

PREDICTION OF THE FATIGUE LIFE TAKING INTO ACCOUNT
RESIDUAL STRESS RELAXATION UNDER CYCLIC LOADING

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This paper presents a model for the prediction of the residual stress distribution during fatigue loading. A finite element software is used for taking the cyclic plasticity into account. With this approach, the treatment of the local plastic yield conditions can easily be made from simple purely elastic analysis. The computed values of relaxed residual stress distributions were compared with experimental results obtained by X-Ray diffraction measurements. The stabilised residual stresses calculated after fatigue testing were introduced in a multiaxial fatigue criterion for life prediction.

INTRODUCTION

All methods of manufacturing introduce residual stresses in components and mechanical structures, and this can considerably modify their performance in service and, in particular, their resistance to fatigue. Taking into account residual stress in estimating fatigue strength poses the problem of analysing components subject to multi-axial stress. The state of stress resulting from applied loads may be uniaxial, but the state of a residual stress is always multi-axial. Where residual stresses are present in a component, the design engineer needs a multi-axial fatigue criterion to be able to evaluate the fatigue resistance of the component [1]. Another parameter is the relaxation of residual stress during fatigue. Thus, it is necessary to develop methods of calculation for prediction of the relaxation of residual stress during fatigue. A model of relaxation using a finite element software is presented which relates residual stress distribution in the depth plane to the number of fatigue cycles, the cyclic stress amplitude, the cyclic hardening behaviour of the material, the notch effect. A multi-axial fatigue criterion is used for estimating fatigue strength when residual stresses should be present, provided that the phenomena of relief of fatigue stresses are taken into account.

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PRINCIPLE OF THE CALCULATION METHOD OF
RESIDUAL STRESSES RELAXATION

For the inelastic analysis of structures, a simple practical approach is proposed by ZARKA and CASIER [2]. For a case of cyclic plasticity (the relaxation of the residual stress due to a local cyclic plastic strain for example), this method can't give the exact solutions, but can predict the response of a kinematic hardening material to the stabilized cycle : elastic shakedown or plastic shakedown. The new approach of ZARKA and CASIER introduces transformed internal parameters into the calculation of the states of stress and the strains. These parameters can be calculated by the following relationship :

$$\hat{\alpha} = \alpha - \text{dév. } \varrho = \alpha - S + S^{\text{el}} \quad (1)$$

where

$\alpha = C \cdot \epsilon^p$ is the tensorial internal variable connected with kinematic hardening

$C = 2/3H$ H is the cyclic modulus of the material

$\epsilon^p =$ is the tensor of the plastic strains

$\varrho =$ is the tensor of the residual stresses

$S =$ is the real stress deviator tensor

$S^{\text{el}} =$ is the elastic stress deviator tensor.

For the material following the Von Mises criterion, the yield condition is :

$$3/2 (S - \alpha)^T (S - \alpha) \leq \sigma_0^2 \quad (2)$$

σ_0 is the real yield limit.

From Equ. 1 and Equ. 2 we can deduct the new plastic criterion :

$$3/2 (S^{\text{el}} - \hat{\alpha})^T (S^{\text{el}} - \hat{\alpha}) \leq \sigma_0^2 \quad (3)$$

This relationship (3) defines a new yield surface (or sphere) using the transformed parameters $\hat{\alpha}$. The center of this sphere is S^{el} and his radius is σ_0 .

In the space of the transformed parameters, the interpretation of the plastic criterion is as follows : if the state of non linearity (plasticity) of the material is defined by the tensor of the parameters $\hat{\alpha}$, the representative point of the state of the stress is in the interior of the sphere with the center S^{el} and the radius of σ_0 at all times.

When the loading condition changes with time, the position (S^{el}) and the size (σ_0) of the sphere changes also, but these changes depend only on the parameters calculated by elastic calculations. During a constant amplitude fatigue test ($F_{min} < F < F_{max}$) the representative point of the state of stress evolves between two convex ($C_{min}(S^{el}_{min}, \sigma_{Omin})$) and ($C_{max}(S^{el}_{max}, \sigma_{Omax})$). According to the ZARKA and CASIER criterion, for a kinematic hardening material, the condition for elastic shakedown (stabilization of the residual stress state for example) is that the convex set $C_p = C_{min}$ and C_{max} is a non void set. In the graphic representation of the parameters α , we need to see if the local convex set C_{min} and C_{max} have a non void intersection (fig. 1).

