SOME PROBLEMS OF PATH DEPENDENCY OF J INTEGRAL FOR WELDED STRUCTURES

Aleksandar Sedmak*, Stojan Sedmak**, Vladimir Ogarević**

Path dependency of Rice´s J integral applied to a bi-material body, has been analysed theoretically and numerically. It has been shown that J integral is path dependent, unless the boundary between two homogeneous parts of bi-material body is parallel to the crack. It is also shown that in general case an additional contour integral "recovers" path independency of J integral. Results of this analysis are given for a tensile panel, which has been previously tested using direct measurement of J integral. It is shown, for the shape of weldment used, that J integral values directly measured, should be corrected on the basis of the finite element results, but not necessarily.

INTRODUCTION

The experimental method for J integral direct evaluation has been applied in a number of situations, like tensile panels and pressure vessels testings, with or without weldments (1-4). The method is based on strain and CMOD measurement along an appropriate path, using the well-known property of path independency of J integral. Anyhow, as it was pointed out by Rice (5), in the case of heterogeneous material, J integral is path dependent, unless the heterogeneity is confined to x_2 direction (normal to the crack). This was recognized also in (3), where the directed measured value of J integral was understood as an approximation of the crack driving force. Besides the heterogeneity of material, the three-dimensional nature of the problem was also pointed out as the "source" of path dependency of J integral (3). Nevertheless, the aim of this paper is two-dimensional elasto-plastic analysis of J integral for bi-material body, such as welded joint, both theoretical and numerical. The theoretical approach will be based on Gurtin´s paper (6), while the finite element method will be used for the numerical analysis.

* Faculty of Mechanical Engng., University of Belgrade, Yugoslavia
** Faculty of Technology and Metallurgy, University of Belgrade, Yugoslavia

J INTEGRAL FOR BI-MATERIAL BODY

Starting point in this analysis is Rice's J integral (5), defined as the measure of strain concentration around a crack tip in a two-dimensional elastic body, without inertial and volume forces. Homogeneity is anticipated "at least in x_1 direction" (5, p. 210), where x_1 lies along the crack. Referring to Fig. 1 J integral can be formulated as follows:

$$J = \int_{\Gamma} (W\, n_1 - T^i\, u^i_{,1})\,ds$$

where W denotes the strain energy density, n_1 is the component of unit outward normal to the contour Γ along x_1 direction, T^i are the components of stress vector, $u^i_{,1}$ is the derivative of displacement vector over x_1 coordinate and ds is an arc element. Before applying J integral to the bi-material body, one can notice that its value becomes zero, if there is no crack surrounded by Γ. It can be writen now (Fig. 1):

$$J = \int_{\Gamma_1} (W\, n_1 - T^i\, u^i_{,1})\,ds$$

$$0 = \int_{\Gamma_2} (W\, n_1 - T^i\, u^i_{,1})\,ds$$

where Γ_1 is the contour in B_1 and Γ_2 its counterpart in B_2. Combination of these two equations gives:

$$J = \int_{\Gamma} (W\, n_1 - T^i\, u^i_{,1})\,ds - \int_{\ell} ([W]\, n_1 - [T^i\, u^i_{,1}])\,ds$$

where ℓ is the boundary between B_1 and B_2, called phase boundary from now on and $[f]=f^+-f^-$ is the jump function. Using Gurtin's approach (6), both path independency and physical interpretation of J integral for bi-material body can be proved, but this is beyond the scope of the paper. We only emphasize here that the second term of J integral for bi-material body eliminates the phase boundary contribution to the crack driving force. Applying this to a welded joint, one can conclude that any shape of phase boundary, not parallel to the crack, produces an additional integral expression, "recovering" path independency of Rice's J integral.

Final remarks in the theoretical considerations are due to the asymmetrical boundary conditions. In such cases, using analogous procedure, one can obtain

$$J_k = \int_{\Gamma} (W\, n_k - T^i\, u^i_{,k})\,ds - \int_{\ell} ([W]\, n_k - [T^i\, u^i_{,k}])\,ds$$

with k=1,2. This expression reduces to Gurtin's expression (6, p. 383, eqn 4) if $n_k=\delta_{k2}$. Anyhow, since asymmetrical problems are not in the scope of this paper, the paper (7) could be applied for such an analysis.

EXPERIMENTAL DATA

The experiment was performed on the tensile panel specimen (Fig. 2), using the direct measurement of J integral (4). Elastic properties of base and weld metal are taken as identical, whereas yield strengths and hardening coefficients are given at Fig. 2, indicating the undermatched welded joint. Results are given in the form of a diagramm J vs. remote stress (Fig. 3) for two specimens: the welded one, with the shape of weldment shown at Fig. 2, and the other one made of base metal alone. All other data and results are given in (4).

