INTERACTION OF BRITTLE FRACTURE AND BUCKLING OF COMPRESSED AND TORSIONED BARS WITH A BISYMMETRIC OPEN CROSS SECTION

Kowal Z.*

INTRODUCTION

This paper is an attempt to construct interaction curves. This can be done by introducing the limit bearing capacity of the cross-section measured by a parametric limiting bimoment B_k as a parametric fuction of the brittle fracture strength R_f and the axial force S

$$B_k = (R_f + S/A)\, J_\omega / \omega \tag{1}$$

where: A - area of the cross-section, J_ω - sector inertia-moment, ω - sector ordinate cross-section.

Bimoment $B < B_k$ as load of cross-section will be determined from differential equation of the form (2), shown by Kowal and Kubica [1].

$$EJ_\omega \phi^{IV} + (Si_o^2 - GJ_s)\phi'' = m_a + M_s \delta_{z=c} \tag{2}$$

where: ϕ - torsion-angle of cross-section, EJ_ω - sector stiffness of cross-section, GJ_s - stiffness of pure torsion, m_s - continuous torsional moment, M_s - torsional moment, δ - Dirac's symbol, $i_o = (J_x + J_y)/A$.

The bar load by bimoment is calculated from the differential equation (2), assuming a zero axial force S. Flexural torsional critical bearing capacity S_{cr} of the bar will also be determined from the differential equation (2) assuming m_s and $M_s = 0$, then we have

$$S_{cr} = (n^2\pi^2 + a^2 l^2)\, EJ_\omega / l^2 i_o^2 \tag{3}$$

where: $a^2 = GJ_s/EJ_\omega$, l - bar span, n - number of half-waves.

Algorithm of the construction of interaction curves is shown by the example of a cantilever compressed bar, loaded by a concentrated torsional moment on the free end of the bar.

* Technological University of Kielce, Poland

EXAMPLEX OF CONSTRUCTING INTERACTION CURVES

Let us take into consideration a bar shown in Figure 1. From the solution of the differential equation (2) for $k^2 = (Si_o^2 - GJ_s)/EJ_\omega > 0$ we have a maximal B

$$B = -M_s(\mathrm{tg}\, kl)/k < B_k \tag{4}$$

We obtain load of support cross-section by bimoment B from the solution of the differential equation (2) for S = 0

$$B = -M_s(\mathrm{th}\, al)/a \tag{5}$$

Taking into consideration the relationships (3,4,5), we obtain the equation of a family of interaction curves (6)

$$B\, al\, \mathrm{tg}\, kl = B_k kl\, \mathrm{th}\, al \tag{6}$$

Argument kl occuring in equation (6) will be transformed to the form (7) taking into consideration the relationship (3) and n = 0.5 for the cantilever bar

$$(kl)^2 = l^2(n^2\pi^2 - GJ_s)/EJ_\omega = (0.25\pi^2 + a^2l^2)S/S_{cr} - a^2l^2 \tag{7}$$

The final form of the interaction curve in the area

$a^2l^2(n^2\pi^2+a^2l^2) < S/S_{cr} \leq 1$, for $k^2l^2 > 0$ had the form

$$\frac{B}{B_k} = \frac{\mathrm{th}\, al}{al}\,\frac{\sqrt{(0.25\pi^2+a^2l^2)S/S_{cr}-a^2l^2}}{\mathrm{tg}\sqrt{(0.25\pi^2+a^2l^2)S/S_{cr}-a^2l^2}} \tag{8}$$

Lower bond of the family of interaction curves only for $a^2 = 0$. Then $B = -M_s l$ equation of the interaction curve assumes the form (9)

$$B\, \mathrm{tg}(0.5\pi^2\sqrt{S/S_{cr}}) = B_k\, 0.5\sqrt{S/S_{cr}} \tag{9}$$

In the interval $0 < S/S_{cr} < a^2l^2 - (n^2\pi^2 + a^2l^2)$, bimoment B determined from differential equation (2) is

$$B = -M_s l(\mathrm{th}\, bl)/bl < B_k \tag{10}$$

where: $(bl)^2 = (GJ_s - Si_o^2)/EJ_\omega = a^2 - l^2 - (0.25\pi^2 + a^2l^2)S/S_{cr}$