So the calculation of the stabilized state of the residual stress after fatigue loading consists in verifying the condition of the elastic shakedown :

$$\sqrt{\frac{3}{2}(\Delta S^{el})^T(\Delta S^{el})} \leq 2 \sigma_0 \quad (4)$$

where : $\Delta S^{el} = S^{el}_{maxi} - S^{el}_{mini}$.

We can find the detail of the different steps of the calculation of the stabilized stress state in the case of elastic shakedown by an approximative method in [2] [3].

This method is very interesting, only a small number of elastic calculations are necessary to find the limiting state of the residual stress (elastic shakedown) using the initial residual stress field, the cyclic hardening characteristics of the material and the cyclic applied loading. With this approach, a software using the finite element method has been developed by the CETIM (CA.ST.OR PC2D software) for resolving the two dimensional cyclic plasticity problems. This software is only applicable in the case of elastic shakedown for a radial cyclic loading (for which the principal stresses are proportional). The model only needs the materials properties : cyclic yield stress, cyclic hardening modulus, Young's modulus and Poisson's ratio - and the initial residual stresses.

APPLICATION OF THE SIMPLIFIED CALCULATION METHOD
FOR THE PREDICTION OF THE RELAXATION OF RESIDUAL
STRESSES DURING FATIGUE

Grinding

Grinding is one of the most used modes of finishing in mechanical engineering. It is very important to study the residual stresses introduced by this machining method, because it often determines the final state of residual stresses in a part, which influences its fatigue strength. The simplified finite element method will be used for the stress relaxation prediction and the calculation results will be compared with the results of measurement using the X-Ray diffraction method. The material used is a 42 CD 4 grade steel.

Fig. 2 shows the results obtained for hard longitudinal grinding. The predicted reduction agrees fairly well with the experimental results in the case of hard grinding, for which the distribution curve of the residual stresses can be measured with fair accuracy. Agreement between the model and the experimental results is worse for soft grinding for which the determination of the stress gradient before and after fatigue is more imprecise due to the low thickness of the prestressed layer. On the other hand, the model of the cyclic behaviour of the layer very close to the surface cannot be represented by the macroscopic cyclic work-hardening curve of a test specimen (because the microstructures are often different). This might be the modeling limit for a microscopic localized phenomenon using the rules of macroscopic behaviour and calculation.

Shot peening

Shot peening is a well known mechanical treatment used to increase fatigue strength due to compression residual stresses introduced on the components surface by the shots impact.

The prediction of residual stress relieving for the shot peened components is largely developed in reference [4], where influence of stress amplitude, mean stress and number of cycle is shown. The relaxation of residual stress increases with the level of the stress amplitude for fully reversed fatigue loading. VOHRINGER [5] used a normalised representation providing the effect of the fatigue stress amplitude on the reduction in the residual stress. Fig. 3 shows the prediction of relaxation of residual stress for a shot-peened 35 NCD 16 grade steel and the comparison with other materials for which the diagram is plotted using experimental results. It can be seen that the prediction gives the same type of curve.

Recent reasearch [7] shows that there are minimum two layers of material for the shot peening case, the shot-peened layer and the no shot-peened layer. So it is necessary to know the real cyclic yield stress of the peened layer which is often greater than the subsurface due to the surface hardening processus by the shot peening.

For the shot peening case a multilayer material must be considered. BERSTROM [6] have studied the relaxation of shot peening introduced compressive stress during fatigue of notched AISI 4110 grade steel samples. Fig. 4 shows an example of the comparison between the results of our calculation and the BERGSTROM experiments. The finite element method is used for the calculation of the real applied stress and the PC2D software is used for the determination of the stabilized residual stress state after fatigue. Here four layers of material are considered. The detail of this method is in [7]. Experimental results and prediction of the relaxation of residual stresses show a good agreement.

