OPTIMUM DESIGN AND CALCULATION OF RELIABILITY FOR STRUCTURES EXPOSED TO FATIGUE LOADING

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Methods to estimate the reliability for the quality protection of complete machines, structures are well-known.

However, as far as we know, this kind of test has hardly been carried out with design methods.

For structures designed for alternating loading (e.g. the frequently used cranes) there is a procedure when the reliability or optimal reliability can be determined by optimal design with a known load spectrum.

Similarly to this method other effects (corrosion, wearing) can also be taken into account or the common influence of the complex mechanism can be tested designing the reliability curve or estimating the optimal reliability curve.

Here we show a system to plot the optimal reliability curve for structures designed for cyclic loading emphasizing the fatiguing effects. According to the Miner-theory

$$\text{if} \quad \sum_{n} \frac{n_i}{N_i} < \delta \qquad (1)$$

the structure complies to cycling loading (Figure 1.). Here n_i, N_i, and δ are random variables, $\delta < 1$, $\delta > 1$, $\delta = 1$ depending on the load spectrum.

If we disregard the variation of the mean stress in the function of time we determine it just at the end of the calculation and only the deviations from the mean stress are recorded, their real chronological order is neglected, a diagram for the number of different stress-cycles can be constructed (Figure 2).

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If this diagram is combined with the Wöhler-curve designed with the intended life probability, p (Figure 3), transposing the formula /1/ to calculate the reliability curve the relation below is obtained:

$$P(n) = \delta - \sum_{n} \frac{n_i}{N_i} \qquad \text{for all } i=1,2,\dots n \qquad /2/$$

So the durability will be

$$N_k = \frac{N_t}{\sum_{n} \frac{n_i}{N_i}} \qquad (2.10^6 < N_o < 10^7 \qquad /3/$$

It should be noted that the curve "2" can also be produced by digital stochastical simulation or by stress-measuring but also on the bases of data from the orderer.

Bolotin (1) deals with this problem in his book and proposes to solve the following equation:

$$\bar{C} = C_o - \int_0^T C_1 (T) \frac{dP(T)}{dT} \cdot dT \Longrightarrow \min \qquad /4/$$

This solution can be well used here by solving the objective function:

$$\bar{C} \cong C_o(P_1, P_2, \dots, P_n) - \sum_{E=1}^{n} C_1(P_1, P_2, \dots, P_n, \frac{t_{k-1} - t_k}{2} \times \qquad /5/$$

$$\times (P_k - P_{k-1}) \Longrightarrow \min$$

where C_o - initial costs

$C_1/T/$ - function of operating costs

$\bar{C}$ - estimated costs in full-service life

P_k, P_{k-1} - given coorinates of reliability function.

The problem was transformed to an optimal design problem, where specifying the minimum of the cost-function the optimum of the reliability curve is also obtained beside the optimal structural dimensions.

In addition the permissible failure probability can be prescribed [2] and other conditions (stiffness, buckling antivibration) can and should be taken into account in solving the objective function.

REFERENCES

[1] Bolotin, V.V.: Metodi teorii verojatnosztei i teorii nadezsnosti v raszcsetah szooruzsenii. Sztroizdat, Moskva 1982.

[2] Konstruiren mit hochfesten und verschleissfesten Blechen. Svenkst Stal AB, Division Grobblech.

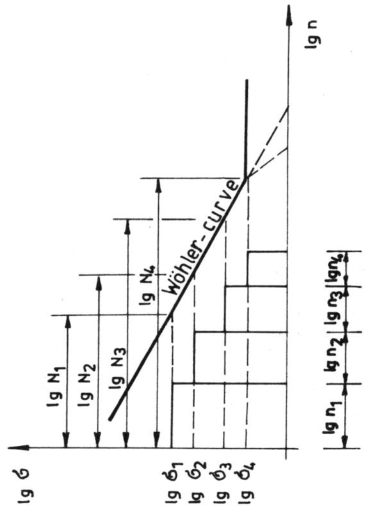

Figure 1 Different loading steps in a programme loading.

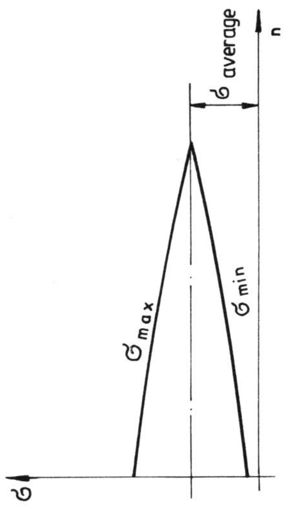

Figure 2 The number of different stress-cycles.

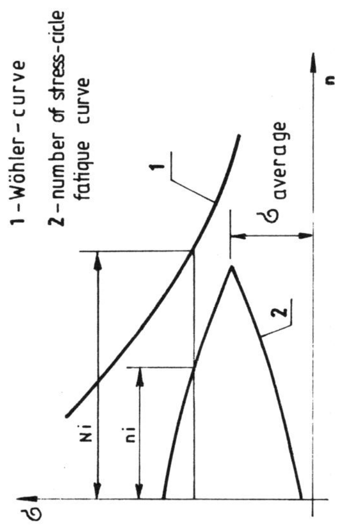

Figure 3 Combining the stress-cycles with the Wöhler-curve.