece. Values of the geometry stability factors for a host of geometrical configurations and loading conditions under load or displacement controlled crack propagation can be found in reference (3).

In the present work the problem of fracture instability of a cracked solid is addressed from a different point of approach based on the strain energy density theory (4). A length parameter defined by the relative distance between the local and global minima of the strain energy density function dW/dV is introduced and associated with the system stability. The case of a cracked plate with two symmetrical notches subjected to a monotonically rising uniform stress perpendicular to the crack plane is used as an example problem to demonstrate the meaning of the length parameter in terms of the system stability.

SYSTEM INSTABILITY

The strain energy density function dW/dV in a solid body subjected to a system of external forces is defined by

$$\frac{dW}{dV} = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \tag{1}$$

where σ_{ij} and ε_{ij} are the components of the stress and strain tensors.

Let O_i be an arbitrary point in the solid at which a local Cartesian coordinate system is attached and consider the fluctuation of dW/dV along the circumference of a circle of radius r_0 centered at O_i. r_0 represents the radius of the core region and reflects a material microstructure parameter. Initiation of fracture from the point O_i takes along the direction of minimum value $(dW/dV)_{min}$ when $(dW/dV)_{min}$ becomes equal to its critical value. In a continuum solid fracture initiates from the point L with the maximum of the local minimum values of dW/dV. When all points of the solid are referred to the same coordinate system the minimum value of dW/dV at point G is the global minimum of dW/dV in the solid. For fast unstable fracture originating from point L the fracture trajectory $LA_1A_2G \ldots A_m$ can be determined from the calculation of the fracture angles for a wide range of radius vectors starting from L (Figure 1). The fracture trajectory would pass from the point G of global minimum value of dW/dV.

Since fracture is dictated by the local minima of dW/dV it would possibly be arrested at point G at which the global minimum value of dW/dV occurs. The length l=(LG) of the arc along the fracture trajectory between L and G should play a role in the fracture stability of the continuum body. The smaller the distance l the more localized the fracture around the point L of fracture initiation. The value of l determines the length of the unstable fracture path and is intimately related to the stability of the body once fracture starts from point L.

APPLICATION

The case of a rectangular panel of width 2b=10.16 cm and height 2h=20.32 cm with two symmetrical notches is considered (Figure 2). Refer the panel to the global Cartesian system Oxy. The notches are circular arcs of varying radius R whose center lies on the Ox axis and intersect the specimen boundary at a distance equal to 1 cm from the Ox axis. The plate contains a crack of length 2a along the x-axis and it is loaded by a stress σ along the Oy direction.

The finite element program PAPST was used for the elastic-plastic stress analysis of the plate. Point L at which the maximum of the local minima of dW/dV occurs is at the intersection of the boundary of the circular core region of radius $r_0=2.54 X 10^{-2}$ cm with the Ox axis. From the stress analysis of the plate the global minimum of dW/dV along the crack ligament, G, is obtained. The distance l=(LG) characterizes the system stability.

The contour lines of the effective stress σ_{eff} for an applied stress σ=204.23 MPa when the yield stress of the material in tension σ_{ys}=295.77 MPa are shown in figure 3. The elastic-plastic boundary is shown by the boldfaced curve. Observe that plastic yield is concentrated around the crack tip and near the notch root. From the stress analysis it was shown that for a monotonically increasing applied stress which results to plastic deformation on the plate the values of dW/dV at points L and G increase continuously while the length parameter l remains constant. The variation of the stability parameter l with the notch radius R for two values of the half crack length equal to a_1 = 1.016 cm and a_2 = 2.032 cm is shown in figure 4.

CONCLUDING REMARKS

A length parameter l defined by the distance between the maximum of the local minima of dW/dV, L, and the global minimum of dW/dV, G, was introduced to characterize the fracture instability of a mechanical system. Fracture initiates from point L and as point G is approached the arrest mechanism is activated. The smaller the distance l=(LG), the more stable the system. When point G lies on the plate boundary the system fractures completely and the situation is unstable. Such a case is the Griffith crack in an infinite plate subjected to uniform stress for which G is at infinity. Thus, when the crack starts to propagate it reaches the point G which corresponds to an unstable situation.

From the analysis of the cracked plate with two symmetrical notches it was shown that the length parameter l is independent on the level of the applied stress but depends strongly on the geometrical configuration of the plate. Large cracks and small notch radii result to more stable situations.

The value of l which dictates the stability of a structural system depends on the interaction of defects with loading, geometry and material type. It serves as a parameter which should be determined in structural design against fracture. For optimal design l should be kept as small as possible. The values of l for the various members of a multi-component structure must be as close to one another as possible. This would avoid premature fracture of a single structural member resulting to failure of the whole structure.

REFERENCES

(1) Gurney, C. and Ngan, K.M., Proc. Royal Soc. London, Vol. A325, 1971, pp. 207-222.

