STRENGTH AND DURABILITY EVALUATION OF STRUCTURAL ELEMENTS USING FRACTURE MECHANICS APPROACHES

A.Ye.Andreykiv*

A general approach to calculations of brittle strength and durability of structural elements in terms of their resistance to crack nucleation and rpropagation is suggested. It is based on the previous theoretical and experimental results obtained by the author [1,2,3,4] .

Application of fracture mechanics methods and approaches to reliability and life assessment of engineering constructions incorporates, as commonly known, the following stages: investigation of in-service conditions of structural elements and shot-pinning of crack-like defects using non-destructive control techniques; the choice of local fracture criterion, corresponding to the fracture conditions of the given material; the choice of fatigue crack nucleation and subcritical growth model; analytical determination of principal parameters of elastic and elastoplastic state of bodies with cracks; determination of crack resistance characteristics of materials; life assessment of structural elements or determination of tolerable dimentions of defects.

The first stage is based on laboratory and in-service observations of the material performance under loading and depends on the qualification of enginee - ring stuff and availability of non-destructive control

*Karpenko Physico-Mechanical Institute of the Ukrainian SSR Academy of Sciences, Lvov, USSR

methods for shot-pinning of defects. In this respect acoustic emission techniques are of special interest.

Local Fracture Criterion. The second stage of calculations is the most important and yet the most complicated one. It practically incorporates all principal concepts of brittle fracture mechanics. Here, universal criterions are of special interest, since they can be successfully applied to describe fracture both of brittle and plastic materials with large or small cracks. The following criterial equation is proposed basing on classical deformational criterion and the theory of cracks of limiting equilibrium [1]

$$K_I^4 K_{Ic}^{-4} + \varepsilon_{nom}^2 \varepsilon_c^{-2} = 1 \qquad \text{.................(1)}$$

This criterion was repeatedly verified using experimental results and calculations obtained due to alternative approaches. Fig. 1 presents calculated (due to criterion (1)) and experimental data of tensile testing of cylindrical specimens with a round crack made of 30KhGSA steel ($\sigma_{0.2}$ = 1540, $\sigma_{\text{в}}$ = 1760, ψ = 0.48, E_1/E = = 0.014, ε_c = 0.616, K_{Ic} = 71).

Formulation of Nucleation and Subcritical Fatigue Crack Growth Model. The problems arising under durable life assessment calculations of structural elements are inadequately studied and analytically developed. The following calculational models are proposed in the present paper for their solution. A body weakened by a stress concentrator in the form of a smooth crevice, symmetrical to plane $z = 0$ is considered. The body is subjected to pulse tension normal to plane $z = 0$. The number of cycles $N = N^{(1)}$ corresponding to the moment of macrocrack formation at the stress concentrator tip (in the prefracture zone) is to be determined. Let deformation ε in the prefracture zone $(R_p - R_0)$ be an invariant fatigue fracture characteristic. Considering the crack propagation rate normal to the contour, we obtain the following differential equation relatively to the mobile contour $R(N, \alpha)$

$$\Phi(\lambda)\left[1 + R^{-2}\left(\frac{\partial R}{\partial \alpha}\right)^2\right]\frac{\partial R}{\partial N} = 1, \quad \left(\lambda = 1 - \sqrt{\varepsilon/\varepsilon_{fc}}\right) \qquad (2)$$

under initial condition $R(0, \alpha) = R_0(\alpha)$.

For ideally elastoplastic material, satisfying Tresk-Saint-Venant plasticity condition for ε determination, we obtain [2]

$$\varepsilon = \varepsilon_0 + (\varepsilon_{fc} K_I^2 K_{Ifc}^{-2} - \varepsilon_0)(R_p - R_0)^{-1}(R - R_0) \qquad (3)$$

The following correlation is proposed for determination of characteristic function of fatigue fracture $\Phi(\lambda)$

$$\Phi(\lambda) = A_0\left[(\lambda_0)^{n_0}(\lambda_0 - \lambda)^{-n_0} - 1\right] \qquad (4)$$

Values A_0, λ_0, n_0 are determined experimentally. In Fig. 2 a curve for a plate with two symmetrical concentrators is plotted according to expressions (4), (5) and (6). Dots correspond to experimental data, which are in good agreement with the calculated curve.

