ON THE RELIABILITY OF FINITE ELEMENT J-INTEGRAL EVALUATION IN FRACTURE ANALYSIS

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Several finite element idealisations are used to determine J values for a number of crack geometries. The specimens are represented by regular and graded rectangular isoparametric elements with or without singularity crack tip elements. The path independency of the J values is investigated and convergence studies on the results are carried out. It is concluded that accurate estimates for J can be obtained using simple meshes which can easily be generated automatically.

INTRODUCTION

The use of the J-integral in linear elastic and elastic plastic fracture analysis has received considerable attention in recent years. Although some expressions have been developed for standard test specimens, obtaining solutions for J in actual components appears to be difficult and it is generally necessary to use finite element methods. The attractiveness of the J-integral approach lies in the fact that the integral is path-independent and thus enables one to choose a path remote from the tip of the crack and avoid the steep gradients of the strains in that vicinity. In the determination of J by the finite element method, therefore, it may be considered unnecessary to employ special elements at the crack tip which more accurately characterise the singularity conditions there. Moreover, Provided that the plastic zone at the crack tip is not

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too large, one may justify an elastic analysis on the grounds that at sufficiently large distances from the crack tip, the material behaves elastically and that it is not affected significantly by the plasticity there. This would be attractive to practicing engineers, as elastic-plastic analysis is much more complicated.

In practice, however, the path independency of J cannot always be guaranteed. This appears to be influenced by the finite element mesh employed in the analysis. Considerably smaller elements are required near the crack tip and many workers have used element topologies requiring a considerable amount of data preparation. It would clearly be advantageous to be able to use uniformly graded meshes which can easily be generated automatically. In the absence of analytical solutions, for guidance, it is necessary to carry out convergence studies before reliance on the results can be assumed. It is felt that this can only be effectively made by the employment of uniform meshes.

THEORETICAL CONSIDERATIONS

Following the Westergaard method of stress analysis, the solution for the stress intensity factor for an infinite plate containing a central, through thickness, sharp crack of length 2a is given by

$$K = \sigma \, (\pi a)^{\frac{1}{2}} \qquad (1)$$

Where σ is the remote applied tensile stress.

For a practical specimen of finite dimensions the above expression is modified by incorporating a correction factor which is obtained from a stress analysis of the cracked geometry. A general form of the expression for the stress intensity factor is

$$K = C \, \sigma \, (\pi a)^{\frac{1}{2}} \qquad (2)$$

Where C is the correction factor which is a function of a/W . W being the width of the plate and 'a' half the crack size for an embedded crack and the full crack size for an edge crack. Expressions for C have been developed for different geometries and some of these will be cited in the next section.

Irwin demonstrated that for linear elastic fracture mechanics (LEFM) conditions, the elastic stress field approach is equivalent to the Griffith energy balance concept and that the strain energy release rate G is related to the stress intensity factor by

$$K^2 = E' G \tag{3}$$

Where $E' = E$ for plane stress

and $E' = E/(1 - \mu^2)$ for plane strain

Another concept as a fracture criterion, which is based on energy balance principles, is the J-integral that was first introduced by Rice. For a two dimensional crack problem this is given by

$$J = - dP/da \tag{4}$$

Where P contains the elastic strain energy of the cracked plate and the work done by the external forces. Under LEFM conditions J may be viewed as the energy available for crack extension which is equivalent to G, the strain energy release rate.

Rice defined J as an integral quantity evaluated along a path, enclosing the crack tip, which has initial and end points lying on the two crack flanks. This is

$$J = \int_{\Gamma} (W\, dy - \underline{T} \frac{\delta \underline{u}}{\delta x} ds) \tag{5}$$

In this expression, W is the strain energy density, $\underline{T}$ the traction vector on a plane defined by the outward normal and $\underline{u}$ is the displacement vector.

