ON A GENERALIZATION OF IRWIN'S FORMULA FOR CURVILINEAR INTERFACE CRACKS IN BRITTLE TWO-PHASE COMPOSITE STRUCTURES

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A calculation of the total strain energy release rate G at the tip of a semi-infinite curvilinear interface crack between dissimilar media has been performed by using complex function theory including the conformal mapping method. The resulting Hilbert problem could be solved in a closed form manner leading to a relationship between the energy release rate and the associated complex stress intensity factors representing a generalization of Irwin's formula for straight cracks in a homogeneous medium.

INTRODUCTION

The fracture behaviour of unidirectionally fiber reinforced composites subjected to mechanical and/or thermal loading is regulated by the appearance of elementary failure mechanisms in the microstructure known as matrix and interface cracks, respectively. Further, as numerous experimental investigations show failure appears very often by branched crack systems consisting of a combination of curvilinear matrix and interface cracks. Thereby from a fracture mechanical point of view the relationship between the total strain energy release rate G at the tip of a curvilinear interface crack and the corresponding stress intensity factor, respectively, is of basic interest. Several investigations have been performed in the past concerning the generalization of Irwin's formula (1) for the case of non-coplanar crack extension, Hussain et al (2), Hayashi and Nemat-Nasser (3), Ichikawa and Tanaka (4). Thereby the references (3,4) contain a detailed analysis of the relationship between the energy release rate G and the stress intensity factors, respectively, for non-coplanar crack growth

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(crack kinking) under combined Mode I, II and III loading also considering the case of the orthotropic elastic solids, e.g. fiber reinforced composites.

FORMULATION OF THE PROBLEM

In this paper, a two-phase compound consisting of two half-planes of homogeneous, isotropic and linearly elastic materials with different elastic properties μ_j, κ_j (j=1,2) is considered containing a curvilinear semi-infinite crack L' in the interface L (cf. Fig.1). Here, κ_j (j=1,2) are the well-known quantities

$$\kappa_j = \begin{cases} 3 - 4\nu_j & \text{for plane strain} \\ \dfrac{3-\nu_j}{1+\nu_j} & \text{for plane stress} \end{cases} \tag{1}$$

Now, the aim of the following calculations consists in the finding of a closed form relationship between the total strain energy release rate G and the corresponding complex stress intensity factor $K_1 = K_{1I} - iK_{1II}$ at the tip of this curvilinear semi-infinite interface crack L' by using the method of conformal mapping known from the complex function theory. Therefore, Fig. 2 shows the conformal mapping of the physical z-plane containing the semi-infinite curvilinear interface crack L' onto the mathematical ζ-plane containing the semi-infinite straight interface crack $\tilde{L}'$ along the negative real ξ-axis with the crack tip located at $\zeta = 0$. Thereby the holomorphic mapping function

$$z = \omega(\zeta) = h(\zeta) + k(\zeta) \quad , \quad \zeta = \xi + i\eta \tag{2}$$

is chosen in sich a way that the curve L is defined by

$$L = \{\omega(\zeta) : \zeta \in \mathbb{R}\} = \omega(\mathbb{R}) \tag{3}$$

with analytical functions h and k, respectively, in each point of L.

Further, the cracked two-phase medium is loaded by external loads such that at the interface L the following boundary and continuity conditions hold true

$$\{\sigma_n(t) - i\sigma_{nt}(t)\}_j = 0 \qquad ; \quad t \in L' \tag{4}$$

$$\{\sigma_n(t) - i\sigma_{nt}(t)\}_1 = \{\sigma_\eta(t) - i\sigma_{nt}(t)\}_2 \quad ; \quad t \in L'' \tag{5}$$

$$\{u_x(t) + iu_y(t)\}_1 = \{u_x(t) + iu_y(t)\}_2 \quad ; \quad t \in L'' \tag{6}$$

Further, the desired complex potentials $\Phi_j(z)$, $\Psi_j(z)$, (j=1,2) which have to be determined from the solution of the mixed boundary value problem (4) - (6) must behave like

$$\Phi_j(z) = -\frac{\{X+iY\}_j}{2\pi z} + O(\frac{1}{z}) \qquad (7)$$

; (j=1,2)

$$\Psi_j(z) = \frac{\{X+iY\}_j}{2\pi z} + O(\frac{1}{z}) \qquad (8)$$

with $\{X+iY\}_j$ as the complex resultant force vector in S_j (j=1,2).

