STRENGTH ANISOTROPY AND THE T-CRITERION

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In the present paper an application of the T-criterion of fracture is presented for the case of materials showing different strengths in tension and compression. The yield locus, necessary for the application of the T-criterion is computed by means of the Paraboloid Failure Condition (PFC). The results obtained agree well with existing experimental data.

INTRODUCTION

Formally plastic flow is connected with shear stresses and fracture with normal stresses. In energy terms the above statement is equivalent to that distortional strain energy density is responsible for yielding whilst dilatational strain energy causes brittle fracture. The first clause of the statement is globally accepted as the Mises yield condition, while a combination of both clauses constitutes the so-called T-criterion [1,2,3]. This criterion states that the distortional part, T_D, of the total strain energy density, T, developed in the material by the external loads creates a plastically deformed enclave around the crack tip defined by the

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equality $T_D=T_{D,0}$ and the remaining dilatational strain energy density, T_V, computed along the elastic-plastic boundary, causes crack initiation, provided that its amount is at least equal to a critical level, $T_{V,0}$, which is considered as a material constant. Hence, two material constants are involved in the phenomenon of failure i.e. $T_{V,0}$ and $T_{D,0}$ describing the inherent tedency of the material to fracture or yield. In addition a conservation principle concerning the mechanical energy density quantities obviously holds, i.e. $T_V+T_D=T(=T_{V,0}+T_{D,0}$ at the onset of crack propagation).

However, an idealization is incorporated in the above discussion. Really, the Mises yield condition necessitates a symmetric behaviour of the materials with respect to tension and compression stresses. But, almost all the materials are not symmetric showing various types of anisotropy. The commonest one is the strength differential effect (SDE) implying that the strength in tension is different (usually smaller) from the strength in compression. This effect is usually strong in brittle materials, weak in ductile ones and negligible in extremely ductile materials which assumingly follow the Mises yield condition.

To account this effect an advanced yield condition must replace the Mises condition. Such a suitable criterion sensitive to the SDE-anisotropy and which was shown to predict with high accuracy the yielding behaviour of a large range of materials, from highly brittle to soft and ductile polymers is the *Paraboloid Failure Condition* (PFC) [4,5]. This condition will be used in the present parer in conjuction with T-criterion to predict the behaviour of a cracked plate of a ductile material showing SDE.

STRENGTH DIFFERENTIAL EFFECT AND THE T-CRITERION

In the 3-dimensional stress space $(\sigma_1,\sigma_2,\sigma_3)$ the yield surface for a material showing the Strength Differential Effect is according to the Paraboloid Failure Condition [5]:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+2(\sigma_1+\sigma_2+\sigma_3)(\sigma_{0C}-\sigma_{0T}) = 2\sigma_{0C}\sigma_{0T} \quad (1)$$

where σ_{0C},σ_{0T} are the yield strengths of the material in uniaxial compression and tension, respectively.

Equation (1) can also be written as:

$$\frac{1+v}{3E}[\sigma_1^2+\sigma_2^2+\sigma_3^2-\sigma_1\sigma_2-\sigma_2\sigma_3-\sigma_3\sigma_1+(\sigma_1+\sigma_2+\sigma_3)(\sigma_{0C}-\sigma_{0T})] = \frac{1+v}{3E}\sigma_{0C}\sigma_{0T} \quad (2)$$

where (E,v) and the Young's modulus and Poisson's ratio of the material, respectively.

Equation (2) in case of materials not showing SDE (i.e. $\sigma_{0T}=\sigma_{0C}$) simplifies to the Mises yield condition $T_D=T_{D,0}$. It is reasonable to assume that in presence of SDE the same as above Eq.(2) represents the amount of strain energy density required for yielding, i.e.:

$$T_y = T_{y,0} = \frac{1+v}{3E}\sigma_{0C}\sigma_{0T} \quad (3)$$

where $T_{y,0}$ is a material constant.

It must be noted that the quantity T_y is not the distortional component of the total strain energy density but it acts on the material in the same, as T_D, way causing yielding. According to the assumptions of PFC, T_y is constant when evaluated along the elastic-plastic boundary in a material defined by the PFC yield condition.

