FAILURE ANALYSIS - THEORY AND PRACTICE - ECF7

PROBABILISTIC FRACTURE MECHANICS: APPROACH TO BRITTLE FAILURE

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The different statistical approaches to brittle failure are discussed. A probabilistic fracture mechanics model (the Muest model) which combines fracture mechanics concepts with sound statistical approach having physical foundation is then presented. Finally this Muest model is compared with the widely used Weibull's statistical method.

INTRODUCTION

With the emergence of new materials such as structural ceramics, new trends in fracture mechanics focus upon statistical approaches.

From the point of view of fracture mechanics the new engineering ceramics are an interesting material. Failure is an erratic event, as a result of the presence of a multitude of fracture inducing flaws having random-like orientation and distribution.

Statistical theories of brittle failure are based upon the weakest link concept which identifies the fracture process to that of a chain, the links of which would be formed by the volume elements. Fracture of the bulk specimen is determined by the local strength of its weakest volume element.

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Several statistical theories have been developed. However, in the case of ceramics where fracture is caused by inherent microstructural flaws, suitable approaches are those considering the microstructural flaws as physical entities.

From this point of view the Weibull approach presents certain shortcomings since no direct use is made of the hypothesis that fractures are due to crack growth from pre-existing flaws. This limits its applicability in particular when analyzing multiaxial failure.

Probabilistic fracture mechanics combine linear fracture mechanics to sound statistical approach. Recently a multiaxial probabilistic model (the Multiaxial Elemental Strength Model, referred to as Muest model) was derived by Lamon and Evans. Fracture mechanics concepts are applied at the microscopic level of flaws. Distribution of the critical flaws is then described by the Elemental Strength Approach which provides failure probability expressions. The Elemental Strength Approach is an alternative to Weibull's theory having fundamental foundation.

The primary intent of this paper is to present probabilistic fracture mechanics, and more particularly the Muest model. The paper thus reviews first the effects of microstructural defects upon failure characteristics. The paper then compares the different statistical approaches to failure and the Muest model.

CERAMIC FRACTURE CHARACTERISTICS: STRENGTH VARIABILITY

Structural ceramics as well as the other brittle materials exhibit two important characteristics:

- first, linear elastic behavior up to fracture and very limited crack arrest capability. As a consequence the elastic limit marks crack growth as well as brittle failure. The corresponding stress measures the resistance to fracture;
- second, the presence of a very large number of microscopic flaws (the critical size is generally smaller than 100 μm). The criticality of these randomly distributed flaws is dictated by parameters such as their nature (pore, void, inclusion, crack, etc.) [1], size, location [2,3] and orientation relative to stresses [4]. As a result fracture is a probabilistic event.

The presence of populations of microstructural flaws has also several implications on failure strength. Strength is not an intrinsic property. It depends upon several parameters such as the specimen size, the stress-state, and the nature of the preexisting flaws.

Several specimens made out of the same material, having the same dimensions and subjected to identical loading conditions fail at different stress levels (figure 1). Because of this scatter inherent to brittle materials, measured strengths are handled like statistical data.

The larger the specimen, the lower the fracture resistance, simply because there is a greater chance that a more severe flaw is present among the greater number of flaws in the large specimen.

This size effect is perfectly illustrated by figure 2 which shows that the strengths measured on tensile specimens are significantly lower than those obtained with 3-point bending specimens having smaller stressed volume.

Sets of specimens which experience different stress-states exhibit different fracture characteristics, as shown by figure 1. In this typical example, mixed-mode loading conditions were enhanced by reducing the span length of flexural beams. It can be seen also that fracture origins were determined by loading conditions. Thus, fracture was dictated by a single population of surface located flaws for specimens subject to uniaxial stress-state (long spans) and by a bimodal population of surface and internal flaws when mixed-mode loading conditions prevailed (shorter spans). The specimen weakening induced by the internal flaw populations is enhanced by large dimensions (figure 3).

PROBABILISTIC FRACTURE MECHANICS AND STATISTICAL - PROBABILISTIC APPROACHES TO BRITTLE FAILURE

If the flaws within the structure could be identified in terms of size, shape and location by non-destructive means, a deterministic-type analysis should accurately predict the strength by way of fracture mechanics relations. In the case of ceramics, the currently available flaw detection techniques are insufficient to screen and identify the critical flaws. Approaches based upon statistical theories should allow the above-mentioned effects to be characterized, e.g. the correlation of strengths measured in different loading conditions and the description of strength variability.

