INFLUENCE OF FIXING PARAMETERS OF ORTOTHROPIC PLATES FROM COMPOSITE UPON QUANTITY OF COEFFICIENT OF STRESS INTENSITY

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In this paper we consider the ortothropic plate from composite when one part of it's contour is fixed and the other part is loaded. We propose the method to determine the coefficient of tension intensity. We discuss an example of the calculation of the intensity factor depending on the plate's parameters and fixing parameters.

In our model we take into account the main properties of the composite, the construction and the elasticity of fixing [1]. The only hypothesis we introduced was the incompressibility in direction y.

Thus, in the fixed part of the plate the displacement component in direction y is equal to null ($w \equiv 0$).

As a result, the next equations are satisfied (Figure 1):

$$\mathcal{L}_1(\tau) = (\partial^2\tau/\partial y^2)_{y=\pm 1} \qquad (x, y) \in G_1, \quad x > 0$$
$$\mathcal{L}_2(u) = 0 \qquad (x, y) \in G_2 \quad x < 0$$
$$\tau = G\frac{1}{h}\frac{\partial u}{\partial y} + \frac{1}{\ell}\frac{\partial w}{\partial x}, \qquad \sigma_x = \frac{E}{\ell} \times \frac{\partial u}{\partial x}, \quad E_y \to \infty (w \equiv 0) \tag{1}$$
$$\mathcal{L}_i = a_i^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \qquad i = 1, 2$$
$$a_i^2 = \left(\frac{E_x}{G}\,\frac{h^2}{\ell^2}\right)_i, \qquad -\alpha \leqslant x \leqslant 1, \ -1 \leqslant y \leqslant 1$$

and the boundary conditions are as follows:

$$y = \pm 1, \quad \tau = 0, \quad x > 0; \qquad m(u) = 0, \quad x < 0$$
$$x = -\alpha, \quad \ell(u) = 0. \tag{2}$$

where m and ℓ are boundary operators, the line $x = 1$ is loaded by Q,P and bending moment, the static conditions are satisfied according to Sen-Benana; u, w are the displacement components in x and y directions; σ_x, τ are normal and shear stresses; ℓ and h are length and half width of plate; E_x and E_y are Young's modulus in x and y directions and G is shear modul.

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The different fixing types are modelled with different boundary conditions or boundary operators m, ℓ.

Note, that this mathematical model corresponds to the first member of the assymptotic approach of the exact solution with the parameter $t = (E_x/E_y)^{1/4} \cdot h/\ell$.

The solution of the problem (1)-(2) appears as a sum of a regular part u_r, a singular part u_s and a partitive solution [1]. The coefficients of intensity are the amplitudes of the singular part of the solution and they can be found on the basis of the necessity of conformity of the boundary conditions at the points $x = 0$, $y = \pm 1$.

For example, the conditions of conformity for the fixing in Figure 2 can be written as

$$(u_r + u_s + u_o)_{\substack{x \to 0 \\ y = -1}} = 0 \qquad (3)$$

The most difficult and important problem is to construct the regular part of the solution. The method of operator's boundary conditions provides the real solution of this problem [2].

This method is based on the analysis of the structure of regular part of solution in the adjacent domains. The operator's boundary conditions, if $x = 0$, are:

$$L_\sigma(\sigma_x) - L_u(u) = \Phi(u_o, \sigma_o) \qquad (4)$$

where L_σ, L_u, Φ are the differential operators with the even derivate of infinite order.

As a result, the regular part of the solution defines a decomposition into row of property functions in each domain. The coefficients of row decomposition are found [1,2].

The conditions (4) give the physical interpretation of the fixing as a generalizable elastic foundation.

In local coordinates, the singular solution $U_s(x,y)$ near to the singularity points can be written as:

$U_s = A_s |z|^\rho \theta(\varphi)$, $Z = x + iay$, $\varphi = \mathrm{arctg}\frac{ay}{x}$, $i = \sqrt{-1}$

where A_s is the coefficient of stress intensity and the parameter is the solution of transcendent equation, which depends on the type of boundary conditions and the local elastic modulus.

In the case when the plate is fixed by two rigid bodies the equation is:

$$tg\rho\frac{\pi}{2} = \sqrt{A/B}, \quad A = a_1/a_2, \quad B = (E_x)_1/(E_x)_2$$

and it can be written as:

$$\beta \sin\pi\rho - \cos\pi\rho = (B/A-1)/(B/A+1), \quad \beta = \left(a\frac{G}{E_x}\right)_2 \cdot \gamma$$

where the plate is fixed by a layer of glue which has a coefficient of elasticity γ, $\sigma_x + \gamma\tau = 0$.

In Figure 3a there is the nomogramm of Koshy type for defining the function ρ of the parameters B and $k = \frac{B}{A}$. The nomogramm is made by method [3]. If we want to define the value of parameter ρ it is necessary to join in a line the given points $B = B_o$, $k = k_o$ on the scales and B, k. On the intersection of line and the scale ρ, we get value ρ_o.

This way it is not difficult to obtain the obvious value of coefficients of intensity.

