LIMITING STATE OF MEMBERS IN NON-UNIFORM STRESS FIELD

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Test results of flat specimens made of D19 AT and BCт3cn5 structural materials investigated for stress-strain state non-uniformity influence on local yield strength are described. A condition of material transition to plastic state in stress concentration zone is derived, which allows for the influence of stress state non-uniformity.

This paper considers flat specimens made of D19AT and BCт3cn5 with elliptical stress concentrators and major semi-axis constant and equal to 10 mm, tested for local yield strength by holographic interferometry (1). Specimens with central holes and stress concentration factors 3-10 were tested for stress values of material transition to plastic state in local zone by the same method.

Figure 1 shows test results of stress state non-uniformity influence on local yield strength in stress concentration zone. The investigation revealed that true initiation of local yield in stress concentration zone doesn't correspond to values, calculated on conventional plasticity theory, particularly, on Mises-Huber-Hencky criterion. The experimental results indicate the rise of local yield stress of material in non-uniform stress field.

As seen from the graph, the yield strength substantially rises even at moderate stress concentration level.

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The relationship between local yield strength of flat specimens with central elliptical holes and stress concentration factor takes the form:

$$\sigma_T^* = (0,9 + 0,1\alpha_\sigma)\sigma_T \tag{1}$$

or in more common form:

$$\sigma_T^* = [1 + \gamma(\alpha_\sigma - 1)]\sigma_T \tag{2}$$

where γ is an experimental constant number (here, by Ref. 2 $\gamma = 0.1$). In Fig. 1 this relationship is shown in linear form; σ_T^* is the "effective" material yield strength, depending on stress state non-uniformity. Relationship (2) may be transformed to the form (3):

$$\sigma_T^* = \left(1 + \sqrt{L_0/L_i}\right)\sigma_T \tag{3}$$

where $L_o = 1.6a\gamma^2$ (here, a is a major semi-axis of ellipse); typical size of non-uniform stress distribution $L_i = 1/\bar{G}_i$ (here, $\bar{G}_i = |\text{grad}\,\sigma_i|/\sigma_i$ is a relative stress intensity gradient). In case of our experiments (2) L_o= 0.16 mm. For elliptical concentrator $L_i = 8a\alpha_\sigma/[(5\alpha_\sigma+2)(\alpha_\sigma-1)^2]$. The relationship (3) and experimental results are shown in Fig. 2. For limiting state criterion of structural materials one may take the yield condition in the form:

$$\sigma_i = \sigma_T^* \tag{4}$$

The criterion (4) with relationship (3) in the right part shows the condition of material transition to plastic state in stress concentration zone, that allows for the influence of stress state non-uniformity.

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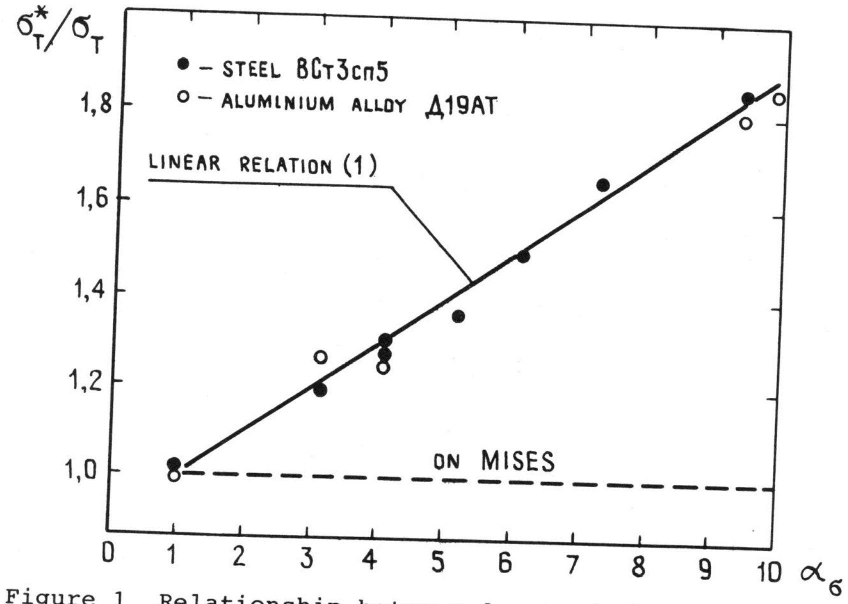