Figure 4 represents the aspects of probabilistic fracture mechanics. It provides the essential relationships between the three criteria for failure of such brittle materials: the flaw populations, the stress and the failure probability. The flaw population and failure probability concepts replace the defect size and fracture toughness concepts pertinent to the deterministic triangle. As with the deterministic triangle, we can go around the triangle in any way we like. For example, we can fix the applied stress and derive failure probability for a specified material. Or, for a given component, we can derive the permitted applied stress from flaw population characteristics.

For isotropic materials, the statistical theories may be essentially grouped into 3 categories: Weibull, the flaw size theories and the elemental strength approach.

The Weibull Approach

In recent years, the failure strength of ceramics has been routinely analyzed in uniaxial conditions using Weibull statistics. An empirical formula of the form given below is used to relate the probability of failure P_f with stress σ:

$$P_f = 1 - \exp\left[- \int_V \left[\frac{\sigma - \sigma_u}{\sigma_o} \right]^m dV \right] \qquad (1)$$

m is an index of the degree of scatter in measured strength values, σ_o a normalized factor and σ_u the stress at which there is a zero probability of failure (σ_u is usually taken equal to 0).

Weibull's formulation for uniaxial stress is straightforward. But his formulation for multiaxial stress states is not readily accepted, and several variations have been suggested.

In the original Weibull treatment of multiaxial failure (also referred to as the normal tensile stress averaging method) failure probability is given by:

$$P_f = 1 - \exp\left[- \int_V k \int_A \sigma_n^m \, dA \, dV \right] \qquad (2)$$

where dA is an elemental area on a unit solid sphere and $k = (2m + 1)/2\pi \, (1/\sigma_o)^m$.

In Eq. (2), integration is performed over half the surface area of the unit sphere where the normal stress σ_n is tensile, neglecting regions where the normal stress is compressive. Eq. (2) is a shear-intensitive description of multiaxial fracture.

Depending on the material considered, predictions using the Weibull formulation may be either conservative or optimistic. Thus, a number of investigators have obtained contradictory results in multiaxial failure prediction based solely on uniaxial tests. Alumina showed both weakening [5] and the opposite effect [6,7] in biaxial tension relative to uniaxial tension. Other materials similarly exhibited either a weakening (titania [8], silicon carbide [9], glass [10]) or the opposite (glass ceramic [11], porous zirconia [12]).

Barnett et al. [13] and Freudenthal et al. [14] suggested an alternative simple approximation for handling multiaxial fracture statistics (referred to here as the Barnett-Freudenthal (BF) approximation). In this approach, the principal stresses are assumed to act independently in each principal direction. As a consequence, the failure probability is calculated from the product of the individual survival probabilities, in the direction of the tensile components.

This assumption leads to the following equation for the probability of failure from volume located flaws:

$$P_V = 1 - \exp\left[- \int \left[\left(\frac{\sigma_1}{\sigma_{OWV}}\right)^{m_V} + \left(\frac{\sigma_2}{\sigma_{OWV}}\right)^{m_V} + \left(\frac{\sigma_3}{\sigma_{OWV}}\right)^{m_V} \right] dV \right] \tag{3}$$

The BF approximation has been criticized by several authors. As it ignores interaction of principal stresses, it should predict lower failure probabilities than the Weibull model [15].

Flaws Size Theories

The flaw size theories are based upon the statistics of flaw size and location. The fracture probability is derived from the flaw size distribution f(a) by applying the fracture mechanics relation ($K_I = \sigma Y \sqrt{a}$):

$$P_f(\sigma) = \iint \frac{2}{\pi} f(a)\, da\, d\beta \tag{4}$$

The major problem in the use of these theories lies in the determination of the expression f(a). These theories have been applied to the tensile biaxial loading of a brittle material [16,17]. This work was based upon a general arbitrary form for f(a). Extensive data on flaw size distribution in ceramics are not available.

Flaw size theories are still in their infancy and require further development. Their future depends on the development of efficient non-destructive techniques.

Elemental Strength Approach

This approach also considers flaws as physical entities. It is based upon the premise that the pre-existing flaws in the material can be characterized by their flaw extension stress or strength S. Failure probability is calculated from the following equation:

$$P_f = 1 - \exp\left[- \int_V dV \int_o^S g(S)\, dS \right] \tag{5}$$

g(S) dS represents the number of flaws with a strength between S and S + dS. g(S) characterizes the distribution of flaws in the material. g(S) can be derived from strength data measured on specimens having well-defined geometry and stress state. The elemental strength approach thus allows the shortcomings of the Weibull solution and the current limitations of the flaw size theories to be overcome.

