# NEW SPECIAL FINITE ELEMENT FOR ANGULAR NOTCHES

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## INTRODUCTION

The elastic analysis of structures containing cracks and re-entrant corners or notches leads to stress singularities (Muskhelishivili (1), Williams (2)). In this paper, a special finite element with global-local interpolation is used to compute the stress singularity arising at the tip of a sharp angular notch.

## ELEMENT FORMULATION

Let us consider the angular notch shown in Fig. 1 ($\pi<\beta<2\pi$). The effect of singularity is included in an eight-noded element by adding a global term in the interpolation of displacements (Benzley (3))

$$u_i = N_j d_i^j + B(F_i - N_j F_i^j) \quad (1)$$

where $N_j$ are local shape functions, $d_i^j$ are nodal displacements, B is a global degree of freedom and $F_i^j$ is the value of $F_i$ at node j, $F_i$ being the specific singular displacement at the vertex of the notch

$$F_i = r^{\lambda} f_i(\lambda, \theta) \quad (2)$$

Here $\lambda$ is the smallest positive root of the characteristic equation

$$\sin(\lambda\beta) = \pm\lambda \sin\beta \quad (3)$$

which varies between $\lambda = 0$ ($\beta=\pi$) and $\lambda=1/2$ ($\beta=2\pi$), and $f_i(\lambda, \theta)$ are known functions.

The singularity analysis of eq. (2) shows that it is useful when dynamic effects are taken into account. In this case, B in eq.(1) has the meaning of a time dependent parameter. Hence velocities and accelerations

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can be expressed as

$$\dot{u}_i = N_j \dot{d}_i^j + \dot{B} (F_i - N_j F_i^j)$$
$$\ddot{u}_i = N_j \ddot{d}_i^j + \ddot{B} (F_i - N_j F_i^j) \quad (4)$$

To maintain inter-element compatibility transition elements have been defined. In these elements the global interpolation is corrected by suitable weight functions such that they equal 1 on boundaries adjacent to special elements and equal 0 on boundaries adjacent to isoparametric elements.

## NUMERICAL RESULTS AND CONCLUSIONS

The present formulation has been compared in static problems for different notch angles with other available results in the literature. For the dynamic problem we consider an elastic body with material properties $\rho=2450 Kg/m^3$, $E=7.561 x 10^{10} N/m^2$, $\upsilon=0.286$ and plane strain conditions (Fig. 2). A uniform tensile stress with a Heaviside step function time dependence is assumed to act along the upper and lower boundaries.

Fig. 3 shows the time variation of the parameter B. It can be seen that B equals the static value at $t=32\mu s$ reaching a maximum at $t=47\mu s$ and then decays in amplitude oscillating about the static solution. The present results show a close agreement with those obtained by Atkinson et al (4) using a path independent integral.

## REFERENCES

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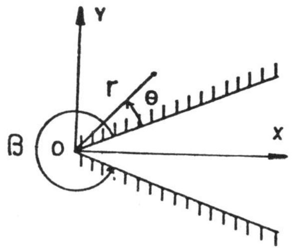


Fig. 1 - Coordinate system for the angular notch.

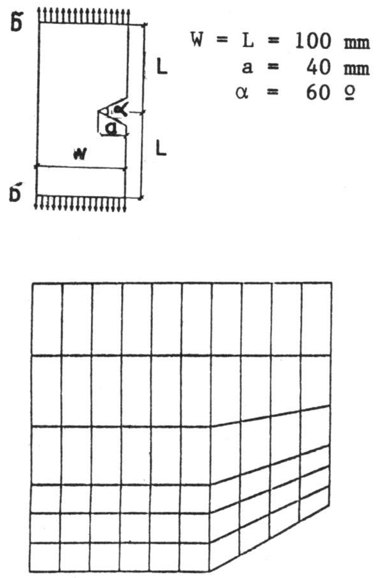


Fig. 2 - Specimen geometry and finite element model.

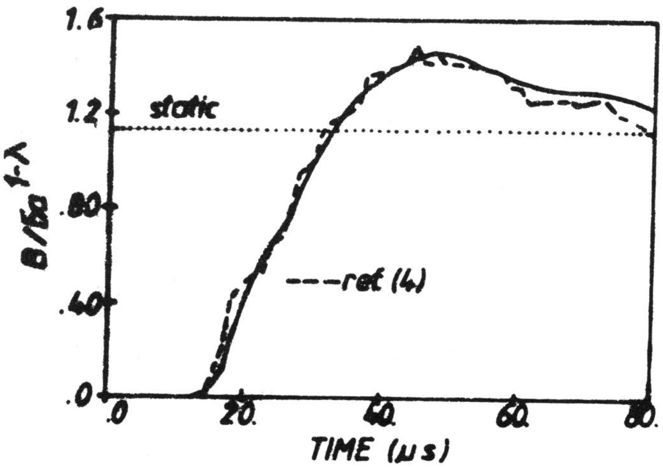


Fig. 3 - Variation of B with time.