(2) Gurney, C. and Mai, Y.W., Engng. Fract. Mech., Vol. 4, 1972, pp. 853-863.

(3) Atkins, A.G., and Mai, Y.W., "Elastic and Plastic Fracture", Ellis Horwood Limited, England, 1985.

(4) Sih, G.C., Mechanics of Fracture, Introductory chapters, Vol. I-VII, Edited by G.C. Sih, Martinus Nijhoff, The Hague, 1972-1982.

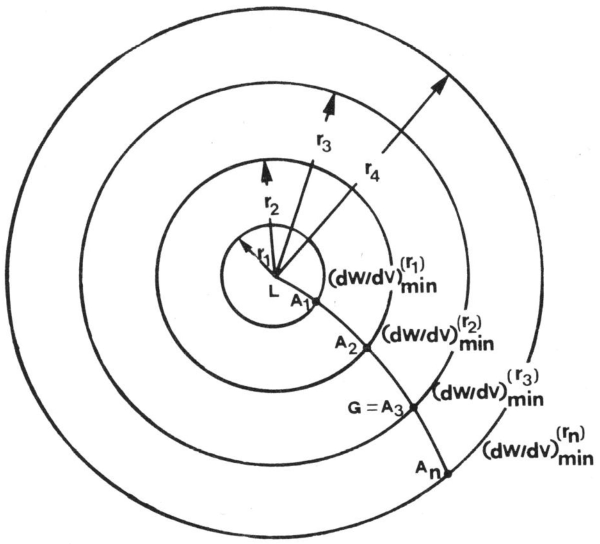

Figure 1 Fracture trajectory starting from point L of maximum of the local minima of dW/dV and passing from point G of the global minimum of dW/dV.

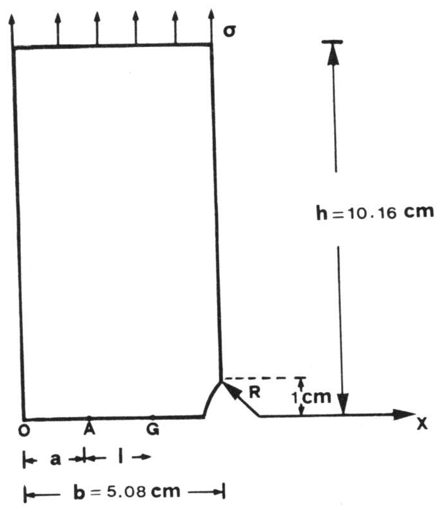

Figure 2 Geometry of the notched cracked plate.

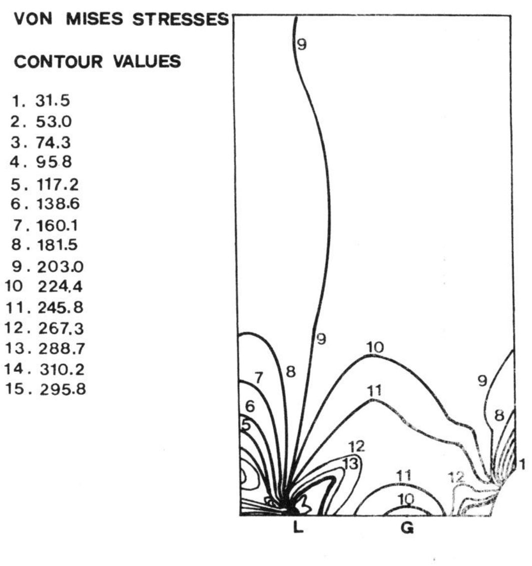

Figure 3 Contours of the effective stress σ_{eff} for R= 2.159 cm and 2a=2.032 cm and σ=204.23 MPa. The bold-faced curves represent the elastic-plastic boundaries.

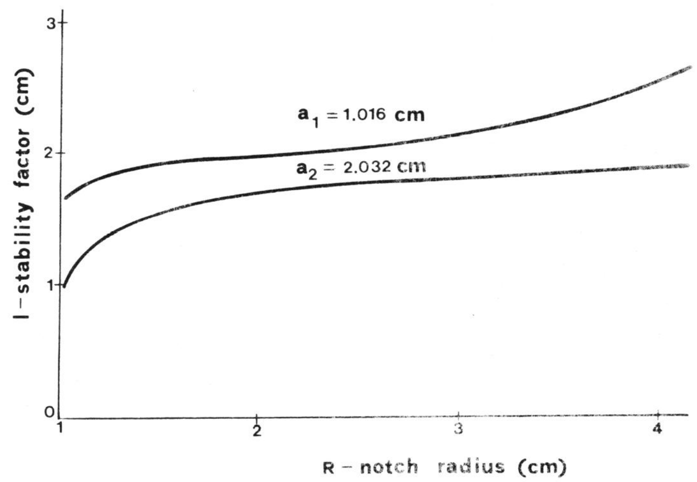

Figure 4 Variation of the length parameter l with R for a_1=1.016 cm and a_2=2.032 cm.