Using the above relations, it is possible to calculate the stress intensity factor K indirectly from G or J. The J-integral concept is useful because it can also be used for nonlinear elastic behaviour provided that no unloading takes place in any part of the material. Furthermore, using the deformation theory of plasticity, J can be extended to model elasto-plastic behaviour of a material. From a computational view point, the J-integral concept is attractive because the value of J is independent of the path chosen provided that the initial and end points of the path are on opposite faces of the crack and that the crack faces are stress free. This path independency allows a calculation to be made along a contour remote from the crack tip, thus avoiding the steep gradients of the strains and any plasticity there. Because J can be considered as an elastic-plastic energy release rate, there must be a critical value, J_c , which predicts the onset of crack extension. This is analogous to G_c in LEFM.

CRACK GEOMETRIES

Finite element analyses, employing 8-noded isoparametric elements, were carried out to compute J values for the crack geometries shown in figure 1. These, together with their corresponding correction factors C (equation 2) , given by Ewalds and Wanhill (1), are given below

(a) Single edge cracked specimen (SECS)

$$C = 1.12 - 0.231\ a/W + 10.55\ (a/W)^2 - 21.72\ (a/W)^3 + 30.39\ (a/W)^4 \qquad (6)$$

This solution which is due to Brown and Srawley is accurate to within 0.5% for $a/W \leq 0.6$.

(b) Double edge cracked specimen (DECS)

$$C = \left[\frac{W}{\pi a} \left(\tan \pi a/W + 0.1 \sin 2\pi a/W \right) \right]^{1/2} \qquad (7)$$

Which is due to Irwin.

(c) Centre cracked specimen (CCS)

$$C = 1 + 0.256\ a/W - 1.152\ (a/W)^2 + 12.20\ (a/W)^3 \qquad (8)$$

Due to Brown and is accurate to within 0.5% for $a/W \leq 0.35$.

SOLUTION PROCEDURE

Numerical solutions for the crack geometries shown in figure 1 were obtained using 8-noded plane stress isoparametric elements and a range of regular and graded meshes. Due to symmetry only one-half of SECS or one-quarter of DECS and CCS was analysed. In order to study the convergence characteristics of the results, the part-specimen analysed was divided into 8x4, 8x6, 8x8, 16x6 and 16x8 rectangular elements. In this description the first figure refers to the number of equally spaced elements in the width direction and the second refers to the number of elements along the half-length of the specimen.

Three solutions were obtained for each geometry. In the first the specimen was divided into elements which were identical in size and were all of the conventional type. The second solution was obtained using the same mesh but with enriched crack tip elements which contain the appropriate singularity associated with LEFM. This was achieved by moving the middle nodes of the sides meeting at the crack tip to the quarter positions. In the third solution a graded mesh with enriched crack tip elements was employed in which the elements reduced in size as the line containing the crack was approached. Figure 2 shows the graded mesh idealisation for the 8x8 mesh. The specimens analysed had aspect ratios $2l/W = 4$ and Poisson's ratio was 0.3 .

RESULTS AND DISCUSSION

J values were computed along several paths such as those shown in figure 2 for the 8x8 mesh. From these correction factors, C, were obtained through the use of equations 2-5 . Convergence curves for the SECS using the three procedures outlined in the previous section are given elsewhere (2). There, it was shown that accurate solutions were obtained using a graded mesh of 8x8 elements with enriched singular crack tip elements. Figure 3 gives the non-dimensional parameter C as the crack size is increased. It can be seen that, except for very small cracks, good agreement with the analytical solution of equation 6 is obtained. This is surprising when considering the large crack tip element sizes employed in the analysis.

In figure 4 the results for the SECS with $a/W = 0.5$ are given for several contours enclosing the crack tip.

Here r is defined as the average distance from the crack tip to the three lines describing half the contour adopted for the evaluation of J. The results demonstrate that path independency of the J-integral can be confirmed using the idealisations (with crack tip elements) employed in the present analysis.

Figure 5 gives the convergence characteristics for the DECS, with a/W = 0.25, for the three idealisations employed in the present study. The results are expressed as a function of the total number of degrees of freedom used in each analysis. The corresponding rectangular meshes are also shown. The results presented here are those obtained from contours furthest away from the crack tip. It can be seen that convergence is rapid and that significant improvement is obtained when singularity elements are employed in the idealisation. In figure 6 the results are shown as a function of the crack size using graded meshes with singularity crack tip elements. It can be seen that good agreement with the analytical solution is obtained.

