PROBABILISTIC FRACTURE MECHANICS - FUNDAMENTALS AND APPLICATION

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The authors deal with probabilistic fracture mechanics as an important branch of probabilistic safety assessment. A brief survey is given about current research activities and trends characterizing this interdisciplinary field. Both microscopical and macroscopical aspects of modeling have been taken into account. Some results are presented applying stochastic finite element method (SFEM) to describe such phenomena as the scatter in crack length within ceramic and heterogeneous materials.

INTRODUCTION

Probabilistic safety assessment is a less developed technical activity than the well-known types of conventional engineering (e. g. stress analysis etc.). We meet exactly the same situation also in the field of probabilistic fracture mechanics compared with the whole field of modern fracture mechanics.
What is the reason?
L. V. Konstantinov, Deputy Director General of the International Atomic Energy Agency writes in his interesting paper presented recently at the SMIRT (1): "Legal decisions and court judgements are mainly made on a deterministic basis and many judges would not accept a decision based purely on a probabilistic argument. For this reason, the licensing system of most countries is based on deterministic decisions. The situation is gradually changing, however...". The last sentence does not only characterize the application of probabilistic safety analysis for nuclear power sta-

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tions but is also typical for a new trend observed in fracture mechanics towards an incorporation of stochastic fracture concepts as an important tool for modern fracture research.

METHODS OF PROBABILISTIC FRACTURE RESEARCH

The fracture process is influenced besides the macroscopic parameters also by the local material behaviour in the near crack tip region. Random material inhomogeneities connected with the defect arrangements are shown to be very important for realistic crack and fracture evaluation. A first kind of probabilistic fracture concepts are "material stochastic fracture concepts" due to the randomness of material behaviour. The defect density, size parameters, position, and orientation of various defects are assumed to be non-deterministic quantities which can be described by means of certain kinds of statistical methods (stochastics, statistical physics) which lead to hypotheses about crack initiation, propagation and crack arrest as well. The second kind of models to be qualified as probabilistic ones arises from deterministic numerical methods of fracture mechanics which will be generalized to random field quantities or parameters (random load, random geometry etc.). The third kind of models can be called "statistical methods". They include the methods of "Mathematical Statistics" to be applied to crack and fracture problems. Here not only theoretical models can be considered but also experimental ones. (Scatter of experimental quantities such as temperature etc.) Another kind of models is due to the application of reliability analysis to fracture mechanics. An important problem in this field is to find out the probability for a fracture quantity (such as K_I) not to exceed the critical value (K_{Ic}). The gap between the models of "fracture mechanics" and "damage mechanics" should become smaller and smaller in this way. There are no sharp "boundaries" between these four kinds of probabilistic methods, as it is of course possible to combine e. g. a certain stochastic model for K_I which may be based on a probabilistic micromechanical distribution of microcracks with a random load model (e. g. seismic loading). The resulting field equations (for displacements, stresses, strains etc.) could then be solved by means of stochastic FEM or BEM. The question could be then whether J remains below J_c for a probability greater than 99 %. The last question is a typical problem of reliability engineering in structural mechanics etc.

Most of the existing probabilistic models, including stochastic finite element methods, have been proved to be more or less "formal" extensions of deterministic techniques in which input parameters are allowed to vary to a certain extent. The following opinion results from this: Probabilistic models as a rule are assumed to suffer also the same limitations as deterministic ones and could be seen only as refinements of existing techniques. But this is not the full truth, though it is valid in most cases. Besides some practical aspects, where in some cases stochastic models also could be much more powerful than deterministic ones with regard of computer time for an evaluation of fracture quantities and their variance terms, the point of view of a stochastic approach to fracture evaluation becomes very important, when collective interaction defects are predominant for crack initiation, crack propagation, crack arrest and crack branching. The reason is that many microphysical and micromechanical activation mechanisms of defects are related to temperature-dependent activation quantities. Often this fact also can be quantitatively described applying a Boltzman-like activation factor $\sim \exp(-U_A/k\hat{T})$, where U is the activation energy, K the Boltzman constant and $\hat{T}$ the thermodynamic reference temperature. The statistical nature of this factor is obvious. We do not go further into detail here, whether $\hat{T}$ should be an "equilibrium" quantity or better a certain kind of generalized non-equilibrium "temperature" characterizing the crack-tip region. This, of course, is also an important question to be answered by solid state physics of deformation and fracture. We only make a small remark about the energy U_A. U_A also contains terms of the local mechanical energy of interaction of defects with each other and with the cracks in the immediate vicinity of the crack. The authors have already reported about these micromechanical phenomena of defects at previous ICF and ECF conferences (31), (32).