The Multiaxial Elemental Strength Model

The Muest model combines fracture mechanics concepts with the Elemental Strength model [2,3]. The fracture criterion incorporates recent concepts of non-coplanar crack extension [2]. It is considered that fracture may occur in a direction depending upon the respective magnitudes of the normal and the shearing components operating on the flaws. This criterion is based upon the maximum in the strain energy release rate G_{max}, in the direction of crack propagation:

$$G_{max} = \frac{(1 + \upsilon)\ (1 + x)}{4E} \left[K_I^4 + 6K_I^2K_{II}^2 + K_{II}^4 \right]^{1/2} \qquad (6)$$

where $x = 3\text{-}4\ \upsilon$ under plane strain conditions, $x = (3 - \upsilon)/(1 + \upsilon)$ under plane stress conditions.

Failure is dictated by the criticality for G_{max}.

Insertion of this fracture criterion and stress distribution in a mathematically convenient form of g(S) then permits the failure probability to be derived as follows in terms of the imposed loading, the specimen size and the flaw strength parameters (m_V and σ_{OMV}) [2,3]:

$$P_V = 1 - \exp \left[- \int_V \left(\frac{\sigma_1}{\sigma_{OMV}}\right)^{m_V} I_V\left(m_V, \frac{\sigma_2}{\sigma_1}, \frac{\sigma_3}{\sigma_1}\right) dV \right] \qquad (7)$$

where I_V (m_V, σ_3/σ_1, σ_2/σ_1) accounts for shear sensitivity of volume flaws and their orientation relative to principal stresses. Expressions for I_V (m_V, σ_2/σ_1, σ_3/σ_1) are given in references 2, 3, 18, 19. σ_2 and σ_3 may be compressive on the condition than σ_1, and the equivalent stress σ_E derived from Eq. (6) are tensile.

A similar equation applies to surface failure origins.

EXPERIMENTAL COMPARISON OF THE WEIBULL AND THE MUEST MODELS

The current capabilities of the Weibull and Muest models for correlating strengths obtained in various conditions were compared on the example of figure 1 involving mixed mode loading conditions and the presence of a bimodal flaw population. The flaw size theories were not considered due to their important limitations.

Failure strengths were predicted for the set of specimens having an intermediate span which experienced failure from bimodal population of surface and volume flaws. The flaw strength parameters pertinent to surface flaws had been determined on long span specimens. Those parameters pertinent to volume flaws had been determined on the short span specimens. The details of the theoretical and computerized analyses are given in references 3 and 19 respectively.

It can be seen on figure 5 that the Muest method calculated failure strengths which are in excellent agreement with experimental results. In contrast, the Weibull method significantly underestimated the failure strengths. It is worth noting that for the 3 loading cases under consideration, the normal tensile stress averaging method and the BF approximation reduced to the uniaxial Weibull's equation (1), thus ignoring the mixed-mode loading generated by the intermediate and short spans.

This analysis thus shows that probabilistic fracture mechanics permits correlation of strengths obtained in various configurations through failure probability computations, and it demonstrates that the Muest model is an improvement over the Weibull method.

CONCLUSIONS

With the brittle materials containing large numbers of microstructural flaws, statistical probabilistic approaches are required for characterizing the probabilistic nature of failure events.

A primary consequence of the presence of the flaws is that fracture strength is not an intrinsic criterion. It depends upon the specimen size, the stress-state, and the characteristics of flaw populations.

Probabilistic fracture mechanics permits the correlation of strengths measured in different conditions. Probabilistic fracture mechanics combines fracture mechanics concepts applied at the microscopic scale of flaws with a sound statistical probabilistic theory. In particular, the Muest model is an improvement over the Weibull's statistical theory.

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SYMBOLS

a = crack size

$2d$ = thickness of bending bars

E = elastic modulus

G_{max} = maximum strain energy release rate

K_I = mode I stress intensity factor

K_{II} = mode II stress intensity factor

$2l$ = span length of bending bars

m = shape parameter

m_V = shape parameter pertinent to volume flaws

P_f = probability of failure

P_V = probability of failure from volume flaws

S = fracture strength (MPa)

V = volume (m^3)

β = crack angle

σ = tensile stress

σ_E = equivalent stress

σ_n = normal stress

σ_o = scale factor

σ_{OMV} = Muest scale factor pertinent to volume flaws

σ_{OWV} = Weibull scale factor pertinent to volume flaws

$\sigma_1, \sigma_2, \sigma_3$ = principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$)

υ = Poisson's ratio.