The results for the CCS are given in figures 7 and 8. It can be seen, from figure 7, that considerable improvement in the results is obtained if the regular mesh is replaced by a graded one with smaller elements near the crack-line.

CONCLUSION

The work presented in this paper shows that, for linear elastic analyses, it is possible to obtain accurate estimates of J using uniformly graded meshes in which the elements are not focussed onto the crack tip. Despite the large size of the elements containing the crack tip, used in the present analysis, considerable improvement in the results is obtained by moving the mid-side nodes of these elements to the quarter positions.

REFERENCES

(1) Ewalds, H.L. and Wanhill, R.J.H., "Fracture Mechanics", Edward Arnold (Publishers) Ltd, London, U.K., 1986.

(2) Sabir, B.B., "On the J-Integral as Computed by Finite Elements", Proceedings, International Conference on Computational Engineering Science, ICES-88, Atlanta, Georgia, U.S.A., In press.

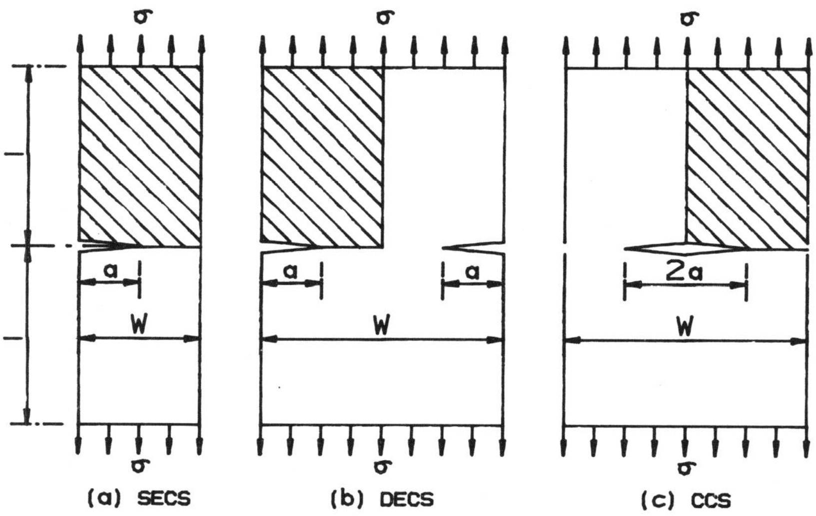

FIGURE 1 CRACK GEOMETRIES

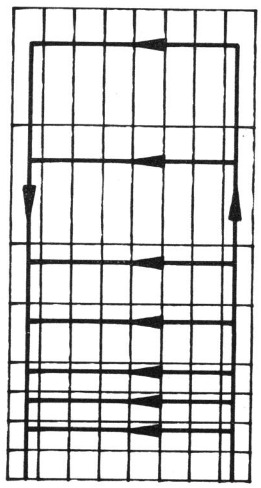

FIGURE 2 GRADED 8X8 MESH

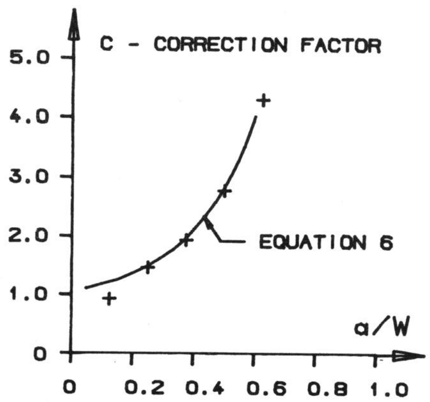

FIGURE 3 VARIATION OF C WITH CRACK SIZE - SECS

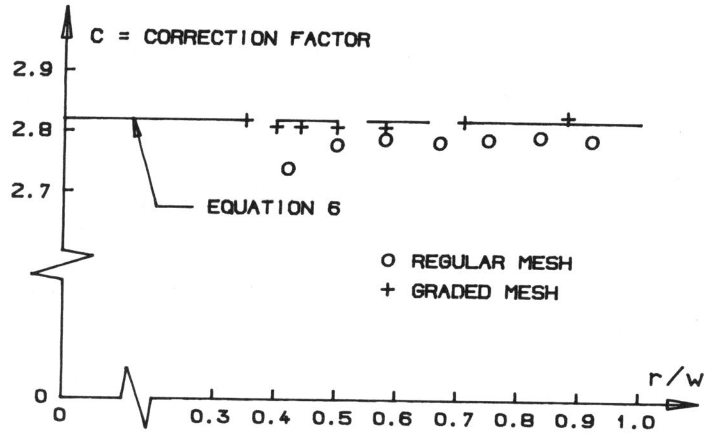

FIGURE 4 VARIATION OF C WITH AVERAGE RADIAL DISTANCE OF THE CONTOUR FROM THE CRACK TIP

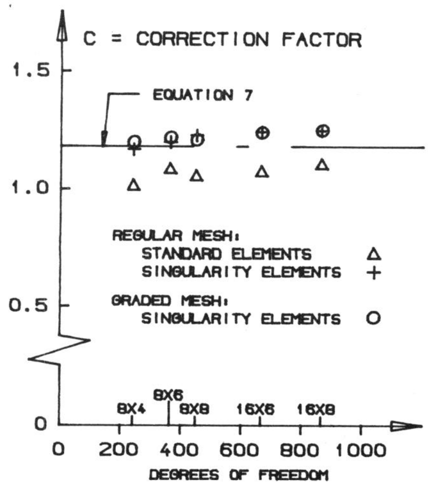

FIGURE 5 CONVERGENCE OF C FOR DECS WITH a/W = 0.25

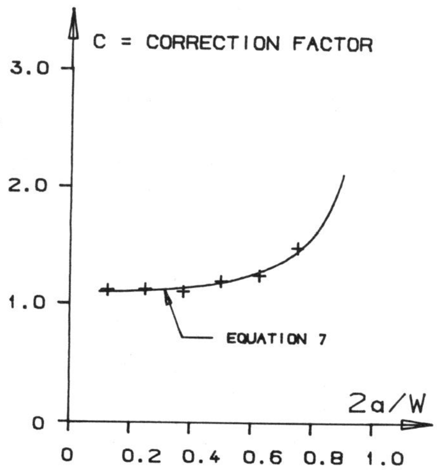

FIGURE 6 VARIATION OF C WITH CRACK SIZE - DECS

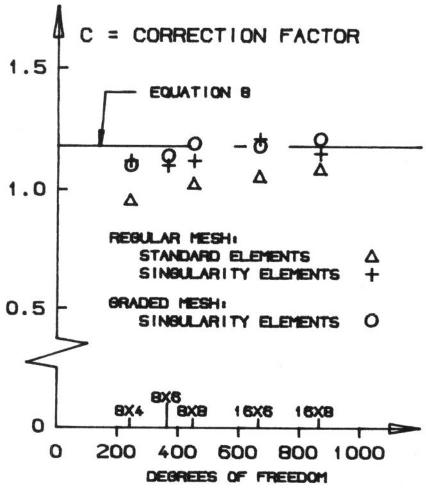

FIGURE 7 CONVERGENCE OF C FOR CCS WITH a/W = 0.25

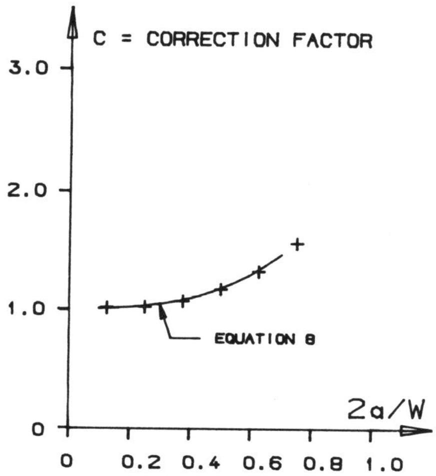

FIGURE 8 VARIATION OF C WITH CRACK SIZE - CCS