Let us give another typical example for the application of fracture mechanics for an evaluation of critical fracture parameters. Fracture mechanics parameter as a rule are used to describe material resistance to fracture. One main advantage of classical fracture theory is the possibility to describe fracture toughness by means of one parameter (K_{Ic}, J_c, $CTOD_c$, C^*_c etc.). By definition these parameters should originally be geometry-independent for a constant state of stresses and strains ahead of a sharp crack. In reality, however, this is normally not exact. Many of the results for describing this "size effect" are contradictory. The systematic investigation of these kinds of size effects for real materials is one special aim of probabilistic fracture theory too. Brittle cleavage fracture which completely differs in its physical mechanism from ductile fracture, has been investigated by WALLIN (20). The influence of thickness effects on the stress intensity factor under certain circumstances can be described in terms of the statistical mechanism which is governed in the case considered by the fracture of brittle precipitates such as carbides etc. The changing probability of finding a crack nucleating weak particle is simulated in terms of the so-called weakest link model. It could of course be replaced by a more advanced one. For a sharp crack for carbide-induced brittle fracture WALLIN, SAARINEN and TÖRRÖNEN obtained for the fracture probability P_f a formula of the following kind (20)

$$P_f = 1 - exp\left\{ \left(\frac{K_I - a}{b - a} \right)^4 \right\} , \qquad (1)$$

where a and b are parameters. b can be interpreted as a "lower limiting fracture toughness", a is a thickness and temperature-dependent normalization factor. Starting from the equation above or a similar one it is possible to arrive at a generalized equation for K_I which allows for comparing fracture toughness experiments obtained from specimens of different thickness. This kind of approach can be considered as a first step provided by probabilistic fracture mechanics towards more realistic K values. Similar steps for other parameters have become known too.

STOCHASTIC FINITE ELEMENT METHOD

The successive perturbation method developed by the Japanese HISADA, NAKAGIRI and MASHIMO (17) for static and eigenvalue finite element analysis has been successfully applied to "Nuclear Fracture Mechanics" too, recently. To make full use of the forte both of numerical analysis and theoretical assessment, a bridge seems to be needed between the FEM and reliability analysis, as any structure or component is designed so that failure is not likely to occur, that is to say, the failure lies far away from the expected points. This calls for the finite element method which is able to cope with variation of parameters over a wide range. Usually an advanced first-order second-moment analysis is applied based upon the method of pertubation theory additionally. For the stochastic finite element analysis, as it has been successfully applied for soil mechanics too (36), material properties are often specified as element means and variations of element means, and as a matrix of covariances among the element means. These statistical description derive from the underlying stochastic model, and are functions of element size, shape and orientation with respect to correlation function. The corresponding element means are obtained by averaging the stochastic process over the element. If the simple assumption of a constant spatial mean is considered, also the element means are equal. But this assumption is not necessarily fulfilled for a crack tip surrounding and requires further investigation in most advanced crack tip models. In the excellent paper (36) a more detailed description of this kind of stochastic FEM can be found.

Stochastic Finite Element Methods (SFEM) in general take into consideration the random nature of the FEM field quantities. Geometric parameters, material parameters and tractions often cannot be regarded as deterministic quantities. To obtain accurate mean values with realistic variances by means of simulation techniques quite a lot of specialized calculations for definite kinds of realizations of the stochastic model would be necessary. We restrict our attention to another stochastic method and give some simple examples for fracture mechanics application applying this powerful technique.

Stochastic FEM as being described now is a generalization of classical FEM for random material behaviour and random crack tip geometry. We restrict our example to the case of two dimensions without loss in generality to avoid difficult expressions here. A linear elastic medium is assumed, but the analysis in similarity could be generalized to non-linear material behaviour.

Let x and y be the initial deterministic co-ordinates of a point P which undergoes small stochastic changes of position. Capital X and Y denote the stochastic co-ordinates, respectively.

$$X_i = x_i\,(1+\tilde{X}_i) \quad \text{and} \quad Y_i = y_i\,(1+\tilde{Y}_i)\,. \tag{2}$$

By means of formula (2) the resulting total stochastic stiffness matrix M can be derived and expanded as a potential series as follows

$$M = M^0 + \sum_{i=1}^{N_s}\left(M_i^{11}\tilde{X}_i + M_i^{12}\tilde{Y}_i\right) + \sum_{i=1}^{N_s}\sum_{j=1}^{N_s}\left(M_{ij}^{21}\tilde{X}_i\tilde{Y}_j + M_{ij}^{22}\tilde{X}_i\tilde{Y}_j + M_{ij}^{23}\tilde{Y}_i\tilde{Y}_j\right), \tag{3}$$

where N -number of all stochastic co-ordinates.

The matrix M is the total stiffness matrix of the initial deterministic system for the case of vanishing $\tilde{X}$, $\tilde{Y}$.