TAKING INTO ACCOUNT THE STABILIZED RESIDUAL STRESS FOR FATIGUE STRENGTH ESTIMATION

The residual stresses always are multiaxial. In order to take a multiaxial residual stress state into account, a multiaxial fatigue criteria is necessary. In ref. [1], several multiaxial fatigue criteria have been compared in order to take into account the residual stresses phenomena. DANG VAN'S criterion is one of the most used criteria. DANG VAN [8] has put forward a criterion which brings into play local variables for a stabilized state. For the fatigue tests with constant amplitude considered here, these variables are :

τ_a the shear amplitude acting in plane of maximum shear stress

P_{max} , the maximum hydrostatic stress.

The limiting curve for the criterion is given by linear equation :

$$\tau_a + \alpha P_{max} < \beta \quad (5)$$

where α and β are material constants. For simple bending or tension-compression fatigue testing, where σ_a is the stress amplitude, we obtain :

$$\tau_a = \frac{\sigma_a}{2}, \quad P_{max} = \frac{\sigma_a + \sigma_m + \sigma_{Rl} + \sigma_{Rt}}{3}$$

Where σ_m is the mean stress due to the fatigue loading and σ_{Rl} and σ_{Rt} are the surface longitudinal and transverse residual stresses.

Fig. 5 shows an example of residual stress taken into account in order to predict the fatigue behaviour of a shot peened 35 NCD 16 grade steel. The DANG VAN diagram is plotted for different fatigue life using reference fatigue tests and S-N curves obtained with soft longitudinal ground specimens (without residual stress influence). The straight lines of this diagram represents the fatigue strength for different fatigue lives.

On this diagram two points corresponding to two levels of stress amplitude (± 50 and ± 600 MPa) are shown for shot peened specimens, without and with residual stress taken into account.

By using the stress relaxation model the infinite life for the ± 550 MPa level and the finite life between $2 \cdot 10^6$ and $5 \cdot 10^6$ cycles for the 600 MPa level can have been predicted. This prediction is successfully compared with the real fatigue behaviour of the shot peened specimens. Of course further work are necessary to validate, this modeling on a great number of results and other tyhpe of fatigue tests.

SUMMARY

In the work carried out, an automatic software using the finite element method to calculate the stabilized residual stress during cyclic loading is developed.

This method is applied to shot peened 35 NCD 16 grade steel, AISI 4140 grade steel and a ground 42 CD 4 grade steel. The different fatigue parameters often used in material research are studied, such as the number of cycles, the stress amplitude, the notch effect. The result is satisfactory enough for the study of the residual stresses relaxation phenomenon. It is a simple method for the prediction of the stabilized multiaxial residual stress state in the depth plane. This study shows a new way for understanding the mechanical relaxation of residual stress. Nevertheless, for better modeling with this method it is necessary to introduce into the calculation the real cyclic stress strain curve corresponding to the materials present in the prestressed layer. This represents a difficulty in the case for example of shot peening. A multilayer method must be used for a better modelisation.

For taking into account the residual stresses in a fatigue strength prediction, the DANG VAN criterion is used. This criterion can predict the fatigue life using the stabilized residual stresses. To validate this criterion for other materials, for other types of residual stresses generated by other manufacturing processes, and for complex fatigue loading, additional tests are required.

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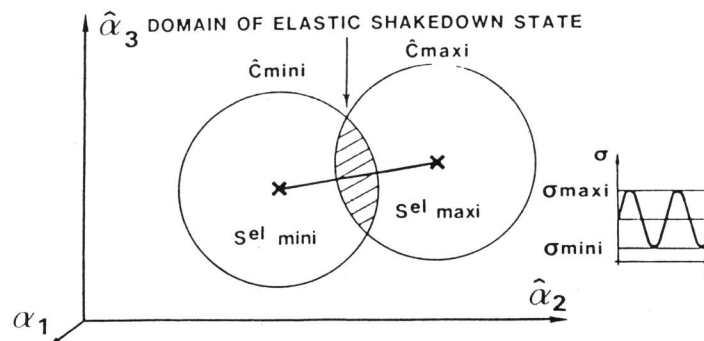


Fig.1 Graphic representation of the domain of elastic shakedown state in the $\hat{\alpha}$ space.