The total amount, T, of strain energy density developed by the external loads is independent of the failure behaviour of the material. It depends only on the generalized Hooke's law. Consequently, it is valid:

$$T = T_D+T_V = T_y+T_f \quad (4)$$

where T_f is the remaining in the loaded material strain energy density, not consumed in plastic deformation. If fracture of the material is to be observed, it must be caused in expenses of T_f. Hence, the statement of the T-criterion suitable for materials showing the strength differential effect is obvious. Namely:

i) A crack will propagate to the direction where T_f possesses a maximum, $T_{f,m}$.

ii) The crack initiates when $T_{f,m}$ is at least equal to a critical quantity, $T_{f,0}$, which is considered as a material constant.

iii) T_f is computed along the elastic-plastic boundary as it is described by the condition $T_y = T_{y,0}$, where $T_{y,0}$ is a material constant.

Algebraically we have:

$$T_y(r,\theta)\Big|_{r=r(\theta)} = T_{y,0} \quad , \quad T_f(r(\theta),\theta)\Big|_{\theta=\theta_0} \geq T_{f,0} \tag{5}$$

Obviously, the above statement is identical to that of the original T-criterion as it was proposed for isotropic materials [2,3,6] , but with T_y in place of T_D and T_f in place of T_V. These two energy-density components (T_y and T_f), although they are not coinciding with distortional and dilatational strain energy densities, act on the material like T_D and T_V causing by definition yielding and fracture. In addition, they reduce to T_D and T_V in the case of an isotropic material.

A different approach to the same problem was presented in refs. [7,8] where the introduction of the constant term in the σ_x-stress component modified slightly the results. In addition, in ref.[7] the connection between T-criterion, PFC and physical mechanisms of ductile fracture was pointed out.

APPLICATION OF T-CRITERION IN CASE OF SDE-ANISOTROPY

Consider a thin plate of a ductile material, showing different strengths in tension, σ_{0T}, and compression, σ_{0C} and the respective ratio, R, being:

$$R = \frac{\sigma_{0C}}{\sigma_{0T}} \geq 1 \tag{6}$$

The plate contains a straight crack of length 2a which is inclined with respect to the axis of the uniaxial loading by an angle β (Drawing embedded in Fig.1). Generalized plane stress conditions are assumed.

The singular stress field around the crack-tips is given by relations of the form:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} f_x(\theta,\mu) \quad , \quad \sigma_y = \frac{K_I}{\sqrt{2\pi r}} f_y(\theta,\mu) \quad , \quad \tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} f_{xy}(\theta,\mu) \tag{7}$$

where (r,θ) are the polar coordinates centered on the crack tip, $\mu=K_{II}/K_I=\cot\beta$ and K_I, K_{II} are the mode-I and -II stress intensity factors. Functions $f_x(\theta,\mu), f_y(\theta,\mu)$ and $f_{xy}(\theta,\mu)$ are well-known and can be found in ref.[1].

The PFC condition, Eq.(1), in the case of plane stress and after introducing the parameter R, takes the form:

$$\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\tau_{xy}^2+(\sigma_x+\sigma_y)(R-1)-R = 0 \tag{8}$$

where, now the stresses $\sigma_x, \sigma_y, \tau_{xy}$ are divided by σ_{0T}.

Equation (8) is equivalent to the first of Eqs.(5) and by solving it, one can derive the function $r=r(\theta)$ which describes the elastic-plastic boundary around the crack-tip. This latter function is:

$$\rho(\theta) = \frac{r(\theta)}{a} = \frac{\sigma_{0T}^2 s^2 \sin^4\beta}{8R^2}\left\{(R-1)f_V+\sqrt{(R-1)^2 f_V^2+4Rf_D}\right\}^2 \tag{9}$$

where:

$$\left.\begin{aligned} s &= \frac{\sigma_\infty}{\sigma_{0T}} \quad , \quad f_V = f_x(\theta,\mu)+f_y(\theta,\mu) \\ f_D &= f_x^2(\theta,\mu)+f_y^2(\theta,\mu)-f_x(\theta,\mu)f_y(\theta,\mu)+3f_{xy}(\theta,\mu) \end{aligned}\right\} \tag{10}$$