SOLUTION OF A MIXED BOUNDARY VALUE PROBLEM

By using the well-known formulae of Kolosov-Muskhelishvili (5) in the z-plane

$$\{\sigma_{xx}+\sigma_{yy}\}_j = 2\{\Phi_j(z)+\overline{\Phi_j(z)}\} \qquad (9)$$

$$\{\sigma_{yy}-\sigma_{xx}+2i\sigma_{xy}\}_j = 2\{\bar{z}\Phi_j'(z)+\Psi_j(z)\} \qquad (10)$$

$$2\mu_j\{u_x+iu_y\}_j = \kappa_j\phi_j(z) - z\overline{\phi_j'(z)} - \overline{\psi_j(z)} \quad ; \quad (j=1,2) \qquad (11)$$

where

$$\Phi_j(z) = \phi_j'(z) \quad ; \quad \Psi_j(z) = \psi_j'(z) \quad ; \quad (j=1,2) \qquad (12)$$

and with holomorphic functions $\Phi_j(z)$ and $\Psi_j(z)$ in the simply connected regions S_j (j=1,2) a conformal mapping by applying the mapping function (2) leads to the following expression for the boundary and continuity conditions in the ζ-plane, Herrmann and Meiners (6)

$$\{\sigma_n - i\sigma_{nt}\}_j = \tilde{\Phi}_j^{\pm}(\xi) + \overline{\tilde{\Phi}_j^{\pm}(\xi)} + \frac{\omega(\xi)}{\omega'(\xi)}\,\overline{\tilde{\Phi}_j'^{\pm}(\xi)} + \frac{\omega'(\xi)}{\omega'(\xi)}\,\overline{\tilde{\Psi}_j^{\pm}(\xi)} \quad ;$$

$$\xi \in \tilde{L} = \tilde{L}' \cup \tilde{L}'' \qquad (13)$$

$$2\mu_j\frac{\partial}{\partial\xi}(u_x+iu_y)_j = \kappa_j\tilde{\Phi}_j^{\pm}(\xi) - \overline{\tilde{\Phi}_j^{\pm}(\xi)} - \frac{\omega(\xi)}{\omega'(\xi)}\,\overline{\tilde{\Phi}_j'^{\pm}(\xi)} - \frac{\overline{\omega'(\xi)}}{\omega'(\xi)}\,\overline{\tilde{\Psi}_j^{\pm}(\xi)} \quad ;$$

$$\xi \in \tilde{L}'' \quad (j=1,2) \qquad (14)$$

where the upper and lower signs remain to j=1 and j=2, respectively. Further, by some lengthy calculations it can be shown that the relations (13), (14) are equivalent to the following Hilbert problem:

$$\tilde{\Phi}_1^+(\xi) + \tilde{\Phi}_2^-(\xi) = 0 \quad ; \quad \xi \in \tilde{L}' \tag{15}$$

$$\tilde{\Phi}_1^+(\xi) - \frac{1}{g}\tilde{\Phi}_2^-(\xi) = 0 \quad ; \quad \xi \in \tilde{L}'' \tag{16}$$

with the definition of the bimaterial constant

$$g = \frac{\mu_2\kappa_1+\mu_1}{\mu_1\kappa_2+\mu_2} \quad ; \quad \beta = \frac{\ell ng}{2\pi} \tag{17}$$

The solution of the Hilbert problem (15), (16) reads as follows

$$\tilde{\Phi}_1(\zeta) = \zeta^{-(1/2+i\beta)}F(\zeta) \tag{18}$$

$$\tilde{\Phi}_2(\zeta) = g\zeta^{-(1/2+i\beta)}F(\zeta) \tag{19}$$

where $F(\zeta)$ is an analytic function with properties defined by the requirements for the complex potentials stated in section 2.

