PROBABILISTIC PECULIARITIES OF CERAMIC BODIES FRACTURE

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Fracture mechanics makes it possible to determine with reasonable accuracy the conditions of crack initiation and propagation and to evaluate the strength and lifetime of ceramic elements. However, application of fracture mechanics is efficient only if the crack initiation location is known. But, the latter is random for ceramics due to statistical nature of its strength and does not necessarily coincide with the maximum stress zone.

The distribution function for random variables which characterize the location and the moment of fracture initiation in a ceramic body can be obtained basing on the probability of a ceramic body fracture which can be written using the work of Weibull (1).

$$P(\sigma) = 1 - \exp\left[\int_{V,\sigma>0} \left(\frac{\sigma}{\sigma_0}\right)^m dv\right] \qquad (1)$$

Here $\sigma = \sigma(\bar{r}, t)$ are stresses at a point $\bar{r}$ at the moment t, σ_0 is the scale parameter, m is the Weibull modulus representing the dispersion in the material's ultimate strength values. The integral is estimated for the part of the body where stresses are positive.

The condition of crack initiation at the point $\bar{r}$ of a body means that only a small volume of the material in the vicinity of this point fractures whereas beyond it the integrity of the body is intact. The problem is solved by the methods of the probability theory (Groushevsky (2)). For the case when $\sigma(\bar{r},t) = f(\bar{r}) \cdot \varphi(t)$ the fracture probability density at the given point for the known stress level is presented as

$$q(\bar{r},\sigma) = \frac{f^m(\bar{r})}{\int_{V,\sigma>0} f^m(\bar{r}) \cdot dv} P'(\sigma) \qquad (2)$$

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while the probability of crack initiation at the given point is

$$q(\bar{r}) = \int_0^\infty q(\bar{r},\sigma)\,d\sigma = \frac{f^m(\bar{r})}{\int\limits_{V,\sigma>0} f^m(\bar{r})\,dv} \tag{3}$$

Hence, the probability of the body fracture at the given point is determined by the stress distribution, by the level of the material homogeneity which is characterized by the Weibull modulus and is independent of its strength (σ_0 parameter). It is clear that the fracture probability density is the largest where the stresses are maximum, however in the less loaded points $q(\bar{r}) \neq 0$. At the uniform stress state $f(\bar{r})$ = const the fracture probability density is the same in all the points and is equal to 1/V .

As an example let us consider a beam in three-point bending (Fig.1a). With the account taken of the beam symmetry, its stress state is described by the function $\sigma(x,y) = \sigma_{max} \cdot \frac{x}{L} \cdot \frac{y}{h}$. Therefore the fracture probability density

$$q(x,y) = \frac{(m+1)^2}{2} \cdot x^m \cdot y^m \tag{4}$$

$$0 \leq x \leq L\,,\ 0 \leq y \leq h$$

is represented by the surface in Fig.2. It follows from Eq.(4) that the function maximum value increases with an increase in m , whereas the probability of fracture beyond the maximum stress zone decreases. With an unrestricted increase in m the function $q(x,y)$ tends to δ -function.

Figure 1b presents a correlation between the empirical function of the coordinate distribution for cracks (symbol " o ") that caused fracture of the ceramic beam specimen tested for strength, and the theoretical function (dash line)

$$\theta(x) = \int_0^x \int_0^h q(\xi, y)\,d\xi\,dy$$

Here parameter m is determined from the scatter of the ultimate strength. Fair agreement of the results proved the possibility of describing the probabilistic regularities of fracture with distribution (2). Thus, distribution (2) which from the probabilistic standpoint determines not only the stress level at which fracture will occur but also the location of fracture, is a generalized Weibull distribution.

REFERENCES

(1) Weibull, W., Proc. Royal Swed. Inst. Eng. Res., No.153, 1939.

(2) Groushevsky, Ya.L., Problemy Prochnosti (in Russian), No.3, 1987, pp.49-52.

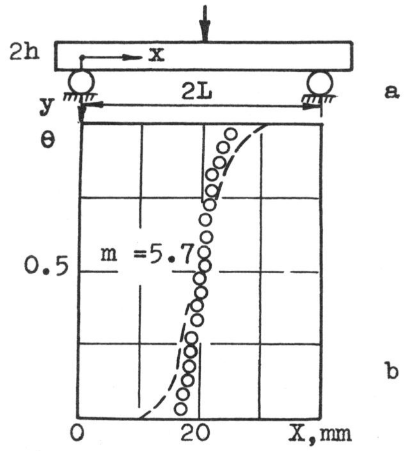

Figure 1 Crack coordinates distribution

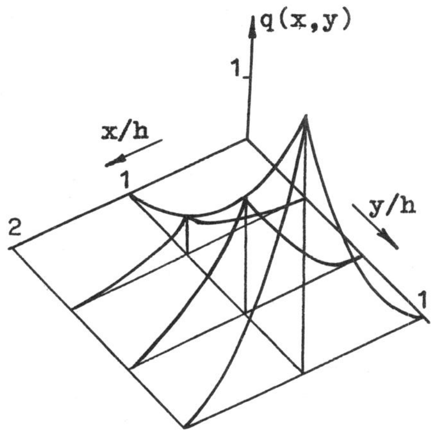

Figure 2 Fracture probability density