PREDICTED FATIGUE FRACTURE PLANES ACCORDING TO VARIANCE OF SHEAR STRESS UNDER RANDOM TRIAXIAL STRESS STATE

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The criterion of the maximum shear stress in a fracture plane belongs to the fatigue criteria proposed in papers (1,2). It is assumed that fatigue fracture is caused by a shear stress $\tau_{\eta s}(t)$ acting in direction $\bar{s}$ on the fracture plane with normal $\bar{\eta}$ (see Fig.1.), where t- time. The direction $\bar{s}$ coincides with a mean direction of the maximum shear stress $\tau_1(t)$ and the fracture plane is determined by a mean position of one of two planes where $\tau_1(t)$ acts. The predicted fracture plane location is described with directional cosines $\hat{l}_n, \hat{m}_n, \hat{n}_n$, ($n$=1,2,3) of the normal principal stress axes $\sigma_1(t) \geq \sigma_2(t) \geq \sigma_3(t)$. According to this criterion equivalent stress $\sigma_{red}(t)$ depends in a linear way on stress state components $\sigma_{ij}(t)$, (i,j=x,y,z) and in non linear way on $\hat{l}_n, \hat{m}_n, \hat{n}_n$, i.e.

$$\sigma_{red.}(t) = \left(\hat{l}_1^2 - \hat{l}_3^2\right)\sigma_{xx}(t) + \left(\hat{m}_1^2 - \hat{m}_3^2\right)\sigma_{yy}(t) + \left(\hat{n}_1^2 - \hat{n}_3^2\right)\sigma_{zz}(t) + $$
$$+ 2\left(\hat{l}_1\hat{m}_1 - \hat{l}_3\hat{m}_3\right)\sigma_{xy}(t) + 2\left(\hat{l}_1\hat{n}_1 - \hat{l}_3\hat{n}_3\right)\sigma_{xz}(t) +$$
$$+ 2\left(\hat{m}_1\hat{n}_1 - \hat{m}_3\hat{n}_3\right)\sigma_{yz}(t) \qquad (1)$$

After writing formula (1) in the form

$$\sigma_{red}(t) = \sum_{k=1}^{6} a_k X_k(t) \qquad \text{where} \qquad X_k(t) \equiv \sigma_{ij}(t) \qquad (2)$$

the variance of the equivalent stress $\mu_{\sigma red}$ can be calculated from

$$\mu_{\sigma red} = \sum_{s=1}^{6} \sum_{t=1}^{6} a_s a_t \, \mu_{xst} \qquad (3)$$

where a_s, a_t are constant coefficient linearly dependent on $\hat{l}_n, \hat{m}_n, \hat{n}_n$ and μ_{xst} are elements of covariance matrix of stress state components.

In this paper it is assumed that the plane on which the variance of equivalent stress (3) is maximum is critical for the material and fatigue fracture plane can be expected in this plane.

For stationary stochastic loadings, i.e. for determined values of elements μ_{xst}, the variance $\mu_{\sigma red}$ depends on 6 directional cosines $\hat{l}_n, \hat{m}_n, \hat{n}_n$, (n=1,3) which must fulfil orthogonality conditions. The conditional maximum of a non-linear function of several variables with non-linear limitations should be found. Unfortunately,

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the problem cannot be analyticaly solved in a general case.

In order to reduced a number of variables of the function (3), the mean cosines of Euler angles $\hat{\vartheta},\hat{\phi},\hat{\psi}$ are introduces intead of $\hat{l}_n,\hat{m}_n,\hat{n}_n$. Hence:

$$\mu_{\sigma red} = f\left(\hat{\vartheta},\ \hat{\phi},\ \hat{\psi},\ \mu_{xst}\right) \tag{4}$$

Searching the maximum of non-linear function (4) in an analytical way is, in many cases, not effective.

The digital simulation method was used in order to investigate courses of variability of function (4) depending on the mean values of Euler angles $\hat{\vartheta},\hat{\phi},\hat{\psi}$, ($\mu_{xst}$ is constant) and to search maxima of variance $\mu_{\sigma red}$. For same selected cases of complex stress state the expected positions of fatigue fracture planes were determined. Simulation calculations were compared with available results of fatigue tests for multiaxial cyclic loadings giving a good agreement. It has been also shown that for each random stress state there is one or some critical planes in the material with reference to fatigue life. In these planes the variance of the equivalent stress, according to the discused criterion, reaches its maximum and fatigue can occur in these planes. In the Figure 2 was presented an example of variance of equivalent stress as a function of $\hat{l}_1,\hat{m}_1$,and $n_1,\hat{l}_3,\hat{m}_3,\hat{n}_3$ are parameters for biaxial random tension-compression with shear stress state ($\sigma_{xx}(t),\sigma_{yy}(t),\tau_{xy}(t) \neq 0$).

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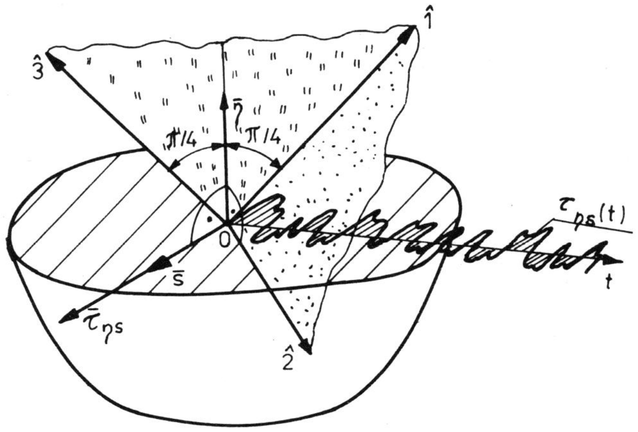

Figure 1 Directions of vectors $\bar{\eta}$, $\bar{s}$ and stress $\tau_{\eta s}(t)$ in relation to axes $\hat{1},\hat{2},\hat{3}$

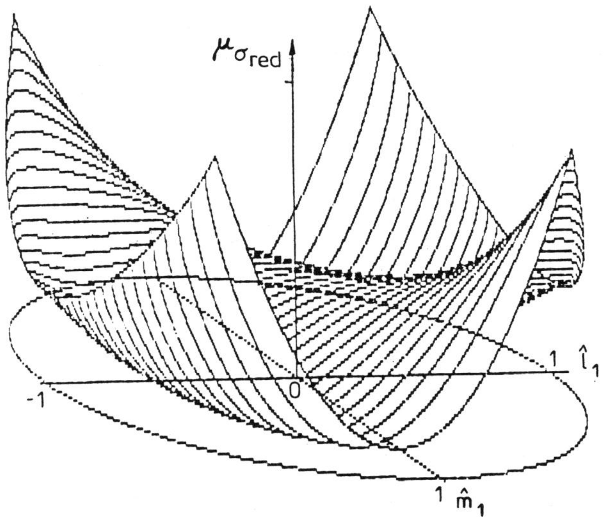

Figure 2 Graph of the function $\mu_{\sigma red}$ according to eq.3 for biaxial random tension-compression with shear