Taking into consideration Eqs.(4) and (2) and that in case of isotropy the dilatational component of the strain energy dnesity is:

$$T_V = \frac{1-2\nu}{6E}(\sigma_x+\sigma_y)^2 \tag{11}$$

we obtain:

$$T_f = \frac{1-2\nu}{6E}\,\sigma_{0T}^2\{(\sigma_x+\sigma_y)^2-(\sigma_x+\sigma_y)(R-1)\} \tag{12}$$

Eqs.(9) and (12) and the second of Eqs.(5) yield:

$$T_f = \frac{\sigma_{0T}^2}{3E}\,\frac{s f_V \sin^2\beta}{\sqrt{\rho(\theta)}}\left\{\frac{(1-2\nu)s f_V \sin^2\beta}{4\sqrt{\rho(\theta)}} - \frac{1+\nu}{\sqrt{2}}(R-1)\right\} \tag{13}$$

Equation (13) for any given pair (β,R) possesses a maximum value $(T^{\beta}_{f,R})_{max}$ at the direction θ_0, towards which the crack is expected to propagate. The corresponding value of the reduced stress at infinity necessary to cause crack propagation in case of ductile materials is according to the T-criterion [9]:

$$\frac{\sigma^{\beta}_{f,R}}{\sigma^{90^0}_{f,R}} = \left[\frac{(T^{90^0}_{f,R})_{max}}{(T^{\beta}_{f,R})_{max}}\right]^{\frac{1}{2}} \tag{14}$$

Hence, from Eqs.(13) and (14) the two quantities θ_0 and $\sigma^{\beta}_{f,R}$ characterizing crack initiation in ductile materials showing SDE-anisotropy can be derived.

Really, in Fig.1 the value of the expected crack propagation direction, θ_0, is plotted versus initial crack inclination β. As it can be seen in this figure, angle θ_0 is a single function of β independent of R. It implies that in materials showing the specific anisotropy we are studying, the expected direction of crack propagation is insensitive to the different strengths in tension and compression. It is easily understood since the location of $(T^{\beta}_{f,R})_{max}$ depends on the location of the minimum value of the radius of the elastic-plastic boundary, which remains constant regardless the value of R, as it can be seen in Fig.2, where the shape of the elastic-plastic boundary (Eq.(9)) is plotted. An interesting remark derived from Fig.2 is that the size of the plastically deformed zone increases with R, the other parameters remaining constant. This conclusion seems paradoxical since increasing R corresponds to an increase of the brittleness of the material. However, the failure loci (Fig.3) according to PFC for plane stress conditions (Eq.(8)) imply that in the first quarter of the plane (σ_1,σ_2) the yield condition is fulfilled with lower stresses when R increases and, consequently, for the same stresses the extent of the plastically deformed zone increases with R.

In the next Fig.4 the value of critical stress for fracture

$\sigma^{\beta}_{f,R}$, reduced to the respective value for $\beta=90^0$ is plotted versus crack inclination, β, for four different values of R. This quantity increases as angle β decreases for all values of R. The increase is stronger for higher values of R, a conclusion conforming with the expected behaviour of materials of increasing brittleness. The above results fit well with the experimental data for two ductile materials, one with $R\approx 1$ and another one with $R\approx 1.05$ [3].

A final remark is worth at this point. From Eq.(12) or (13) it is easily concluded that T_f may become negative for certain combinations of β,R,θ. The upper limits of R for non-negative $(T^{\beta}_{f,R})_{max}$-values are plotted in Fig.5 versus Poisson's ratio, ν, for three representative β-values. From this figure it can be concluded that for ν tending to 0.5, $(T^{\beta}_{f,R})_{max}$ tends to zero and, so, fracture of such a material is impossible. In fact such a material, with $\nu=0.5$, is in a state of perfect plasticity. On the contrary materials with relatively small values of ν, being relatively brittle, can fracture in a more wide range of R-values. However, the implied by Fig.5 inability of materials with certain characteristic parameters (β,R,ν) to fail by fracture deserves its own study.