Interaction curve assumes the form:

$$\frac{B}{B_k} = \frac{\mathrm{th}\, al}{al}\,\frac{\sqrt{(0.25\pi^2+a^2l^2)S/S_{cr}-a^2l^2}}{\mathrm{th}\sqrt{(0.25\pi^2+a^2l^2)S/S_{cr}-a^2l^2}} \tag{11}$$

Figure 1 shows limiting curves determined from interaction equation derived for selected examples. Curve 1 for al = 0 refers to: a cantilever bar loaded by a moment concentrated at the end of the bar , a bar with a forked fix at the ends and a bar rigidly fixed, loaded by a concentrated torsional moment in midspan. Curves 2 refers to the same bars for al = 1. Curves 3 and 4 refer respectively to the cantilever bar loaded unformly along the bar's length, for al = 0 and al = 1.

REMARKS AND CONCLUSIONS

The introduction of the concept of limit bearing capacity of the cross-section of a non-free torsioned bar, measured by bimoment B_k as a parametric function of axial force S and brittle fracture strength R_f provide the possibility to construct interaction curves in dimensionless coordinates. The interaction curves depend on the coefficient al. Lower bond of the interaction curves is obtained for al = 0.

The characteristic feature of the interaction curves in dimensionless coordinates is their similarity for many cases of bars in spite of their different limit bearing capacity.

REFERENCES

(1) Kowal Z., Kubica E., Second Order Torsioning of the Thin-Walled Bars with a Bisymmetric Open Cross-Section, TU Wroclaw, PNIB 4, Metalowe Dzwigary Specjalne 1, Wroclaw 1971.

(2) Kowal Z., Interaction of Brittle Fracture and Stability Loss in Bended and Compressed Bars, 6-th European Conference on Fracture ECF, Amsterdam 1986.

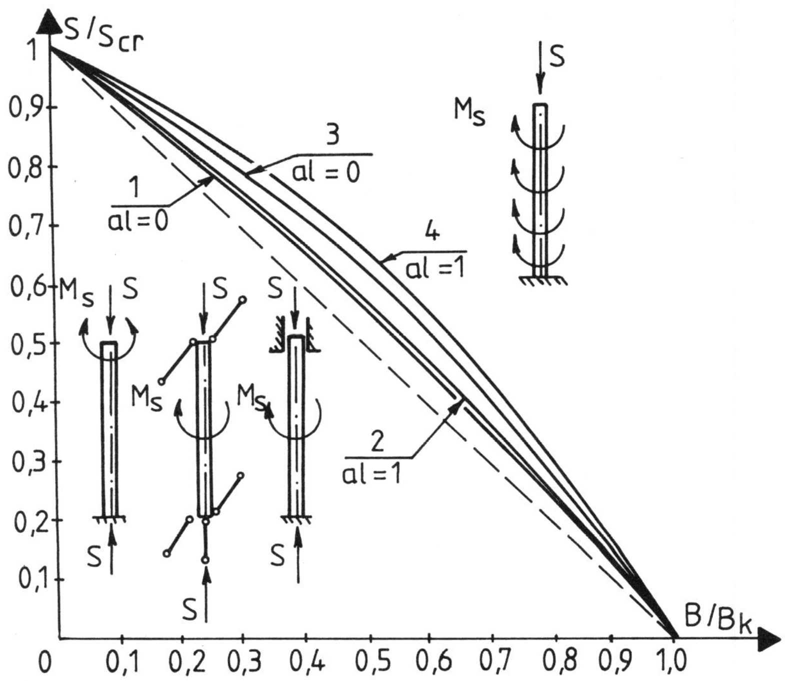

Figure 1 Examples of interaction curves