After macrocrack nucleation the task is to determine the number of cycles of its growth up to critical size. Basing on geometrical (Fig. 3) and physical [2] preconditions, determination of the fatigue crack growth kinetics is described by the following set of equations

$$\Phi(\lambda)\left|\frac{\partial \vec{r}}{\partial N}\right| = 1 \quad (\lambda = 1 - K_I/K_{Ic}), \tag{5}$$

$$\frac{\partial \vec{r}}{\partial N} \cdot \frac{\partial \vec{r}}{\partial \alpha} \cdot \frac{\partial^2 \vec{r}}{\partial \alpha^2} - \left|\frac{\partial \vec{r}}{\partial N}\right| \cdot \left|\frac{\partial \vec{r}}{\partial \alpha} \cdot \frac{\partial^2 \vec{r}}{\partial \alpha^2}\right| \sin\beta = 0, \qquad \vec{r} = \vec{r}_0(0,\alpha).$$

The angle β is described by relation

$$\frac{\partial}{\partial \beta}\left[\cos^2(\beta/2)\left(K_I\cos(\beta/2) - 3K_{II}\sin(\beta/2)\right)^2 / \left(1 - \eta \sin 2\gamma_*\right)\right] = 0 \tag{6}$$

Starting from its initial configuration $\vec{r}_0(0,\alpha)$, the crack grows up to its critical size $\vec{r}_* = \vec{r}(N^{(2)},\alpha)$ determined from expression

$$\frac{\cos^2(\beta/2)}{1 - \eta \sin 2\gamma_*}\left(K_I(\vec{r}_*)\cos(\beta/2) - 3K_{II}(\vec{r}_*)\sin(\beta/2)\right)^2 = 0.2222 K_{fc}^2. \tag{7}$$

As an example, basing on differential equations (5), growth kinetics of an elliptical fatigue crack under pulse tension of infinite body of 40Kh steel (K_{fc} =43.4) is determined. Crack surfaces are subjected to cyclic loading P_{max} = 172.6. Crack growth curves 1,2,3,4 in Fig. 4 correspond to N_1 = 0, N_2 = 128·10^3, N_3 = 193·10^3, $N_4 = N^{(2)}$ = 212·10^3 respectively.

Interpolational Method of K_I, K_{II}, K_{III} Determination. Multiparametric Problems. The subsequent stages of strength and life assessment of structural elements in terms of brittle fracture mechanics are connected with determination of stress-strain state parameters of bodies with cracks. The following method is proposed. Let us consider a body containing a system m_1 of cracks. Suppose, the size of the body and distances between cracks are determined by m_2 linear parameters C_1, C_2,..., C_{m_2} and configuration of each i-th crack (i = 1,2,...,m) - by $b_i^{(j)}$ values (j = 1,2,...,n_i). Assuming $\lambda_{il} = b_i^{(1)}/C_l < 1$ (l = 1,2, ...,m_2), $\lambda_n^{(q)} = b_q^{(n)}/C_1 < 1$ (n = 1,2,..., n_q; q = 1,2, ...,m_1; $q \neq i$) we obtain [4] approximate expressions for $K_S^{(i)}$ (S = I, II, III) determination

$$K_S^{(i)} = K_{S0}^{(i)}\psi_S^{(i)}(\lambda_{i1},\ldots,\lambda_{im_2},\lambda_1^{(q)},\ldots,\lambda_{n_q}^{(q)}),$$
$$\psi_S^{(i)} \approx 1 - m_2 - \sum_{q=1(\neq i)}^{m_1} n_q + \sum_{l=1}^{m_2}\psi_S^{(i)}(\lambda_{il},0,\ldots,0) + \sum_{q=1(\neq i)}^{m_1}\sum_{n=1}^{n_q}\psi_S^{(i)}(\lambda_n^{(q)},0,\ldots,0), \tag{8}$$

where $\psi_S^{(i)}$ - continuous and continuously differentiated functions $K_{S0}^{(i)} = K_S^{(i)}$, under $C_1 \to \infty, \ldots, C_{m_2} \to \infty$. As follows from (8), solution of multiparametric problem consists in solution of corresponding monoparametric problems.