For example, expression of coefficient of intensity of the plate bending by load Q is given by:

$$As = (d_1 + d_2 + d_3)/(C_1 + C_2 + C_3)$$

$$d_i = B_i/\lambda_i^2, \quad B_i = th(\alpha\lambda_i/a_2)/(A + Bth(\alpha\lambda_i/a_2)\mu_i$$

$$\mu_i = 1 + a_2^2/\alpha\lambda_i^2, \quad tg\lambda_i = \lambda_i, \quad C_i = ZI_i, \quad I_i = \pi/(\lambda_i^2 - \pi^2)$$

$$Z = -24(D + D_o - C - C_o)/\pi^3 \cdot \alpha(1 + a_2^2/\alpha^2)$$

$$D = (4a_2^2 + \alpha^2)^\rho \sin(\pi - \varphi)\rho, \quad D_o = (4a_2^2)^\rho \cdot \sin\frac{\pi\rho}{2} \qquad (5)$$

$$C = \rho(4a_2^2 + \alpha^2)^{\rho-1} \sin\left[(\pi - \varphi)\rho + \varphi\right],$$

$$C_o = \rho(4a_2^2)^{\rho-1} \sin\pi(\rho+1)/2, \quad = arctg\ (2a_2/\ ).$$

The type of fixing is showed in the Figure 2a. The approximate formula (5) is obtained from the exact solution in the case $\alpha \approx h$, $a_2 \ll 1$, when the main members of the expression are taken into account.

The formula is complex for the practical calculations, therefore a circular approximate nomogramm was constructed as follows:

$$A_s = A_s(a_2, \alpha, k) \qquad (6)$$

Preliminary the expression (6) can be approached by the expression in the form

$$A_s = f_{12}(a_2, \alpha)\cdot f_{13}(a_2, k)$$

according to the method of G.S. Hovansky [4]. In Figure 3b there is a working nomogramm constructed by method [3]. If we want to use the nomogramm, it is necessary to find the given points a_o and k_o in the binary fields (a,α) and (a,k) of values $\alpha = \alpha_o$ and also to measure the segment of line between these points and to lay it from the given point on the scale A_s. The left end of segment is the requested value A_o.

It is important to note, that the nomogramms provide a comfortable and efficient method for obtaining the solution of the correspongind inverse problems, in defining the optimal parameters.

REFERENCES

[1] Vasiliev, V.V., Lurie, S.A., Mechanica polimerov, 1972., 4.
[2] Vasiliev, V.V., Lurie, S.A., DAN CCCP., 1977.,233., No. 5.
[3] Borisov, S.N., VC AN CCCP, 1982.
[4] Hovansky, G.S., VC AN CCCP, 1985.

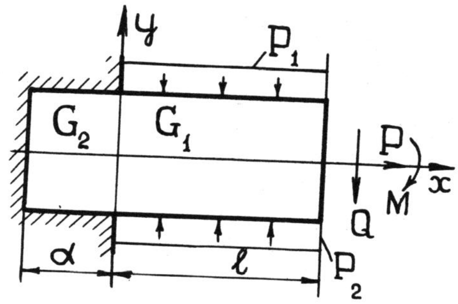

Figure 1

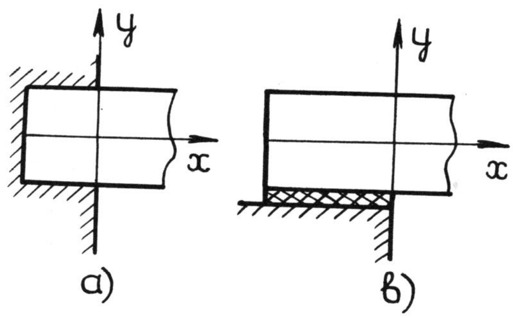

Figure 2

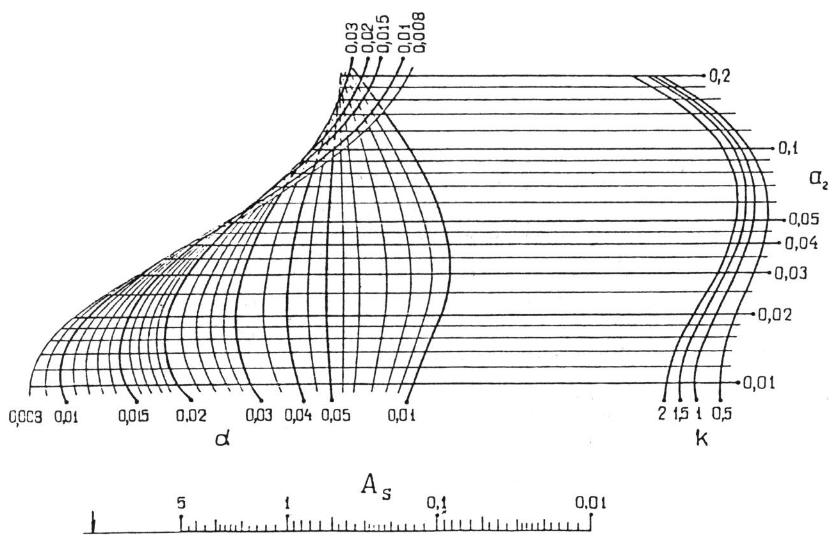

Figure 3a

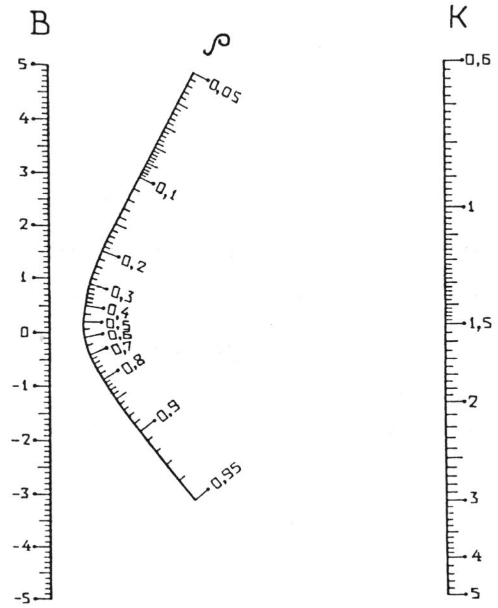

Figure 3b