NUMERICAL PROCEDURE AND ANALYSIS OF THE RESULTS

An elasto-plastic analysis of J integral in welded joint has been performed using the finite element method. Toward this end the constant strain triangles are used and an extrapolation technique applied, as described in (8). One of the meshes used is shown at Fig. 4, together with representative integration path. Elasto-plastic problem is treated in the frame of deformation theory of plasticity, using the special technique for nonlinear problem solution (9).

Two specimens are analysed (welded one - WS and the one of base metal alone - BS), the later one in order to check the numerical procedure. The results for BS are given at Fig. 5, where J integral (directly measured values, Rice´s definition and expression given in this paper) is plotted against path distance from the crack. It should be noticed that Rice´s J integral and J integral for bi-material have the same value for BS, so that they are marked by one symbol only, namely ○ . The same is valid in Fig. 6, for the cases when their value is identical. As it is clear from Fig. 5, the agreement between numerical and experimental results is up to the accuracy of 95%, what enables reliable use of the finite element method.

Fig. 6 shows the results for WS, presented in the same way as at Fig. 5. As it can be seen, Rice´s J integral loses its path independency for paths crossing the phase boundary, but only for the remote stress higher than a certain value (i.e. when the properties of weld and base metal become different). It is important to notice that Rice´s J integral actually overestimates the crack driving force (represented by J integral for bi-material body). Therefore, it can be suggested that for an outer path, Rice´s J integral becomes the conservative measure of the crack driving force. It should be also mentioned that the difference between Rice´s J integral and J integral for bi-material body (at largest 15%) depends on the shape of weldment and material properties. Since the welded joint used here could be regarded as an extreme case in both sences, it is still possible to use directly measured J integral as an engineering estimation of the crack driving force.

DISCUSSION AND CONCLUSIONS

The analysis of results revealed the fact that directly measured J integral gives an overestimation of the crack driving force for the undermatched welded joint. The similar results were obtained in (10), where undermatched welded joints were numerically analysed for three different weldment geometries. It should be noticed that the results in (10) are in agreement with the theoretical analysis given here and in (6, 7), since there was no discontinuity effect of the phase boundary for the rectangular strip weldment, and the other two weldment geometries, double and single V, have shown path dependency of Rice´s J integral according to their deviation from the parallel shape to the crack. Finally, based on the results from (7), one can conclude that the same conclusion about direct measured Rice´s J integral holds for an overmatched welded joint.

ACKNOWLEDGEMENT

This paper is based on work supported by U.S.-Yugoslav Joint Fund for Scientific and Technological Cooperation, in cooperation with the NBS, Boulder, Colorado under Grant NBS (G) - 278.

REFERENCES

(1) Read, D.T., ASTM STP 791/II, 1983, pp. 199-213.

(2) Sedmak, S., Petrovski, B., Drenić, D., ECF 6 Proceedings, Amsterdam, The Netherlands, EMAS, London, 1986, pp. 999-1013.

(3) Read, D.T., Petrovski, B., IV International Fracture Mechanics Summer School Proc., Dubrovnik, Yugoslavia, 1986, pp. 119-139.

(4) Sedmak, S., Petrovski, B., Ogarević, V., Int. Conf. on Cracks in Welded Joints, Bratislava, Czechoslovakia, 1985, pp. 243-249.

(5) Rice, J.R., Fracture, Vol. II, Academic Press, New York, 1972.

(6) Gurtin, M.E., J. Elasticity, Vol. 9, 1979, pp. 186-195.

(7) Miyamoto, H., Kikuchi, M., II Int. Conf. on Numerical Methods in Fracture Mechanics, Swansea, Wales, 1980, pp. 359-370.

(8) Sedmak, A., III International Fracture Mechanics Summer School Proceedings, Arandjelovac, Yugoslavia, 1984, pp. 421-426.

(9) Berković, M. et al, VI Yugoslav Aeronautical Conference Proc., Belgrade, Yugoslavia, 1984.

(10) Bleackley, M.H., Jones, R.D., Luxmoore, A.R., ECF 6 Proceedings, Amsterdam, The Netherlands, EMAS, London, 1986, pp. 643-654.