In the case of linear elastic deformation the stochastic system of equations

$$M\,U = f \tag{4}$$

is to be solved, where U denotes the random displacement vector. Suppose only small changes $\tilde{X}$, $\tilde{Y}$, i.e.

$$\| (M^{o})^{-1}(M - M^{o}) \| < 1 \tag{5}$$

with probability one. Then after certain mathematical requirements have been fulfilled the system (4) can be solved by means of pertubation theory. The random variable U is expanded as

$$U = U_o + \sum_{i=1}^{N_s} \left(U_i^{11} \tilde{X}_i + U_i^{12} \tilde{Y}_i \right) + \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \left(U_{ij}^{21} \tilde{X}_i \tilde{X}_j + U_{ij}^{21} \tilde{X}_i \tilde{Y}_j + U_{ij}^{23} \tilde{Y}_i \tilde{Y}_j \right). \tag{6}$$

The unknown variable U can be determined after replacing U in (4) by relationship (6).

The components U_o, U_i^{jk}, U_{ij}^{kl} etc. can be obtained in successive manner. U_o represents the FEM solution of the normal deterministic boundary value problem. From the series (6) the displacements and all the other field quantities may be derived. Consider now two independent stochastic quantities $\tilde{X}$ and $\tilde{Y}$. Neglecting higher order terms we write

$$U_i = U_i^{0} + U_i^{11} \tilde{X} + U_i^{12} \tilde{Y} + U_i^{21} \tilde{X}\tilde{X} + U_i^{22} \tilde{X}\tilde{Y} + U_i^{23} \tilde{Y}\tilde{Y}. \tag{7}$$

Suppose that we only know the values of expectation

$$EU_i = U_i^{0} + U_i^{11} E\tilde{X} + U_i^{12} E\tilde{Y} + U_i^{21} E\tilde{X}^2 + U_i^{22} E\tilde{X} \cdot E\tilde{Y} + U_i^{23} E\tilde{Y}^2.$$

The variance is given by

$$D^2 U_i = EU_i^2 - (EU_i)^2 \tag{8}$$

For continous density function $f_{\tilde{X}}(x)$ and $f_{\tilde{Y}}(y)$ of the random variables $\tilde{x}$ and $\tilde{y}$ it is no problem to derive the necessary expressions.

STOCHASTIC FEM AND FRACTURE QUANTITIES

The authors have applied the method shortly outlined above to calculate fracture quantities. In ref. (9), (10) a plate with a hole and two surface cracks in a tensile stress field has been dealt with in detail. The length of the cracks was assumed to vary randomly within certain limits, dependent upon the material. This kind of assumption is important for ceramic and composite materials as well. The authors have varied the crack length (randomly) and the geometry of the specimen (in a deterministic manner) to simulate realistic conditions. For Al_2O_3 ceramic also the model of randomly distributed Young's modulus dependent on porosity has been discussed applying the stochastic FEM. Different kinds of probability density functions have been studied. Most important was the comparison of the results obtained by the stochastic simulation procedure and several deterministic calculations. From this it follows that the method of stochastic FEM leads to very good results. To get a full curve for the stress intensity factor K or the J integral the computer time was approximately twice the time needed for one single deterministic calculation ot these quantities. From this one may draw the conclusion that the stochastic FEM is a powerful method as about 10...20 deterministic calculations and more should be required to get realistic results for real scatter of crack configurations. From this it follows that the stochastic FEM under certain conditions may lead to a reduction in computer costs by a factor 5...10. This might be important in three-dimensional cases for generalized J etc. ($\hat{J}$ by MIYAMOTO/KIKUCHI or C^*, ΔT_k in the case of visco-plastic stochastic FEM etc.). Extended numerical investigations for two-dimensional and three-dimensional cases have been carried out by the author applying the computer code ATOLL of The Institute of Mechanics of G.D.R. The results obtained favour the stochastic FEM both for a scatter of materials properties and random crack length distributions for single cracks. The phenomenon of interacting cracks has been investigated too. Some experts consider (and so do the authors of this paper) that in the future the subdivision of safety analysis (fracture analysis) into deterministic and probabilistic approaches will gradually disappear and an integrated approach will merge as it is already to be seen for nuclear safety analysis. While the probabilistic concepts become more widely used they will loose some of their "flexibility" too and must also be standardized to allow a better comparison of the results. In this process the probabilistic methods should acquire a more "deterministic flavour" (38).

PROBABILISTIC FRACTURE MECHANICS AND EXPERIMENTS

Experiments for probabilistic fracture mechanics at the first glance should most carefully take into consideration the mean values, variances and higher order moments of all the important quantities. From this one should think that "probabilistic" experiments always should be more expensive as more extended in quantity than deterministic ones. But this is not generally the case. In real experiments all the measuring quantities also vary from one measuring point to another. Thus it will be more a philosophy whether using the results of the experiment of a certain number of measurements of the same physical quantity to determine a "deterministic" or a stochastic fracture concept, as a certain scatter is also present for each deterministic component due to the real experimental procedure. The difference, of course, can increase considerably for such kind of probabilistic models, where a sufficiently large ensemble of individuals in the distribution will be necessary. The research group of the authors mainly use laser experiments, X-ray diffraction, acoustic and election microscopical methods to obtain field quantities, distributions of fracture quantities etc. which enter into the probabilistic fracture concepts. Some results will be reported in more detail at the conference.

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