Figure 1 Relationship between local yield strength and stress concentration

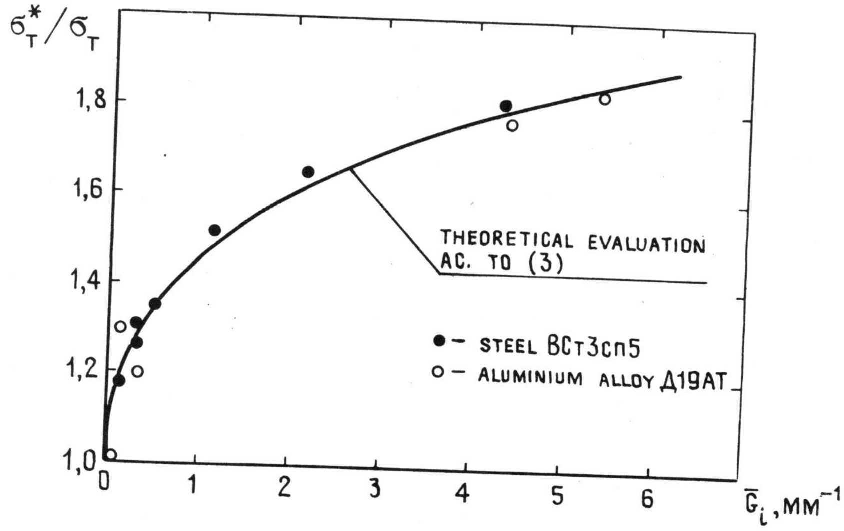

Figure 2 Relationship between local yielding stress and relative stress gradient level

A PARTLY UNBONDED RIGID FIBER INCLUSION IN AN INFINITE MATRIX

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The plane elastostatic problem of a rigid fiber inclusion which is perfectly bonded along one of its faces to an elastic matrix, while its other face forms a crack is studied (Figure 1). The matrix is subjected to a biaxial stress state at infinity, while the crack faces are loaded by a uniform internal pressure. Using the method of complex potentials the problem is reduced to a Hilbert problem, the solution of which gives the stress and displacement fields in the matrix in closed form. The various unknown coefficients entering in the solution of the problem are determined by satisfying the boundary conditions.

For the analysis of the fracture behavior of the body the local stress field in the neighborhood of the crack tips is investigated. It is obtained that

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{bmatrix} = \frac{e^{\lambda\theta}}{\rho^{3/4}}\left\{K_I\begin{bmatrix}3c_1-\hat{e}\,s_2-2l_1\,\gamma_2\\ c_1+\hat{e}\,s_2+2l_1\,\gamma_2\\ s_1+\hat{e}\,c_2+2l_1\,\gamma_1\end{bmatrix} - K_{II}\begin{bmatrix}3s_1-\hat{e}\,s_2+2l_1\,\gamma_1\\ s_1+\hat{e}\,c_2-2l_1\,\gamma_1\\ -c_1-\hat{e}\,s_2+2l_1\,\gamma_2\end{bmatrix}\right\} + \frac{e^{\lambda\theta}}{\rho^{1/4}}\left\{S_I\begin{bmatrix}3c_3+\hat{e}\,s_4-2l_2\,\delta_2\\ c_3-\hat{e}\,s_4+2l_2\,\delta_2\\ s_3-\hat{e}\,c_4+2l_2\,\delta_1\end{bmatrix} - S_{II}\begin{bmatrix}3s_3+\hat{e}\,c_4+2l_2\,\delta_1\\ s_3-\hat{e}\,c_4-2l_2\,\delta_1\\ -c_3+\hat{e}\,s_4+2l_2\,\delta_2\end{bmatrix}\right\} \quad (1)$$

where

$$c_1=\cos(\tfrac{3}{4}\theta+r),\quad c_2=\cos(\tfrac{3}{4}\theta-r),\quad c_3=\cos(\tfrac{\theta}{4}+r),\quad c_4=\cos(\tfrac{\theta}{4}-r)$$

$$s_1=\sin(\tfrac{3}{4}\theta+r),\quad s_2=\sin(\tfrac{3}{4}\theta-r),\quad s_3=\sin(\tfrac{\theta}{4}+r),\quad s_4=\sin(\tfrac{\theta}{4}-r)$$

$$\gamma_1=\sin\theta\,\cos(\tfrac{7}{4}\theta+r-\tan^{-1}\tfrac{4\lambda}{3}),\quad \gamma_2=\sin\theta\,\sin(\tfrac{7}{4}\theta+r-\tan^{-1}\tfrac{4\lambda}{3}) \quad (2)$$

$$\delta_1=\sin\theta\,\cos(\tfrac{5}{4}\theta+r-\tan^{-1}4\lambda),\quad \delta_2=\sin\theta\,\sin(\tfrac{5}{4}\theta+r-\tan^{-1}4\lambda)$$

$$l_1=(\tfrac{9}{16}+\lambda^2)^{1/2},\quad l_2=(\tfrac{1}{16}+\lambda^2)^{1/2},\quad \hat{e}=e^{2\lambda(\pi-\theta)},\quad r=\lambda\ln\frac{\rho}{2\alpha},\quad \lambda=\frac{\ln\kappa}{4\pi}$$