Monoparametric Problems. If in the above discussed problem we assume $m_1 = m_2 = n_1 = 1$, it becomes monoparametric. Then, the size of the body and that of the crack are expressed as C_1 and $b_1^{(1)}$ respectively, and $\lambda_{11} = b_1^{(1)}/C_1 < 1$. In this case K_S is suggested to be determined by the following interpolational relations

$$K_S = \sigma_{S\,nom}\,\alpha_S, \quad \alpha_S = \alpha_S^{o}\alpha_S^{(1)}\left[(\alpha_S^{(0)})^2 + (\alpha_S^{(1)})^2\right]^{-1/2}; \tag{9}$$

$$\sigma_{S\,nom} = \left(\sqrt{\sigma_{S\,nom}^{(0)}} + \sqrt{\sigma_{S\,nom}^{(1)}} - \sqrt{\sigma_{S\,nom}^{(1)}}\,\Big|_{\lambda_{11}=0}\right)^2. \tag{10}$$

The effacy and accuracy of the above method were verified basing on numerous problems. As an example, we present results of solution for a problem of tension of a rectangular plate with a central crack (Fig. 5) and of torsion of a cylinder with semielliptical surface crack (Fig. 6). Solid lines in Figures are plotted due to equation (8), dotted lines - due to exact solution [2]. In Fig. 8, the curve corresponds to expression (9) and dots are obtained experimentally. Here, $F_1 = K_I \rho^{-1}(2\pi b_1^{(1)})^{-1/2}$; $F_2 = K_I M^{-1} D^{5/2}$.

Experimental Techniques. At present, numerous experimental techniques for crack resistance determination are available. An attempt has been made to develop a universal technique for experimental determination of these parameters under static and cyclic loading based on cylindrical specimen with a round external crack. The use of the above specimen is advantageous in several respects as compared with other methods, namely: a) due to the absence of the crack contour access to the free surface of specimens fulfilment of the predetermined state (plane deformation) in the prefracture zone along the contour is provided, which corresponds to calculational Griffith-Irwin model; b) due to sircumferential position of the crack contour around the specimen cross-section, the plastic yield of material decelerated which renders the testing conditions more strict thus increasing the strength resource; c) manufacturing of cylindrical specimens, formation of round cracks [3] and experimental technique are simple and widely accessible which is very important under static and cyclic crack resistance evaluation of bars. Using the above technique, K_{IC} parameter, as well as A_0, λ_0, n_0, K_{Ifc}, of the function $\Phi(\lambda)$ are determined in (4) coordinates.

Tolerated defects sizes and critical values of external loading. The performed investigations at all stages of strength and life assessment are interconnected. There are two possible sequences. The first - life assessment of structural elements at a given loading and possible presence of defects; The second - determination of tolerated defects size at the given life and loading values. Realization fof the above sequences of problems incorporates achievements of the mentioned approaches of fracture mechanics.

The proposed approach was used in investigation of various structural elements and verified by results of in-service and laboratory experiments. Results and experimental verification of serviceability of several structural elements are given for illustration. Basing on criterial equation (1) dependency of tolerated pressure upon the size of semi-elliptical surface crack P_* was investigated for thin-walled pipe. The same dependency was determined experimentally due to testing of pipes made of 17G1SU steel. Calculated values (solid line) and experimental data (dots) are compared in Fig. 7. In Fig. 8, calculated (solid line) and experimental (dots) results were obtained for bending of rail weakened by an oval crack at the head. As evident, the proposed approach to calculation is sufficiently effective.

SYMBOLS USED

K_I, K_{II}, K_{III} = stress intensity factors for mode I, II and III cracks (MPa$\sqrt{m}$)

K_{Ic} = fracture toughness under static loading (MPa$\sqrt{m}$)

ε_c = maximum strain under tension at the moment of fracture

ε_{nom} = nomunal strain determined due to standard tension testing

$\sigma_{0.2}$ = yield strength (MPa)

σ_b = ultimate tensile strngth (MPa)

ψ = relative trnsverse narrowing

E = Young modulus (MPa/m^2)

E_1 = linear strengthening modulus (MPa/m^2)

$N^{(1)}$ = macrocrack nucleation period (cycles)

$R_p - R_o$ = the size of initial plastic zone near the concentrator (m)

ε_{fc} = maximum tensile strain under cyclic loading at the moment of fracture

P_* = fracture loading under tension and bending of cylindrical specimens (Fig. 9)

ε_o = maximum strain in the intial prefracture zone near the concentrator

K_{Ifc} = fracture toughness under cyclic loading (MPa$\sqrt{m}$)

$\vec{n}_\alpha$ = normal to the crack surface

σ_{nom} = normal stresses in netto-section (MPa)