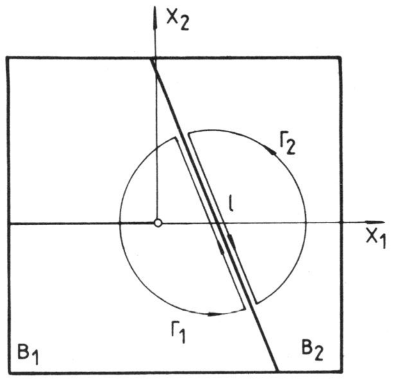


Figure 1 Bi-material body with an edge crack

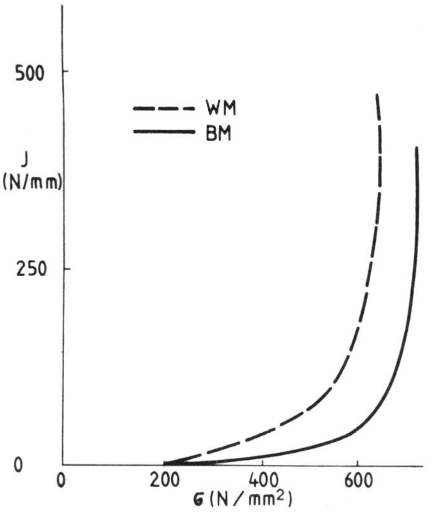
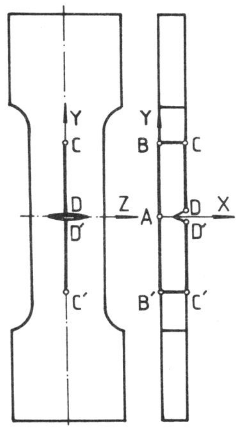
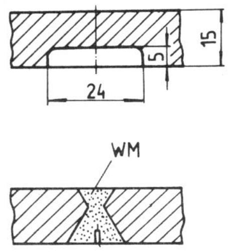


Figure 3 Directly measured J int. vs. remote stress





	Yield strength (MPa)	Hardening coefficient (-)
WM	600	0.05
BM	800	0.02

Figure 2 Specimen used for direct measurement of J integral

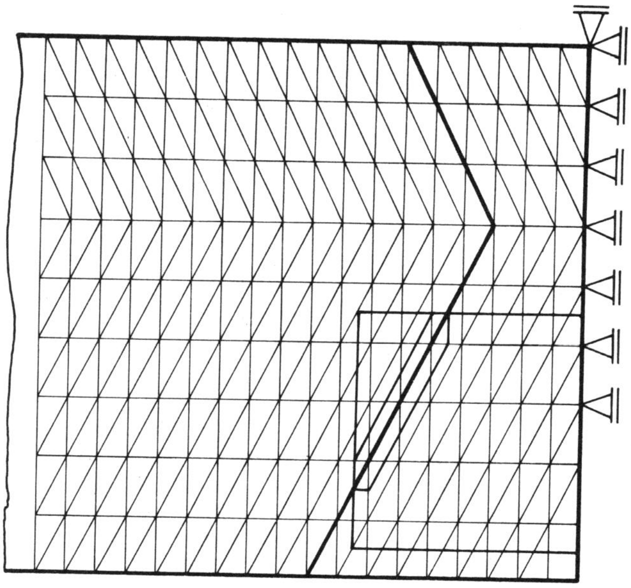
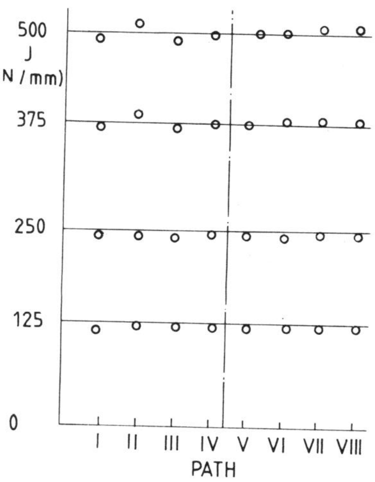
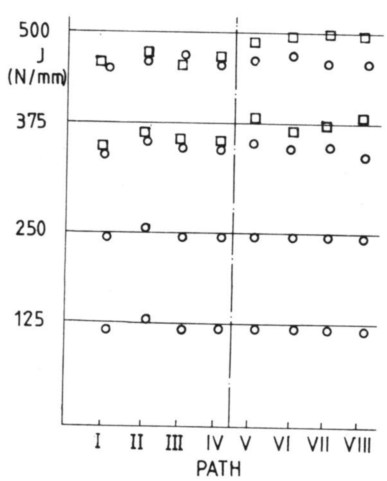


Figure 4 Finite element mesh with an integration path



— direct measurement
○ J integral

Figure 5 J integral vs. path distance for base metal



— direct measurement
○ J integral for bi-material body
□ Rice's J integral

Figure 6 J integral vs. path distance for welded specimen