Then the asymptotic near-tip solution of the Hilbert problem (15), (16) can be obtained from the general solution (18), (19) by the complex potentials

$$\Phi_j(\zeta) = K_j\zeta^{-(1/2+i\beta)} \quad ; \quad (j=1,2) \tag{20}$$

with the definitions

$$K_1 = F(o) \quad , \quad K_2 = gK_1 \tag{21}$$

and

$$K_j = K_{j,I} - iK_{j,II} \quad ; \quad (j=1,2) \tag{22}$$

DETERMINATION OF THE ENERGY RELEASE RATE

By using the asymptotic near-tip solution (20) of the Hilbert problem (15), (16) in the ζ-mapping plane the complex stress vector along the interface $\tilde{L}''$ can be calculated according to

$$\sigma_n(\xi) - i\sigma_{nt}(\xi) = (1+g)K_1\xi^{-(1/2+i\beta)} \quad ; \quad \xi \in \tilde{L}'' \tag{23}$$

Further, the complex crack opening displacement is defined as follows

$$\Delta(u_t(\xi)+iu_n(\xi)) = \{\frac{\kappa_1+1}{\mu_1} + \frac{\kappa_2+1}{\mu_2}\}\frac{K_1}{1-2i\beta}\,\xi^{1/2-i\beta} \quad ; \quad \xi \in \tilde{L}' \tag{24}$$

Finally, the total energy release rate G at the tip of the straight interface crack in the ζ-plane can be obtained by use of Irwin's modified crack closure integral

$$G = \lim_{\Delta a \to o} \frac{1}{2\Delta a} \int_{\xi=o}^{\Delta a} \mathrm{Re}\{i[\sigma_n(\xi,0) - i\sigma_{nt}(\xi,0)][\Delta u_t(\xi,\Delta a) - i\Delta u_n(\xi,\Delta a)]\}d\xi \tag{25}$$

By inserting equations (23), (24) into equation (25) the following closed form expression for the total strain energy release rate is obtained

$$G = \frac{\pi}{4}(1+g)^2 (\frac{\kappa_2}{\mu_2} + \frac{1}{\mu_1}) K_1\bar{K}_1 \tag{26}$$

Further investigations concerning the relationship between the total energy release rate and the corresponding complex stress intensity factors by consideration of the anisotropy of the dissimilar materials are under way.

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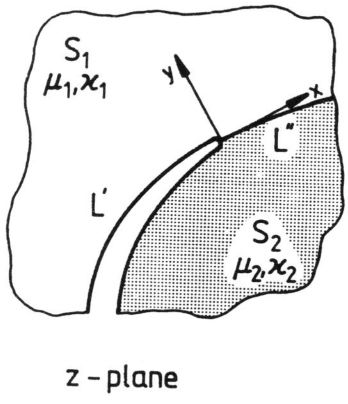

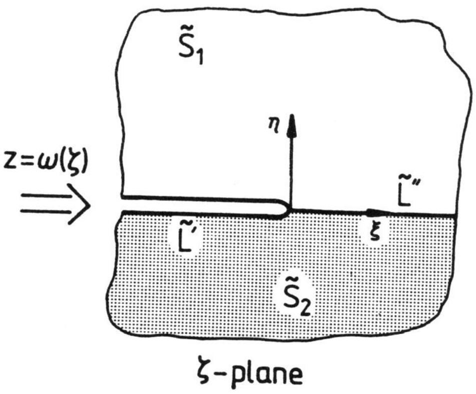

Figure 1 Semi-infinite curvilinear interface crack between dissimilar media

Figure 2 Straight interface crack in the mathematical ζ-plane after conformal mapping