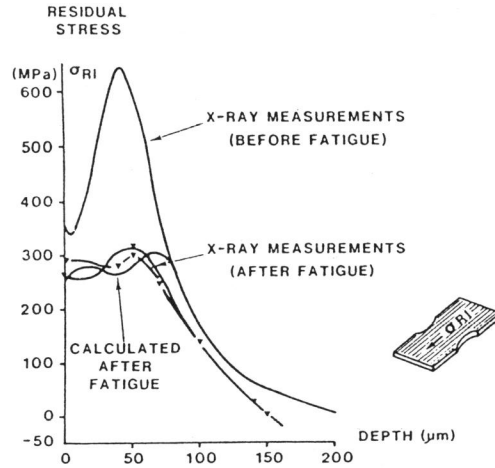


Fig. 2 Predicted reduction of residual stresses produced by hard longitudinal grinding, X-Ray diffraction measurements made on two test specimens after fatigue testing.

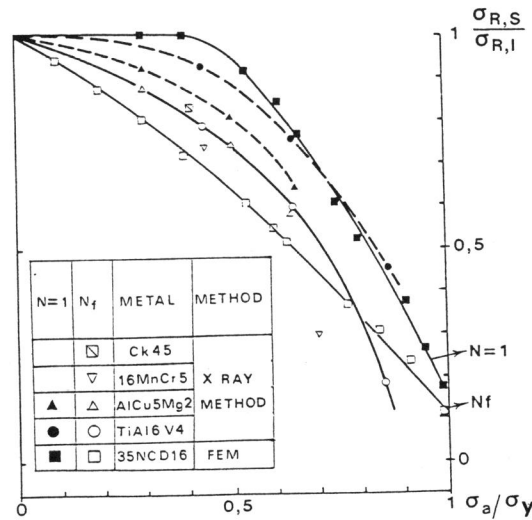


Fig. 3 Prediction of shot peening residual stresses relieving on a 35 NCD 16 grade steel, due to tension-compression fatigue as a function of stress amplitude σ_a resp. ratio $\frac{\sigma_a}{\sigma_y}$ (σ_y = surface yield stress) and comparison with the experiential results for other materials (N = number of cycles of the fatigue loading, N_f = number of cycles to obtain fracture).

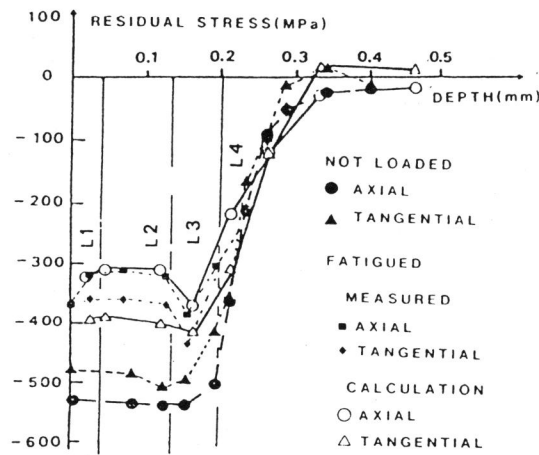


Fig. 4 Comparison of the results between the calculation and the experiment (residual stress profiles for axial and tangential directions at bottom of notch of AISI 4110 samples shot peened with F30-35A (mm/100) almen intensity conditions. Data are shown for shot peened and shot peened plus fatigued. ($R = -1$, nominal stress amplitude = 283 MPa, 10^6 cycles states, notch factor = 3).

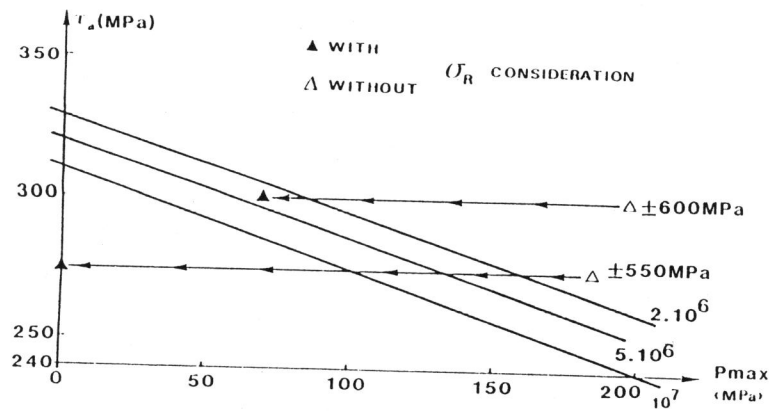


Fig. 5 Prediction of fatigue life using a DANG VAN criterion for shot peened 35 NCD 16 grade steel.