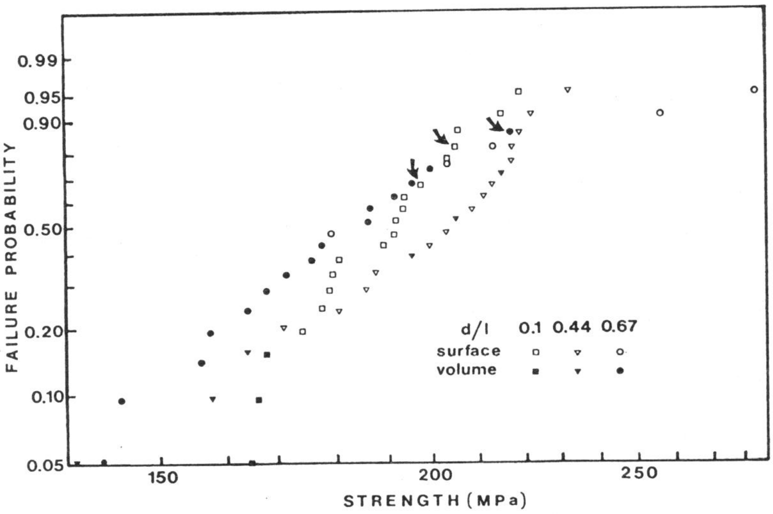


Figure 1 - 3-Point bending strengths at various span lengths (Silicon Nitride Ceramic).

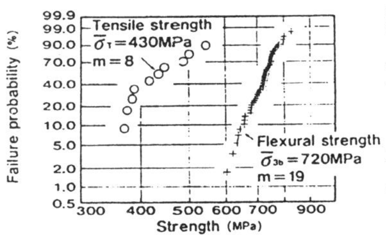


Figure 2 Strength distributions (silicon nitride ceramic) [20].

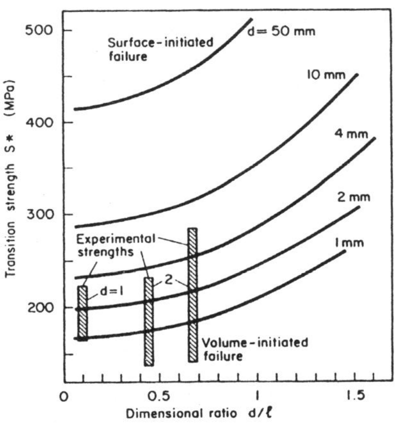


Figure 3 Influence of specimen size upon fracture origins and flexural strengths.

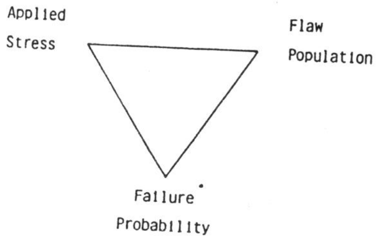


Figure 4 Probabilistic fracture mechanics triangle.

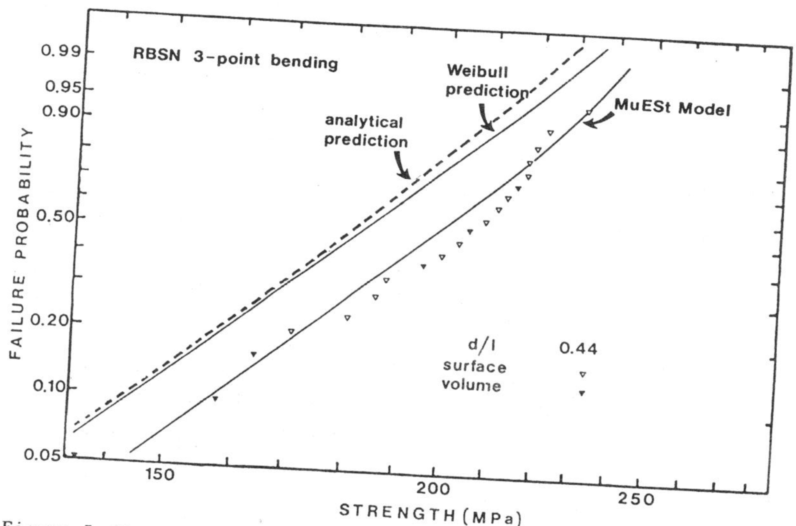
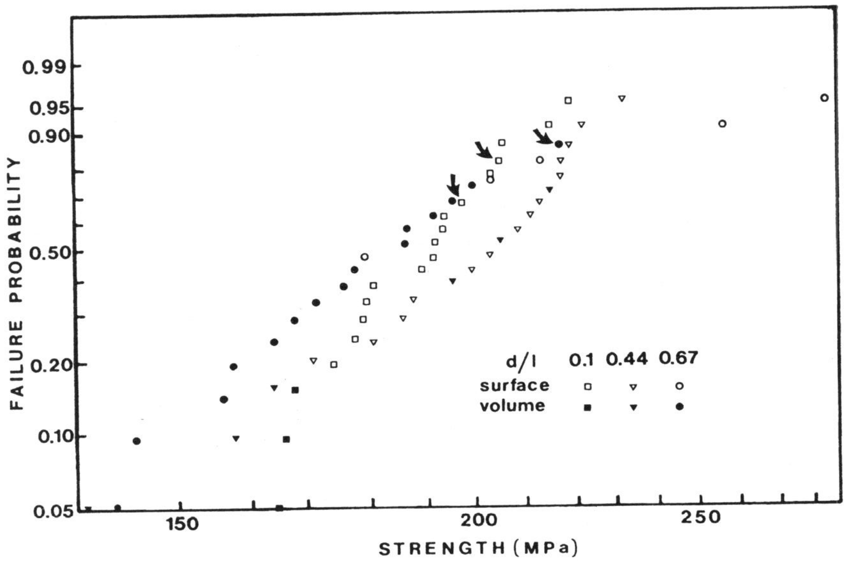