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In equation (1) and (2) ρ and θ are the polar coordinates of a point in the neighborhood of the crack tip and $\varkappa=3-4\nu$ or $(3-\nu)/(1+\nu)$ for plane straim or generalized plane stress, respectively. For the case of an internal pressure p the values of K_I, K_{II} and S_I, S_{II} are given by

$$K_I = \frac{P(2\alpha)^{3/4}}{8(1+\varkappa)}\left(1+\frac{\sqrt{\varkappa}}{\pi}\ln\varkappa\right), \quad K_{II} = \frac{P(2\alpha)^{3/4}}{8(1+\varkappa)}\left(\frac{\ln\varkappa}{\pi}-\sqrt{\varkappa}\right)$$
$$S_I = \frac{P(2\alpha)^{1/4}}{8(1+\varkappa)}\left(3-\frac{\sqrt{\varkappa}}{\pi}\ln\varkappa\right), \quad S_{II} = \frac{P(2\alpha)^{1/4}}{8(1+\varkappa)}\left(\frac{\ln\varkappa}{\pi}+3\sqrt{\varkappa}\right) \quad (3)$$

For a biaxial stress field N,T at infinity we have

$$K_I = \frac{T(2\alpha)^{3/4}}{32}\left\{\left(1+\frac{3\ln\varkappa}{\pi\sqrt{\varkappa}}\right)s-\frac{\ln\varkappa}{\pi\sqrt{\varkappa}}+1\right\}$$
$$K_{II} = \frac{T(2\alpha)^{3/4}}{32}\left\{\left(\frac{\ln\varkappa}{\pi}-\frac{3}{\sqrt{\varkappa}}\right)s+\frac{1}{\sqrt{\varkappa}}+\frac{\ln\varkappa}{\pi}\right\}$$
$$S_I = \frac{T(2\alpha)^{1/4}}{32}\left\{3\left(1-\frac{\ln\varkappa}{\pi\sqrt{\varkappa}}\right)s+\frac{\ln\varkappa}{\pi\sqrt{\varkappa}}+3\right\} \quad (4)$$
$$S_{II} = \frac{T(2\alpha)^{1/4}}{32}\left\{\left(\frac{\ln\varkappa}{\pi}+\frac{9}{\sqrt{\varkappa}}\right)s+\frac{\ln\varkappa}{\pi}-\frac{3}{\sqrt{\varkappa}}\right\}$$

where s = N/T.

The variation of the normalized stress components $\bar{\sigma}_{\rho\rho}$, $\bar{\sigma}_{\theta\theta}$ and $\bar{\sigma}_{\rho\theta}$ with respect to polar angle θ for $\varkappa=2$, s=2 and $\rho/\alpha=5\mathrm{X}10^{-4}$ is shown in figure 2.

Based on the results of stress analysis the minimum strain energy density criterion developed by Sih was used to study the fracture behavior of the composite plate. The fluctuetion of the normalized strain energy density $(d\bar{W}/dV) = (dW/dV)\mu/T^2$ with the angle θ for the case of figure 2 is plotted in figure 3. The fracture path follows the angle at which $d\bar{W}/dV$ becomes minimum, while unstable fracture occurs when this minimum reaches a critical value. The variation of the angle θ_c that the fracture path subtends with the fiber and the critical quantity $(d\bar{W}/dV)_{min}$ versus the biaxiality factor s is shown in figure 4.