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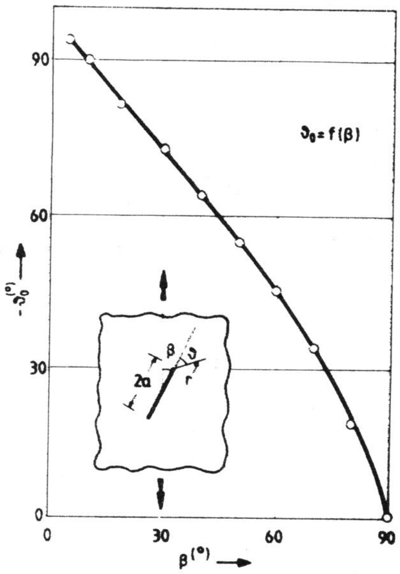


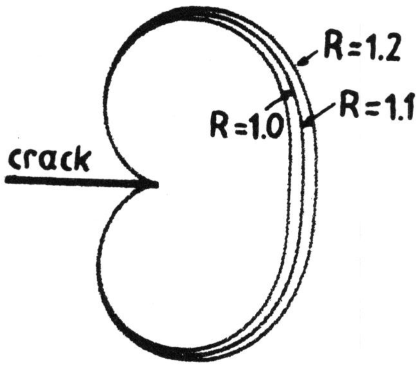
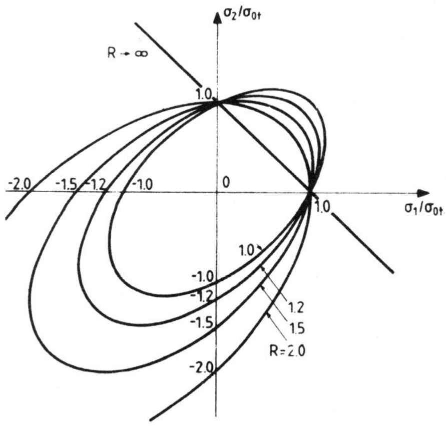


Fig.1 Expected angle, θ_0, versus crack inclination β.

Fig.2 Elastic-plastic boundary around the crack tip ($\beta=90^0$).



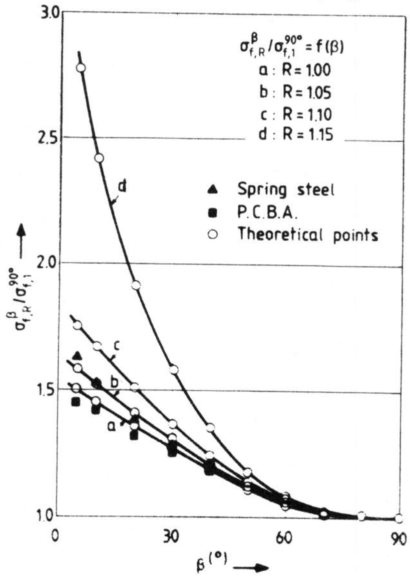


Fig.3 Plane stress failure loci according to PFC.

Fig.4 Fracture stress versus crack inclination β.

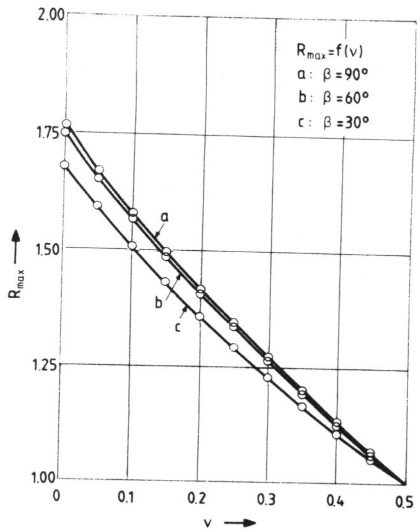


Fig.5 Upper limits of R versus Poisson's ratio for $T_f \geqq 0$.