Figure 5 Strength predictions using the Muest and the Weibull's models.



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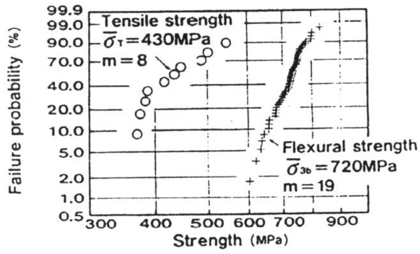

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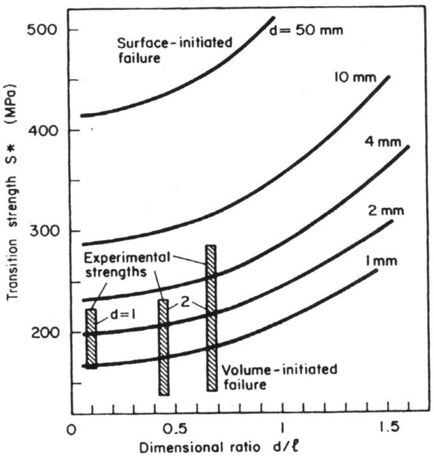

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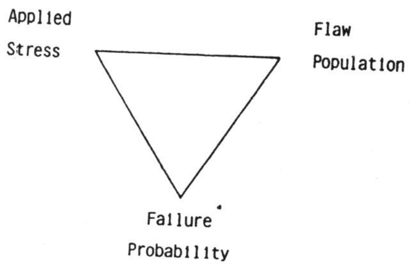

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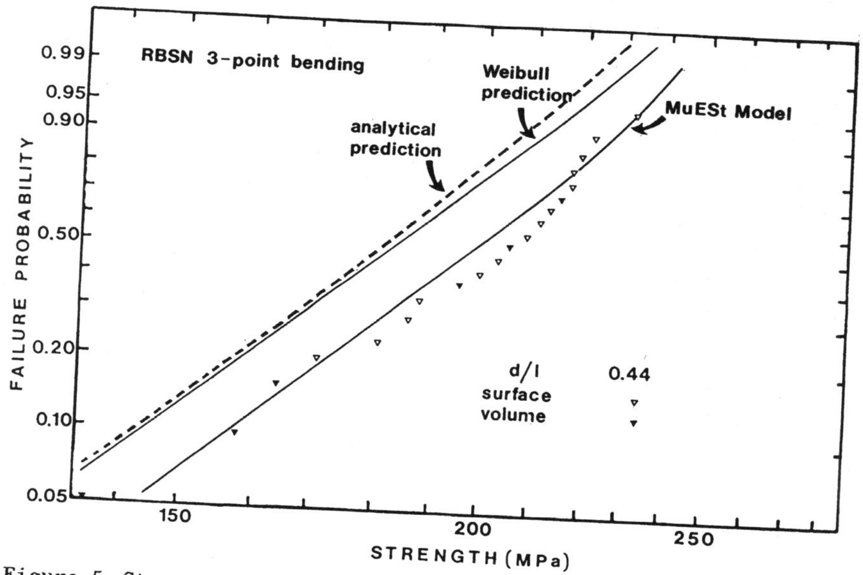

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