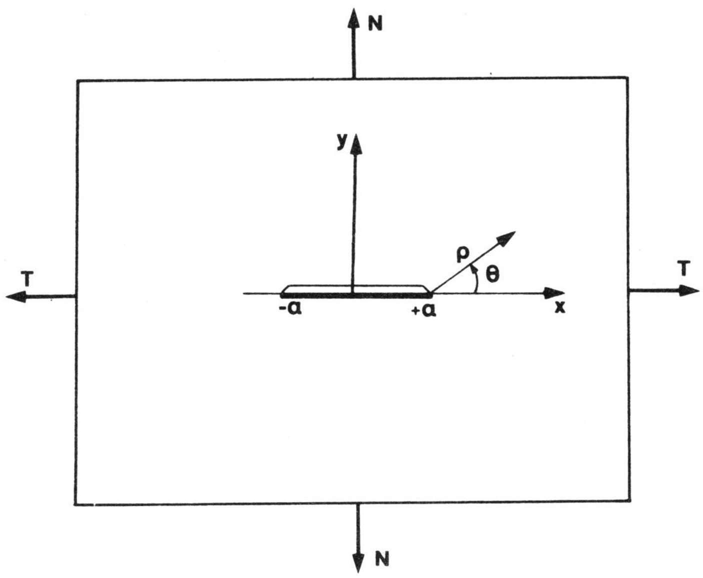

Figure 1 An infinite plate with a fiber inclusion.

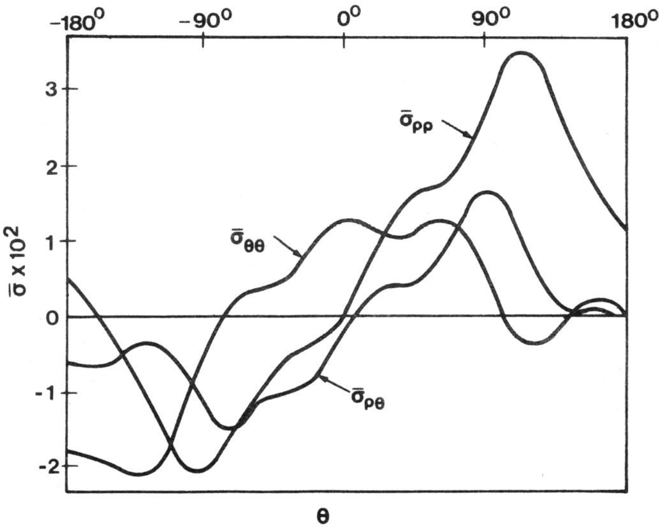

Figure 2 Variation of the local stress field with θ.

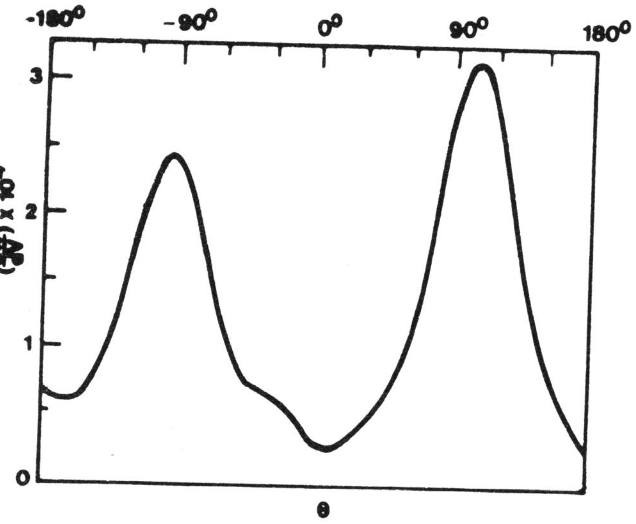

Figure 3 Variation of $(d\bar{W}/dV)_{min}$ with θ.

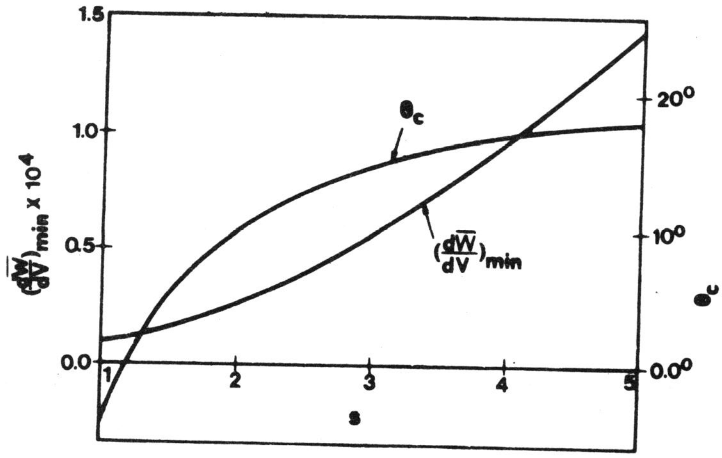

Figure 4 Variation of fracture angle and critical $(d\bar{W}/dV)_{min}$ with s.