$N^{(2)}$ = subcritical fatigue crack growth period (cycles)

τ_S = yieldstrength under shear (MPa)
$P^{*(e)}_{nom}, P^{*(T)}_{nom}$ = experimental and calculated fracture of netto-section (MPa)
$\sigma_{Inom}, \sigma_{IInom}, \sigma_{IIInom}$ = normal and maximum netto-stresses
A_1, A_2, A_3 = experimentally determined coefficients
K_c = crack resistance of the material with the investigated crack (MPa$\sqrt{m}$)
S_2 = crack area at the head (m^2)
S_3 = rail head area (m^2)

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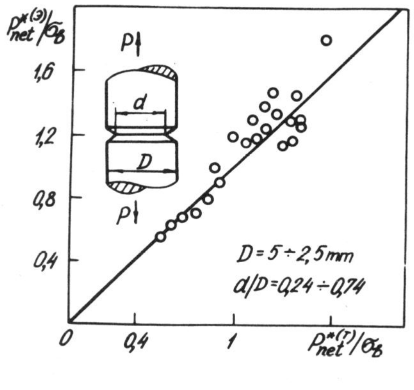


Figure 1

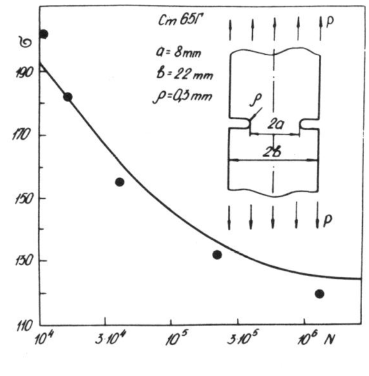
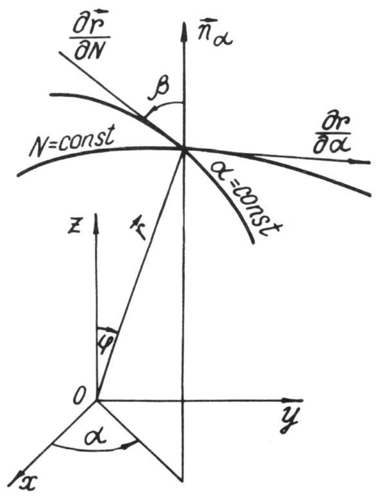


Figure 2



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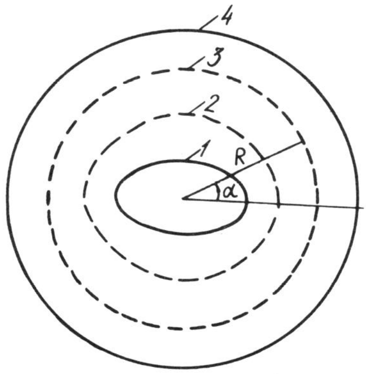

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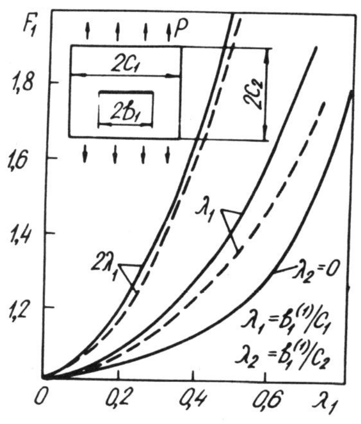
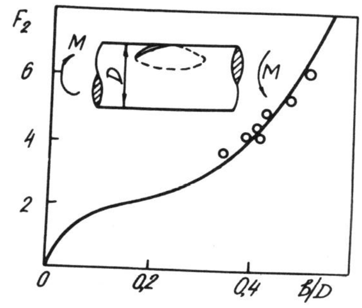


Figure 5



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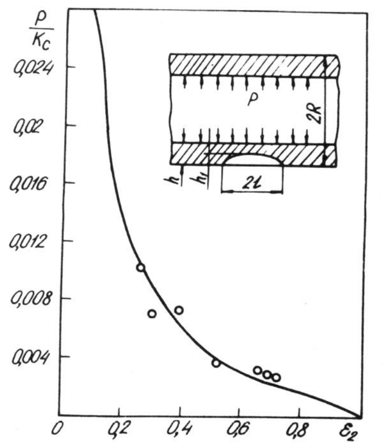

Figure 7